

# The M4 tide from a numerical model constrained by tidal estimates from TOPEX/Poseidon on the European Shelf.

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## Abstract

We have estimated the M4 tide from TOPEX/Poseidon (TP) data in the northeast Atlantic Ocean and on the European continental shelf, and assimilated these estimates into a nonlinear, finite difference, barotropic tidal model. Versions of the model have been run using the bathymetry datasets ETOPOS [Sloss, 1988], Smith and Sandwell [1997], and Terrain Base [National Geophysical Data Center, 1994]. Comparison to 57 tide gauges along coasts and on the shelf show a significant reduction in RMS from the results of Flather [1976] from 5.46 cm to 3.05 cm. The different bathymetry databases do make a large difference in the results. The data contained in ETOPOS and Terrain Base are similar, but ETOPOS has some problems with depths less than 10 meters. The Smith and Sandwell database [1997] does not match coastlines well and thus tidal amplifications at coasts are not present.

These results are encouraging but more validation needs to be done to additional tide gauges and cross-over differences to unassimilated TP and ERS-1 and ERS-2 data. From here, the method can be applied to other nonlinear tides in this and other regions that are large enough to be extracted from TP signal.

## Introduction

During the last 10 years, there has been a dramatic increase in the accuracy of global ocean tide models. This has been possible due to the availability of increased computing power for improvements in barotropic ocean tide models and the availability of T/P altimetry sea surface height data. Most of the improvement has come in waters deeper than 1000 meters. In the deep ocean, the current tidal models agree to within 2-3 cm in the deep waters [Shum et al., 1998]. In the shallow waters, there is much less agreement. Some of the reasons for this are due to the increase in model resolution needed to resolve the short tidal wavelengths, errors in the bathymetry, parameterization of bottom friction, and the existence of nonlinear tides which current tidal models ignore (See Andersen, this conference, for some results on global empirical estimates of nonlinear tides).

Tierney et al. [1998, 1999] has shown that accurate TP tidal estimates of the linear tides are possible in shallow waters, and these estimates can improve numerical models in shallow water. Andersen [1999] showed that although the wavelengths of nonlinear tides are very short on the continental shelf, estimation of the larger nonlinear tides (M4, MS4, M6) is possible from TP data. Gridded estimates were created by interpolating along-track and crossover estimates with a sophisticated statistical function.

The M4 tide is generally the largest nonlinear tide. It is induced through the advection term by the M2 current interacting with itself. The period of M4 (6.21 hours) is half that of M2. There have been several investigations of the M4 tide with numerical models the European Shelf [Andersen [1999] has a good review of those works]. The M4 tide is on average over 8 cm on the European Shelf [Andersen, 1999], a significant amount that must be modeled accurately if the accuracy of global models in deep water is to be extended into shallow waters.

Here, we attempt to model the M4 tide on the European Shelf with a nonlinear, finite difference, barotropic tidal model. TP estimates of M2 and M4 are assimilated into the model. The model resolution is 1/8 degree (approximately 9 km). The model domain extends from 32 W to 12 E, and 35 N to 64 N. The domain is significantly larger than the European Shelf, and was selected so that the M4 tide along the model boundary is small (near zero). The model will be validated by comparisons to another M4 model [Flather, 1976, 1981] and tide gauges.

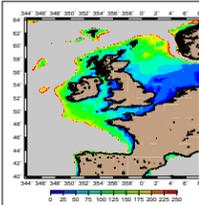


Figure 1: The bathymetry on and around the European Shelf. This data comes from the TBASE dataset, but at this scale ETOPOS and SS look similar. Units are in meters.

Since bathymetry plays a significant role in the shape of the tides in shallow waters, we will run the model with three different bathymetry databases. Two of the sets, ETOPOS [Sloss, 1988] and Terrain Base (TBASE) [National Geophysical Data Center, 1994], have been created from ship depth soundings. TBASE is based on ETOPOS, but several regions have been improved with more accurate data, including the European Shelf. The third set Smith and Sandwell (SS) [1997] was created using ship depth soundings as well as inverting gravity anomalies calculated from satellite altimetry.

Figure 1 shows the bathymetry on and around the European Shelf. The shelf is less than 200 m deep, and in the southern part of the North Sea, the depth is less than 100 m. At this scale, all three bathymetry databases look similar.

## Altimetry Processing

In this work, we have followed a procedure similar to Andersen [1999] to estimate the M4 tide along the ground track of T/P. Data from TP cycle 8 through 223 are used, excluding the Poseidon data. The elastic tide (ocean + load) tide is estimated from TP at each point along the ground track within the model domain. The linear tidal components (diurnal, semi-diurnal, and long period bands) are described by the response method [Munk and Cartwright, 1960] using 22 orthotides. The nonlinear tide, M4, is an additional harmonic component added to the observation equation.

The linear tide is again estimated using the response method, but in bins sized at the resolution of the numerical model [Tierney et al., 1999] to obtain the best possible estimates. The amplitude and phase of M2 are inferred from the estimated orthotides. Linear estimates are computed in all grid cells that contain data.

