SSB estimation using satellite and *in situ* data

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ABSTRACT

The largest single source of error for mean sea level estimation is the electromagnetic (EM) bias. Most models that are widely used are derived using a linear combination of wind speed and significant wave height. These models still leave much room for improvement. In an effort to improve the EM bias estimation a nonparametric technique has been applied to the satellite and *in situ* tower data. Using this technique on both types of data sets yield results that are very similar. Additionally, another parameter, the wave slope, improves the EM bias estimates. The best case of the nonparametric technique used with the wave slope provides a standard deviation of error of under .21 cm.

NONPARAMETRIC ESTIMATION

Nonparametric regression (NPR) is a method to statistically smooth a data set such that a valid estimate for one variable is available over a chosen spacing of different variables. For the derivation in two dimensions, **U** and **H** are used to estimate β.

$(\mathbf{U}_1,\mathbf{H}_1,\beta_1)$, $(\mathbf{U}_2,\mathbf{H}_2,\beta_2)$, ... , $(\mathbf{U}_n,\mathbf{H}_n,\beta_n)$ form an independent, indentically distributed sample from a population $(\mathbf{U},\mathbf{H},\beta)$.

It is desired to estimate the regression function

$$\beta(\mathbf{x}) = E[\beta|\mathbf{x} = (\mathbf{U},\mathbf{H})]$$

This equation is expanded about the point (U_o,H_o) using a Taylor's series expansion

$$\beta(\mathbf{x}) = a_0 + a_1(U-U_o) + a_2(H-H_o)$$

where $\mathbf{a}_1$ and $\mathbf{a}_2$ are partial derivatives of $\beta(x_o)$ with respect to **U** and **H** respectively. The NPR estimate is found by solving the least squares problem

$$\min_{a0,a1,a2}\Sigma\{\beta_i - a_0 - a_1(U-U_o) - a_2(H-H_o)\}^2 K_h(X_i-x_o)$$

where $\mathbf{K}_h$, is the kernel function.

The nonparametric regression chosen for this paper is the Local Linear Regression (LLR). Let

$$\mathbf{A} = (a_0\ a_1\ a_2)^T,$$

$$X = \begin{bmatrix} 1 & U-U_1 & H-H_1 \\ 1 & U-U_2 & H-H_2 \\ & \cdots & \\ 1 & U-U_n & H-H_n \end{bmatrix},$$

and

$$\mathbf{W} = \mathrm{diag}\{K_h(x-X_n)\}$$

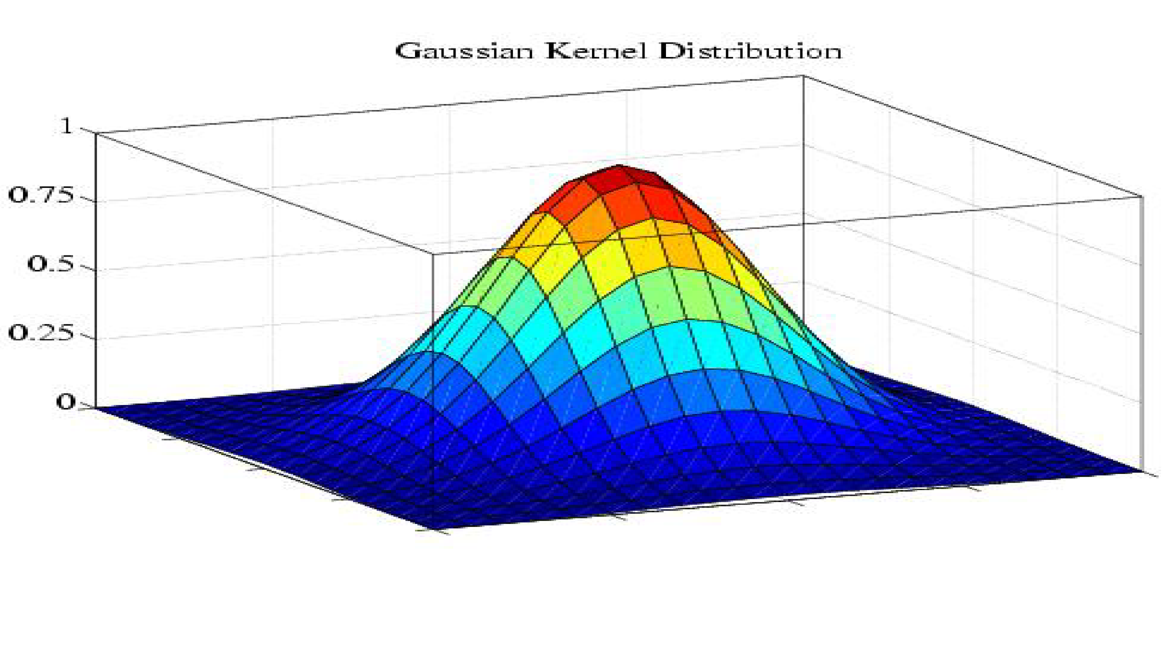
The minimum least squares solution for the LLR is then

