

# Area dependency of estimation errors in geostrophic velocity

Kaoru Ichikawa and Shiro Imawaki

Research Institute for Applied Mechanics, Kyushu University, JAPAN

(e-mail: [ichikawa@riam.kyushu-u.ac.jp](mailto:ichikawa@riam.kyushu-u.ac.jp))

## 1. Introduction

Satellite altimetry provides good observations of not only sea surface height but also geostrophic velocity component normal to the satellite tracks. It is, however, the surface velocity field, or horizontal distribution of velocity vectors, that many oceanographic applications requires. In the present study, estimation error of geostrophic velocity field is to be focused: Area dependency of the error is discussed with respect to spatial/temporal sampling patterns of altimeters by comparing with advection velocity of surface drifter buoys.

## 2. Methods

### 2.1 Geostrophic velocity field from altimetry data

Geostrophic velocity field is calculated from composite sea surface dynamic topography (SSDT) which is the combination of the climatological mean SSDT and the temporal fluctuation SSDT's obtained from altimetry data. The former is calculated from 0.25-degree grided density filed (Boyer and Levitus, 1997) referring to 1000 db surface. The latter is determined from both T/P and ERS-1/2 altimetry data (AVISO, 1996) by a simple optimal interpolation (OI).

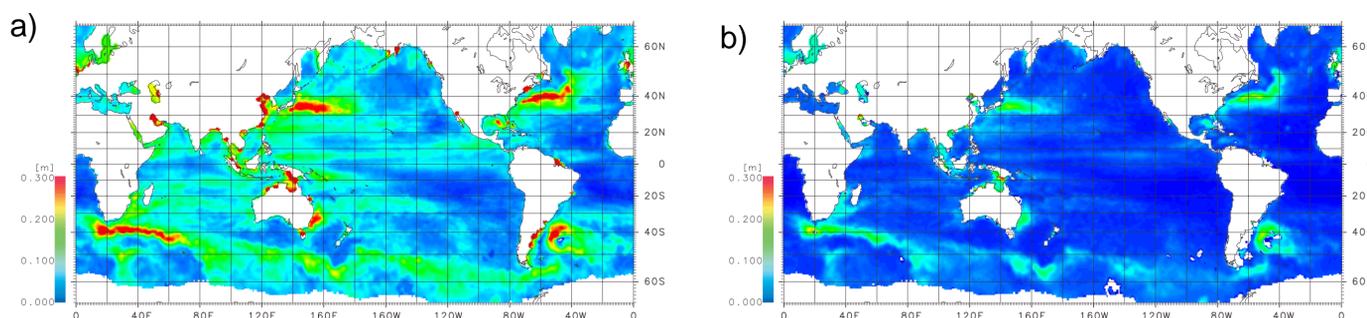
Covariance functions for signal and noise in the OI are as follows: For the signal covariance  $W$ , we use the Gaussian function with location dependent covariance magnitude  $S_t$ ; namely,

$$W(\mathbf{R}_1, \mathbf{R}_2) = S_t(\mathbf{r}_1)S_t(\mathbf{r}_2) \exp\left(-\left(\frac{\Delta x}{L_x}\right)^2 - \left(\frac{\Delta y}{L_y}\right)^2 - \left(\frac{\Delta t}{L_t}\right)^2\right) \quad \text{where } \mathbf{R} = (\mathbf{r}, t) = (x, y, t)$$

The covariance function for noise consists of 3 terms as

$$\psi(\mathbf{R}_1, \mathbf{R}_2) = \sigma_{alt}^2 \delta(\mathbf{R}_1 - \mathbf{R}_2) + E_0(\mathbf{r}_1)E_0(\mathbf{r}_2) \exp\left(-\left(\frac{\Delta \mathbf{r}}{a_R(y)/2}\right)^2 - \left(\frac{\Delta t}{0.1day}\right)^4\right) + E'_t(\mathbf{r}_1)E'_t(\mathbf{r}_2) \exp\left(-\left(\frac{\Delta \mathbf{r}}{a_R(y)}\right)^2 - \left(\frac{\Delta t}{0.1L_t}\right)^4\right)$$

where  $a_R$  is the internal Rossby radius. The first delta-function comes from altimeter's noise and both the second and the third comes from SSDT variations with frequencies higher than  $L_t$ . All those  $S_t$ ,  $E_0$ ,  $E'_t$  and  $\sigma_{alt}$  are estimated from temporally lagged covariance of along-track TOPEX, POSEIDON ERS-1 and ERS-2 altimeter data; Figure 1 shows spatial distributions of  $S_t$  for time scales longer than 10 days and  $E'_t$  for 35 days.



