Assimilation of sea level using a new assimilation method for diagnostic variables

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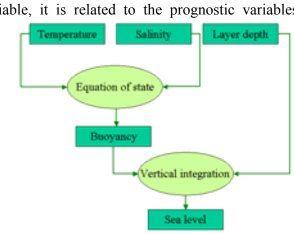
INTRODUCTION

In an ocean general circulation model using the rigid lid condition, the sea level is a diagnostic variable. That is, while it may be retrieved from the model, the evolution of the system is not modified by any correction of the values of the sea level. In this context, assimilation of sea level observations requires a method able to modify the state of the variables affecting the time evolution of the system. The methodology proposed here is derived from the continuous form of the Kalman filter. In the case of maximum simplification of the error covariance matrices, an equation combining the concepts of nudging and the adjoint of the observation operator arises. The latter allows a natural projection from a diagnostic variable, e.g. sea level, into corrections of dynamically active variables, e.g. temperature and salinity. **The novelty of the methodology proposed here is that model corrections to temperature and salinity are not parameterized by any empirical/statistical relationship between diagnostic and prognostic variables, rather the methodology exploits the physical relationships of the model itself to vertically extrapolate the sea level information**. The ability and limitations of the method to assimilate sea level observations into an ocean circulation model are examined here.

MODEL

The model used here is the a reduced gravity, primitive equation model, twenty layers, and explicit evolution of temperature and salinity. Each time step, sea level is calculated as shown below. Thus, even if sea level is a diagnostic variable, it is related to the prognostic variables via the state equation.



THE DATA ASSIMILATION

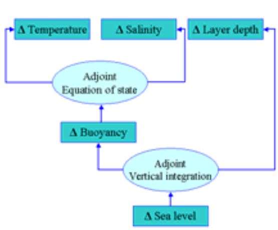
The equations of the continuous Kalman filter are

$$\mathbf{x' = Ax + PH^T R^{-1} (y - Hx)}$$
$$\mathbf{P' = AP + PA^T + Q - PHT R^{-1} HP}$$

Where **x** is the vector of prognostic variables**, A** is a linear transition matrix**, P** is the expected error of the state of the system**, Q** is the expected error of the transition matrix, **R** is the expected error of the observations, and **H** is the observation operator. The prime indicates time derivation. The method investigated here is based on the maximum simplification of these equations. Setting aside the time evolution of **P**, and supposing that only a rough estimate of the order of magnitude of the errors is known, $\mathbf{P} = p^2\, \mathbf{I}$ and $\mathbf{R} = r^2\, \mathbf{I}$, the time evolution of the system is given by

$$\mathbf{x' = Ax + \mu H^T (y - Hx)} \quad \text{with} \quad \mu = p^2 / \tau r^2$$

where τ is a relaxation time scale. This equation may be used to combine the adjoint of the observation operator with the nudging technique. In the approach used here, the adjoint of the observation operator is used to project the information about the misfit in sea level to corrections in temperature, salinity and layer depths:



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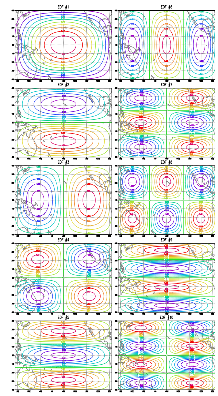
The method is tested using twin experiments. A **reference** simulation of the model sets the ocean to be reconstructed. Then, two simulations start from false initial conditions (using the same forcing). The **assimilation** of sea level is applied to one but not to the other (**free**). Tropical dynamics is strongly controlled by winds. To reduce the convergence between the reference and free simulations, a stochastic noise is added to the wind field of **reference**. The spatial structure of the noise is

$$c(i,j) = \exp\,[-\,(x_i - x_j)^2/L_x^2 - (y_i - y_j)^2/L_y^2]$$

With Lx=10°, Ly=4°. The first ten modes of such a covariance matrix are shown on the right. The stochastic wind noise is constructed as

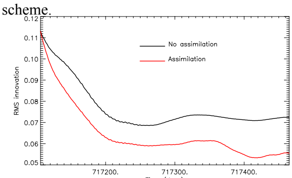
$$\delta W = w_o\, \Sigma\, \lambda_k^{\,1/2} N_k\, \eta_k$$

Where λ are the eigenvalues of the covariance matrix, N are the eigenvectors, and η is a Gaussian noise with zero mean and unit variance.

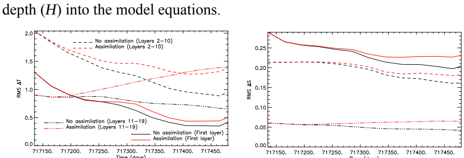


RESULTS

The convergence of the sea level (*ssh*) for one year is shown below. The black line corresponds to the **free** simulation. The red line corresponds to the simulation with **assimilation** of sea level. The results display the convergence of the system related to the fact that these simulations are forced with the same fields as the **reference** simulation. The larger convergence of the assimilation indicates the beneficial role of the data assimilation scheme.



However, the method is shown to fail in the vertical extrapolation of the vertical profiles of temperature and salinity (Figures below). That is, the method modifies the deep ocean in order to match the sea level, but imposes invalid corrections on the vertical structure of the water column. Such a behavior is found to be insensitive to different choices of initial conditions, nudging amplitude, amplitude of the stochastic wind forcing, or the technique used to introduce the corrections of temperature (*T*), salinity (*S*), and layer depth (*H*) into the model equations.



CONCLUSIONS

A method aiming to assimilate diagnostic variables has been tested by assimilating *ssh* into a model of the tropical Pacific Ocean. The method is based on the maximum simplification of the continuous version of the Kalman filter equations. While the method is able to reduce the error in *ssh*, it increases the error of the deep ocean. That is, the adjoint of the observation operator is not enough, by itself, to identify the unique solution of the system, and an estimate of the error of the prognostic variables, matrix **P**, is required. Note that the information in **P** refers to the errors of *T*, *S*, and *H*, but not to the relationship between these variables and *ssh*. That is, the extrapolation of the *ssh* information is decoupled in a vertical propagation using the adjoint of **H**, and a weighted correction based on the information in **P**. Therefore, a more realistic parameterization of the error matrix **P** is required that captures the 3-D structure of the error covariance.