$$\mathbf{A} = (\mathbf{X}^T\mathbf{W}\mathbf{X})^{-1}\mathbf{X}^T\mathbf{W}\beta T$$

The purpose of the kernel is to smooth the EM bias estimation over the **(U,H)** plane. The two dimensional Gaussian kernel chosen for this study is of the form

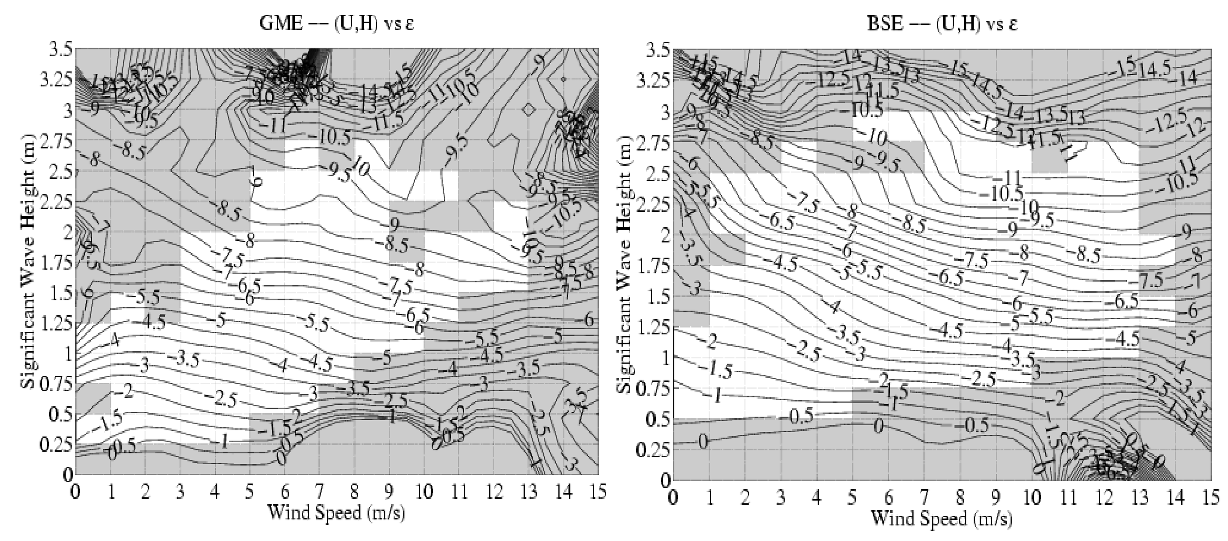
$$K_h(x-X_i) = C(\mathit{exp}[-(U-U_i)^2/2h_U^2-(H-H_i)^2/2h_H^2])$$