**Figure 1:** The RMS variability of SSDT with time scales longer than 10 days (a), and that of SSDT for time scales 10-35 days (b). The latter is included as "signal" in the former, but it will be treated as "noise" for estimation of SSDT variations with time scales longer than 35 days.

For the sets of parameters ( $L_x$ ,  $L_y$ ,  $L_t$ ), we choose values based on typical sampling patterns of T/P or ERS-1/2 (Table 1) in order to keep OI estimation errors homogeneous.

**Table 1:** Two sets of parameters used in OI.

T/P pattern	$L_x=445\cos(y)\text{km}$ , $L_y=360\cos(y)\text{ km}$ , $L_t=5\text{ days}$ along-track 20 points average
ERS pattern	$L_x=91\cos(y)\text{km}$ , $L_y=135\cos(y)\text{ km}$ , $L_t=15\text{ days}$ along-track 10 points average

From pressure gradient determined from the SSDT, geostrophic velocity is calculated by dividing by Coriolis parameter  $f$ , except in the equatorial region (within 2 degrees) where wind stress terms becomes comparable to or greater than the Coriolis terms in the equations of motion. In this region, we need to remove pressure gradients induced by wind stress, but they can be approximated by the values at the equator where the pressure gradient is balanced with wind stress in a steady state. Namely, we use

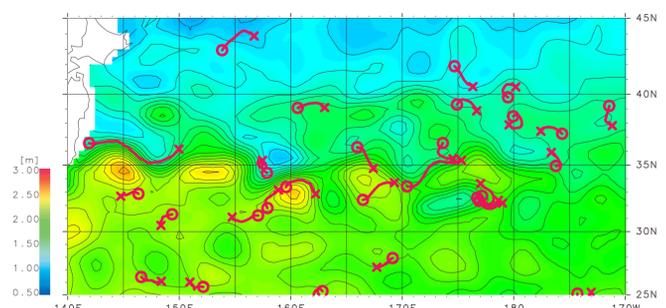
$$u = -\frac{g}{f} \left\{ \frac{\partial \eta}{\partial y} - \frac{\partial \eta_0}{\partial y} \exp\left(-\frac{y^4}{L_u^4}\right) \right\} \quad v = \frac{g}{f} \left\{ \frac{\partial \eta}{\partial x} - \frac{\partial \eta_0}{\partial x} \exp\left(-\frac{y^4}{L_v^4}\right) \right\}$$

where  $L_u$  and  $L_v$  indicates the area where the Coriolis force become dominant in the equations of motion; we set  $L_u=0.2\text{ deg.}$  and  $L_v=5\text{ deg.}$  In addition, velocity is calculated at the equator by dividing 2nd order derivatives of SSDT by beta.

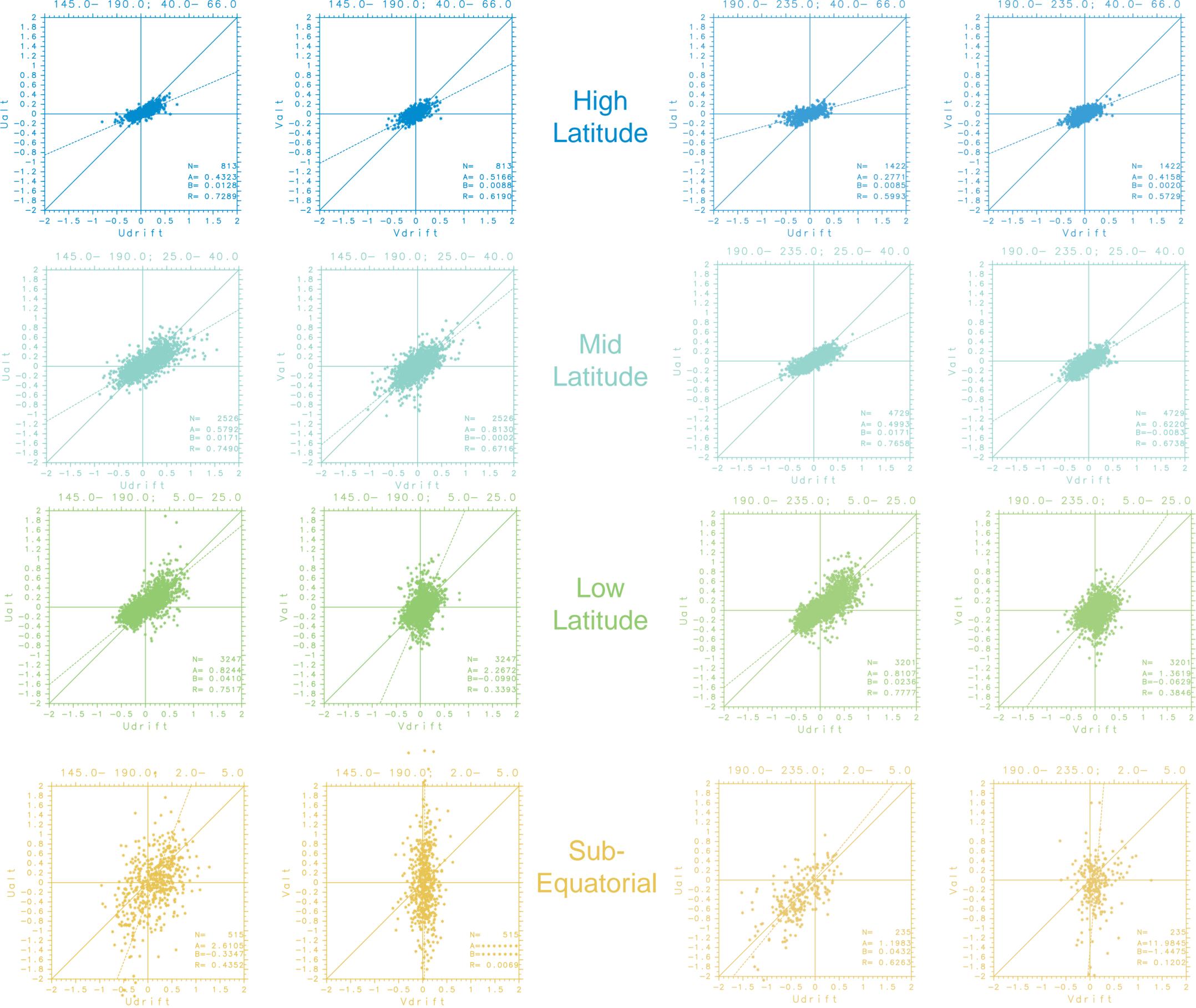
### 2.2 Drifter velocity from tracks of drifting buoys