The along-track estimates of M4 are then interpolated to the locations of the gridded linear estimates in a similar manner to Andersen [1999]. The main difference is that estimates at crossover locations are not considered in these results. The load tide from Desai and Wahr [1995] is used to remove the load tide from the M2 estimates. We assume that the loading due to the M4 tide is negligible.

Figure 2 shows the distribution and magnitude of the M4 estimates that are to be assimilated into the model. There are very large estimates south of the Celtic Sea, in the North Sea along the French, Belgium, and Netherlands coasts, and in the Bay of Biscay. In some areas, such as the Dover Straits and the Irish Sea, there are little or no TP observations with which to compute estimates.

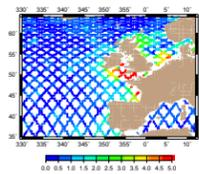


Figure 2: Estimates of M4 computed from TP data. Units are in centimeters.

## Numerical Modeling

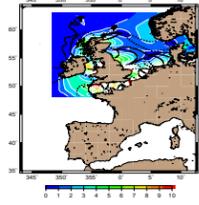


Figure 3: Amplitude/phase map of the M4 tide using the TBASE bathymetry. Units are in cm.

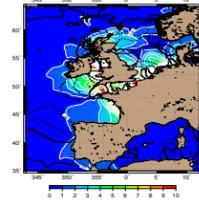


Figure 4: Amplitude/phase map of the M4 tide using the ETOPOS bathymetry. Units are in cm.

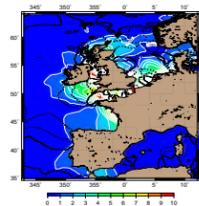


Figure 5: Amplitude/phase map of the M4 tide using the SS bathymetry. Units are in cm.

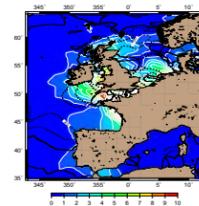


Figure 6: Amplitude/phase map of the M4 tide using the Flather bathymetry. Units are in cm.

The empirical M2 and M4 results are assimilated into a nonlinear, finite difference, barotropic model by nudging [Tierney et al., 1999]. The assimilation weight varies between 0.30 and 0.50, depending on the fit of the tidal estimate to the original TP data. Although these weights give good results, they are preliminary and are not necessarily optimal.

Figure 3 shows the M4 results from Flather [1976, 1981]. This model has a resolution of approximately 12 km. Figures 4, 5, and 6 are the model results using the TBASE, ETOPOS, and SS bathymetry sets respectively. At first glance these three results are similar, and show some of the same structure in the Flather results. All model results show large tides in the English Channel, along the French, Belgium, and Netherlands coasts in the North Sea. Differences exist on and near coasts. The amplitudes in the English Channel differ. Flather has less amplification along the southern coast of England. The TBASE and ETOPOS results have more amplification in the Irish Sea than SS results or Flather. The SS results don't show any amplification along coasts in the North Sea.

Most of the differences are along the coasts. There is one significant feature away from the coast that is present in the three different models, but not present in the Flather results. There is a large amplitude (up to 5 cm) extending into the North Sea starting at about 5 E, 55 N in figures 4 through 6. Later, tide gauges will be used to determine if this feature is real.

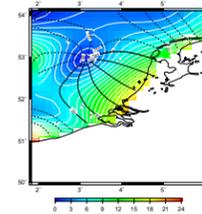


Figure 7: Amplitude/phase map of M4 from Flather centered on 4E, 52 N. Units are in cm.

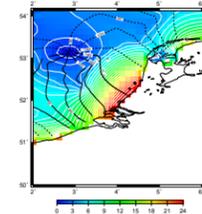


Figure 8: Amplitude/phase map of M4 using TBASE bathymetry centered on 4E, 52 N. Units are in cm.

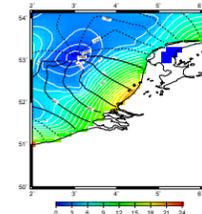


Figure 9: Amplitude/phase map of M4 using ETOPOS bathymetry centered on 4E, 52 N. Units are in cm.

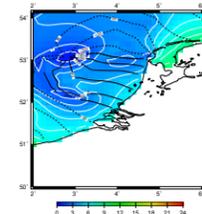


Figure 10: Amplitude/phase map of M4 using SS bathymetry centered on 4E, 52 N. Units are in cm.

Most of the differences are along the coasts. There is one significant feature away from the coast that is present in the three different models, but not present in the Flather results. There is a large amplitude (up to 5 cm) extending into the North Sea starting at about 5 E, 55 N in figures 4 through 6.