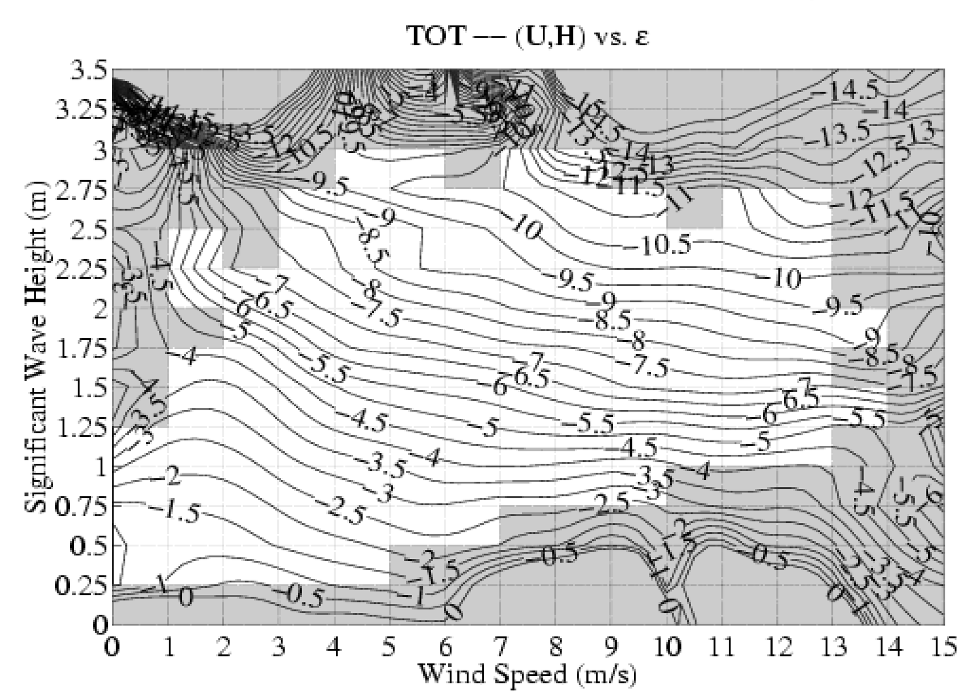
where **C** is a constant to normalize the kernel function.