The drifter data are provided from Marine Environmental Data Service, Canada, for 5 years starting from Oct. 1992. (figure 2) For each drifters, daily advection velocity is first calculated from their tracks, and then averaged over 3 inertia days to reduce effects of ageostrophic velocity such as wind drifts and inertia oscillations.

Meanwhile, the altimetry-derived geostrophic velocity is extracted at the position (both in space and time) of the drifter observations. In the present analysis, these two velocities are compared only when the radius of curvature of the drifter tracks during given 3 inertiadays is greater than 75km (ERS pattern) or 300km (T/P pattern) in order to keep consistency with OI.



**Figure 2:** Tracks of drifter buoys (red lines) in mid October, 1992, superimposed on the composite SSDT with ERS sampling pattern. In the figure, tracks are plotted only for drifters moved more than 50km during 10 days. The contour interval of SSDT is 10cm.



**Figure 3:** Comparisons between drifter velocity and geostrophic velocity determined from altimetry SSDT with ERS-pattern smoothing. The comparisons are made separately for zonal (u) and meridional (v) components. In each panel, the regression line is shown by a dotted line. In addition, the number of comparisons (N), slope (A) and intersect (B) of the regression line, and correlation coefficient (R) are indicated in the panels. For the convenience of interpretation, all panel are painted by colors shown in the map (right).