Along the coast of Belgium and the Netherlands there seems to be differing amounts of amplification between the three models presented here. Figure 7 shows the results for Flather centered at 4E, 52 N. Figures 8 through 10 show the M4 tide using the TBASE, ETOPOS, and SS bathymetry datasets respectively centered at the same location as in Figure 7. In each case, the maximum amplitude along the coast varies. Flather has a maximum of about 18 cm. The maximum amplitudes for TBASE, ETOPOS, and SS are 25, 19, and 6 cm respectively. The SS model is not reproducing the amplification here, as well as other areas shown in Figure 6. The indicate locations of five tide gauges. The amplitude of the M4 tide at these gauges ranges between 24 and 25 cm. The RMS of the vector amplitude difference between these tide gauges and the Flather model, and the TBASE, ETOPOS, and SS results are 12.24, 3.97, 8.80, and 13.28 cm respectively. The TBASE based model performs better than Flather and the other cases here since it is able to better reproduce the tidal amplification along the coast.

Figures 11 through 13 may explain why the model using TBASE does best in this case. Figure 11 shows the bathymetry of this region from TBASE. Along the whole coast there is a very defined decrease in depth approaching the coast. In Figure 12, ETOPOS shows the decrease expected in some locations, but it is not as defined as TBASE, and at 4.75E, 52.75N, it appears as if ETOPOS is showing a depth of 30 m at the coast. Figure 13 shows the bathymetry predicted by SS. There are large-scale differences away from the coast as compared to ETOPOS and TBASE. Also, SS does not match the coastline at all. The depth of the water where these gauges are is shown to be 30-40 m by SS.

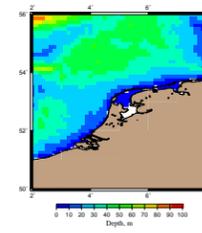


Figure 11: Bathymetry from the TBASE database. Depths are in meters.

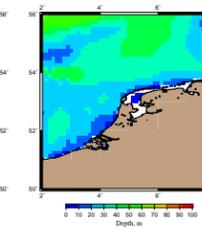


Figure 12: Bathymetry from the ETOPOS database. Depths are in meters.

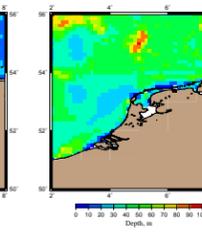


Figure 13: Bathymetry from the SS database. Depths are in meters.

## Error Analysis

We compared the Flather results to 57 tide gauges on the Shelf. These gauges came from the data collected by the Marine Information Centre (MARIS) in Rijswijk, The Netherlands, and the International Association for Physical Sciences of the Oceans (IAPSO) [Smithson, 1992]. Figure 14 shows the locations of the gauges on the shelf. The red circles indicate gauges that are along the coast, and the blue circles indicate gauges away from the coast.

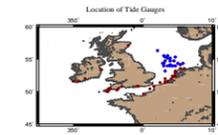


Figure 14: Location of the 57 tide gauges used for model validation.

Table 1 shows the RMS of the vector amplitude difference between the models and the tide gauges. Bathymetry seems to make a significant difference in the results. ETOPOS and TBASE improve the results from Flather. The results from SS are worse than Flather. This is because the SS bathymetry does not provide the tidal amplification expected along many of the coasts (See Figures 8-10).

Table 1: RMS of the vector amplitude difference between the model results and 57 tide gauges on the European shelf for M4. RMS is in cm.

Model	RMS
Flather	5.46
SS	7.26
ETOPOS	4.69
TBASE	3.05

In Figures 3-6, a feature in the North Sea away from the coast was identified in the models presented here that is not in the Flather model. The tide gauges marked in blue in Figure 14 sample this feature. Table 2 shows the RMS of the vector amplitude difference with these 23 gauges. It appears that the feature is real, as all the models perform significantly better than the Flather results. TBASE is doing worse than SS or ETOPOS, although one can question if the differences are significant. What this does show is that the differences in Table 1 are probably caused by differences of each of the models along the coast, and that the models perform similarly away from the coasts.

Table 2: RMS of the vector amplitude difference between the model results and 23 tide gauges for M4 on the European shelf away from the coasts. RMS is in cm.

Model	RMS
Flather	2.93
SS	1.26
ETOPOS	1.21
TBASE	1.28

Figure 15 is the vector amplitude difference between TBASE and ETOPOS. Note that away from the coasts, the models are performing similarly. It is only along the coasts where differences are apparent. The difference between TBASE and SS is not shown, but the results are the same (differences only along coasts).

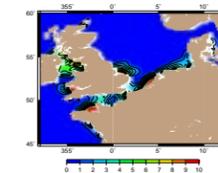


Figure 15: Vector amplitude difference between TBASE and ETOPOS results. Units are in cm.

## Conclusion

Andersen [1999] demonstrated that it is possible to estimate the M4 tide from TP data. We have estimated the M4 tide in a similar manner and assimilated these results into a finite difference, nonlinear, barotropic tidal model. Results show improvement over the results from Flather. Bathymetry seems to play a very important role in tidal amplification along the coasts. Current datasets give drastically different results along coasts. Tide gauges indicate that using the TBASE bathymetry gives the best estimates of M4. This does not mean that TBASE is better globally, but gives the best results for this investigation. The gridded estimates of M4 can be improved by including estimates at crossover locations. More validation is needed in this region before we can investigate other regions and other nonlinear constituents.

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