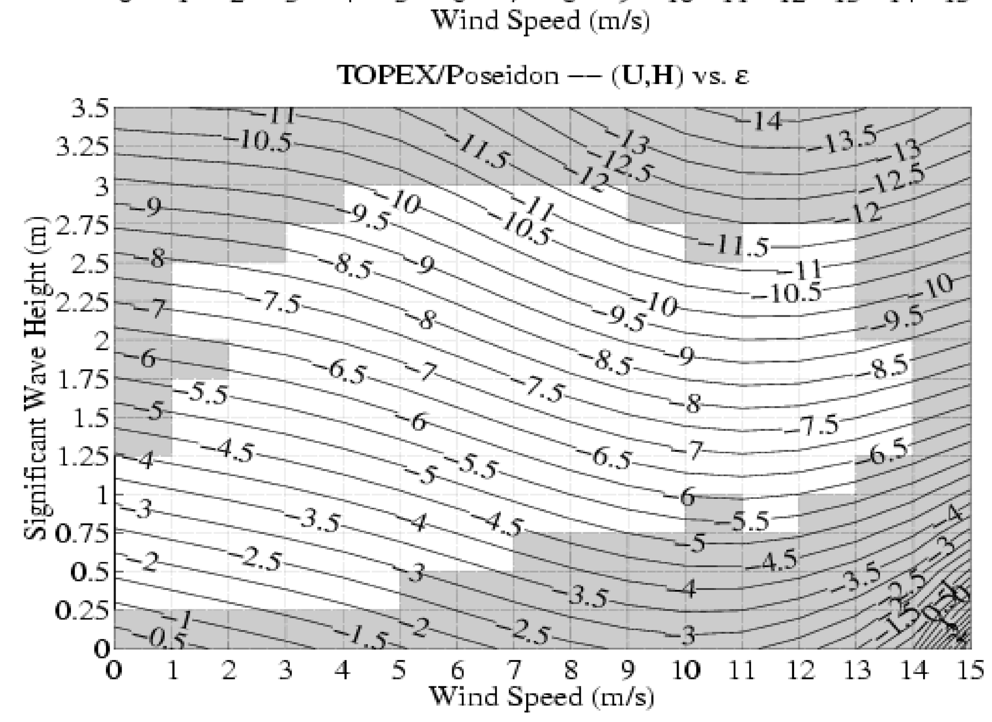


SATELLITE vs. *IN SITU* DATA

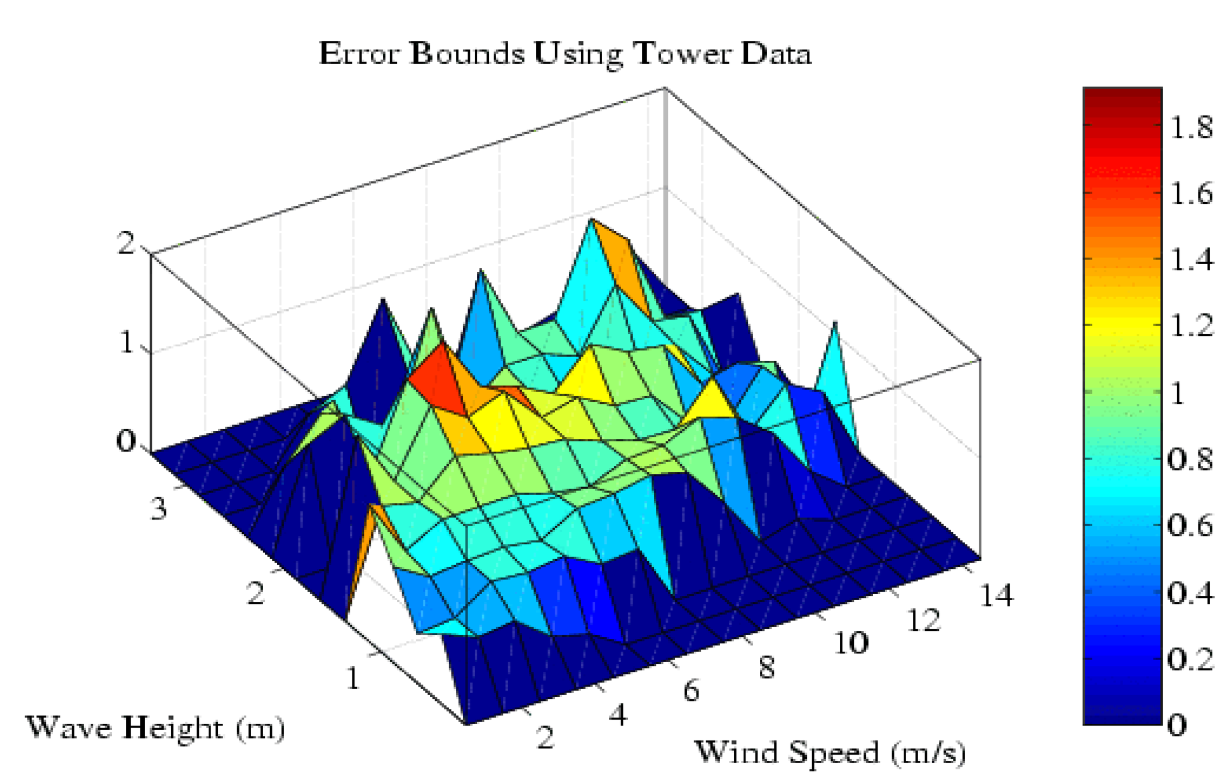
Non-parametric regression was applied to data from two tower experiments, the Gulf of Mexico Experiment (GME) and the Bass Strait, Australia Experiment (BSE), as well as the combined data set, TOT. Additionally, non-parametric analysis was applied to data from the TOPEX/Poseidon satellite. Using the typical parameters of wind speed, U, and significant wave height, H, for the EM bias estimation yielded the following models.







As the plots show, the in situ data and satellite data create models that are very similar. The general forms of the graphs show strong correlations, as well as the actual values presented. It can also be seen that the plots from the individual experiments show regional variations. This regional variation will cause models from global data sets to be in error proportional to the regional differences.



The in situ data provides a means to put "error bars" on the individual and combined data sets. The standard deviation of the bias for the data from each of the grid spaces in the previous plots is shown above. It can be seen that the typical values for the "error bars" tends to be close to 1 cm, with very few being larger than 1.5 cm.