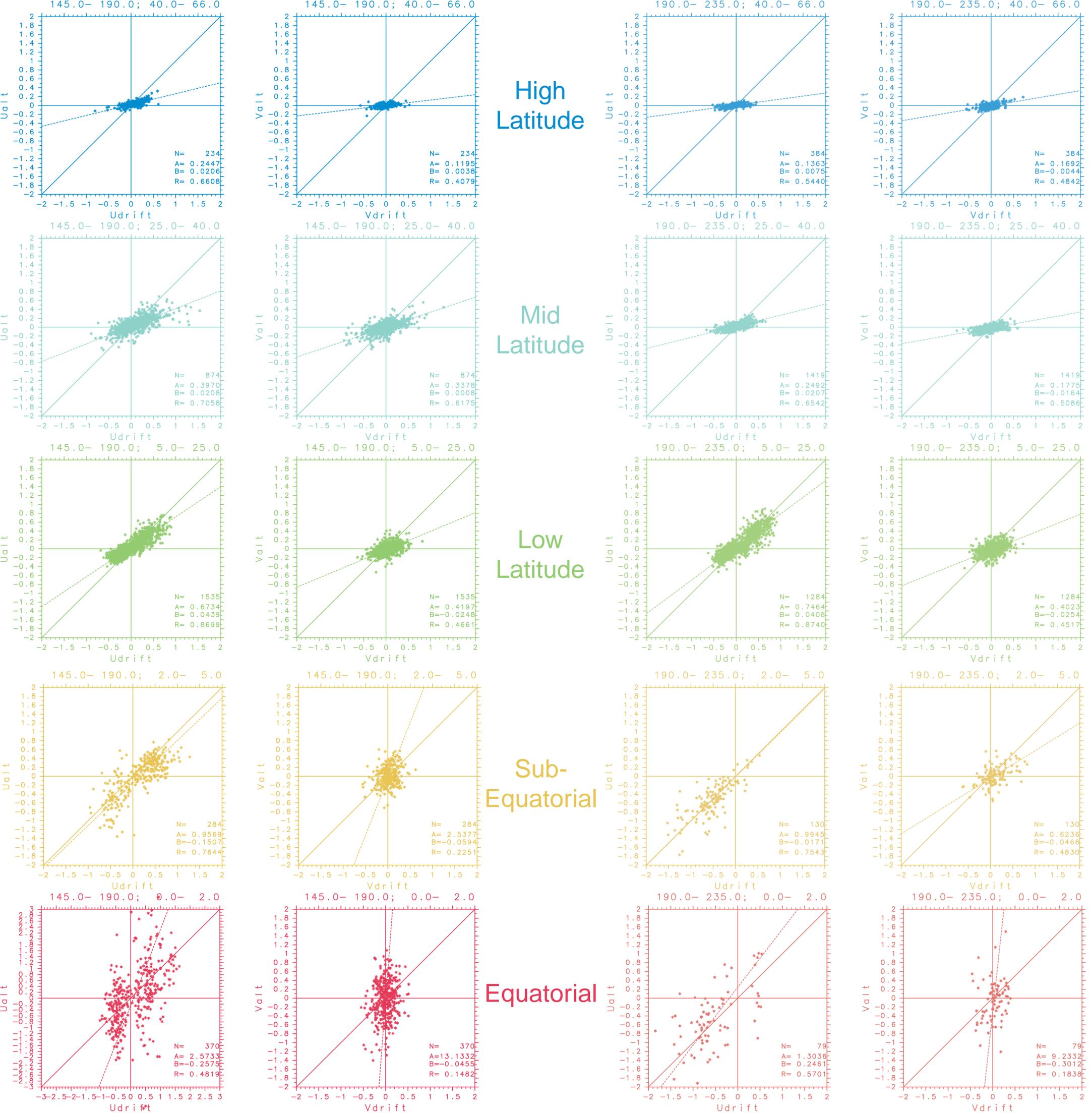
### 3. Results

Comparisons are made in the North Pacific. In order to study the area dependency of errors in geostrophic velocity, we divide the area into several latitude bands, and also eastern and western areas. The results are shown in Figure 3 (for SSDT with ERS pattern) and Figure 4 (T/P pattern).

Obvious tendency is found in both figures that slopes of regression lines become small as the latitude increases, especially in T/P sampling pattern (Figure 4). This would be explained that smoothing scales in the present OI is much larger than typical spatial scales at higher latitudes, so that the amplitudes of SSDT variations are reduced. Note that correlation coefficients are statistically significant even for those areas, which suggests that the phase of SSDT variations are estimated well. Also note that ERS pattern (Figure 3) becomes worse near the equator where fast and large-scale phenomena are known to be dominant.

At lower latitudes, zonal component shows better results than meridional one, while the discrepancy is not clear at higher latitudes. Wind drift which is not explicitly removed in the drifter velocity is a candidate of the reason since it would be stronger at lower latitudes.

Area dependency between the eastern and western regions are not significant in spite of distinct difference in eddy kinetic energy. Some part of the reasons would be because location dependency of the variance is already accounted in the present OI.



**Figure 4:** Same as Figure 3, but for altimetry SSdT with T/P pattern.

## 4. Summary

Sea surface dynamic topography (SSdT) is estimated from T/P and ERS-1/2 data by a simple optimal interpolation (OI); temporal and spatial smoothing scales are set for T/P or ERS sampling patterns. Then, geostrophic velocity calculated from the SSdT is compared to drifter velocity.

Altimetry-derived velocity with ERS sampling pattern SSdT performs best at low to mid latitudes (say, 5-40N), but results in very bad near the equator. On the contrary, that with T/P pattern SSdT works good at lower latitude (say, 2-25N), but becomes significantly underestimated at higher latitudes, although its correlation to the drifter velocity is still high. The latter case is due to larger sampling scales of T/P with respect to the typical spatial scales at high latitudes, so that velocity field at higher latitudes may not be quantitatively estimated from T/P altimetry data alone with simple OI. Some other sophisticated interpolation methods such as data assimilation techniques are especially necessary for those area.