WAVE SLOPE PARAMETER

EM bias models using the wind speed and significant wave height leave undesirably large amounts of error. In an effort to improve the bais estimation, the correlation of the wave slope parameter was calculated from the in situ data sets.

To calculate the wave slope, first the deep water dispersion relation is used to estimate the wave number, k,

$$k = \frac{(2\pi f)^2}{g}$$

where $g = 9.8\ \mathrm{m/s^2}$ is the gravitational constant. Then the wave displacement spectrum, is obtained by computing the power spectral density, $\Phi(f)$, of the wave displacement vector. Using the wave number and $\Phi(f)$ the slope spectrum is

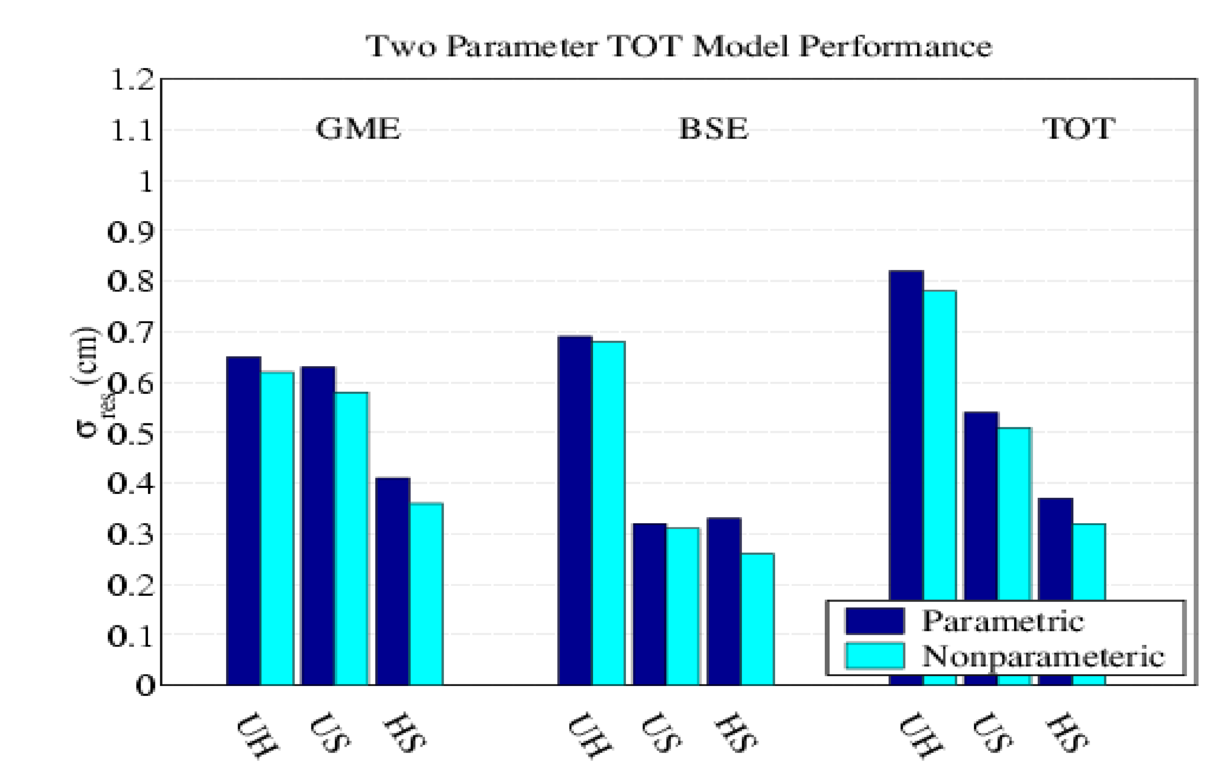
$$\Psi(f_i) = \frac{(2\pi f_i)^4}{g^2}\Phi(f_i)$$

The rms slope is then calculated using a discrete method,

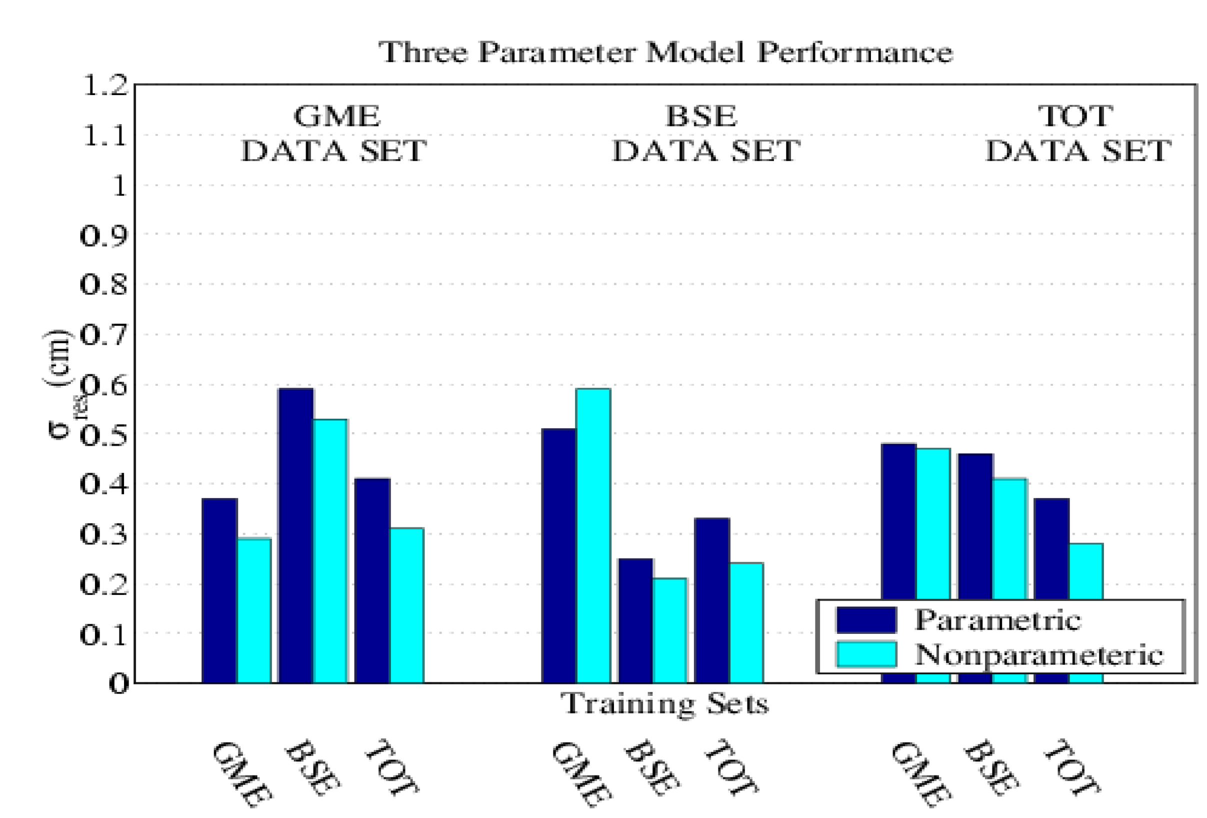
$$S = [(f_s/N_{FFT})\Sigma_i\{[(2\pi f_i)^4/g^2]\Phi(f_i)\}]^2$$
$$= [(f_s/N_{FFT})\Sigma_i[\Psi(f_i)]]^2$$

where N_{FFT} corresponds to the number of points in the FFT and f_s is the sample frequency.

Using the combined data set, the ability of the wave slope to create accurate estimates of the EM bias is demonstrated in the following graphic. In similar manner as the plots of the wind speed and significant wave height plot against the contours of the bias, the combination of any two of the three parameters, wind speed, U, significant wave height, H, and wave slope, S.



The improvement is more apparent when all three parameters are used to estimate the EM bias.



SUMMARY

Using **S** with the nonparametric estimation technique provides excellent estimates of the normalized EM bias. These estimates are better than estimates using just **U** and **H**, and also show promise for improving the estimates over traditional parametric models.