Using Probability Density Functions to Evaluate Ocean Velocity Statistics from Altimetry

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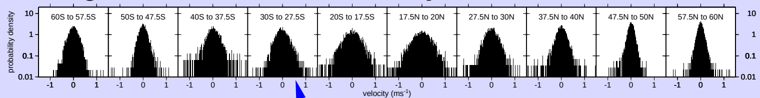
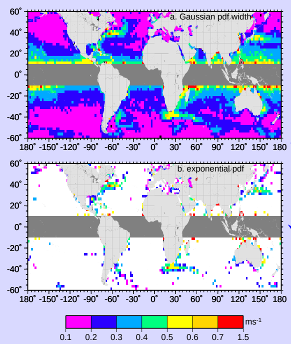
Background: PDFs and Altimetry





Sample probability density functions (PDFs) for TOPEX ocean velocities along the dateline are roughly Gaussian. These PDFs measure the likelihood that a particular value of velocity will be observed. Most physical quantities are expected to have Gaussian distributions.

Widths of best-fit Gaussian PDFs vary geographically like eddy kinetic energy.

Regions where TOPEX data indicate non-Gaussian PDFs are located primarily in highly energetic regions such as the Gulf Stream, and in coastal areas, where TOPEX-derived velocities are likely to be erroneous [Gille and Llewellyn Smith, 2000].

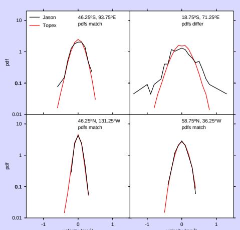
This poster examines two ways in which PDFs can help us interpret altimeter data.

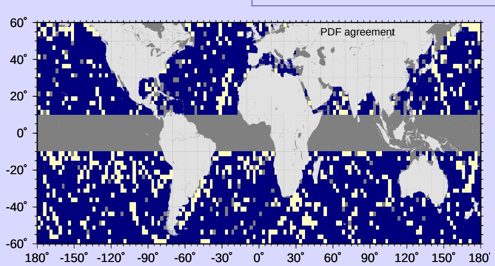
- **Cal/Val: Assessing whether velocity distributions of Jason data match distributions from TOPEX data. Do the instruments observe extreme velocities with the same frequency?**
- **Ocean response to wind: Why are oceanic PDFs more Gaussian than wind PDFs, and what does this tell us about the ocean's dissipation of wind energy?**

Quality Controlling New Observations

Kolmogorov-Smirnov statistics are a formal mechanism to diagnose whether two randomly sampled data sets are drawn from the same PDF. Here, they are used to assess whether PDFs of preliminary Jason IGDR data match PDFs of older, well-calibrated TOPEX data.

Sample pdfs from TOPEX and Jason geostrophic velocity anomalies show similar shapes but Jason observations are much less numerous than TOPEX observations. For these comparisons, the TOPEX mean was subtracted from Jason height fields. Data processing methodology adapted from Yale et al. [1995].

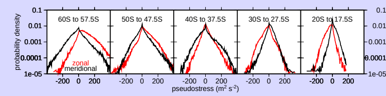




Global map based on Kolmogorov-Smirnov statistics shows that Jason and TOPEX observations are statistically indistinguishable (blue) over most of the ocean. However, more than 17% of data boxes have PDFs that differ (yellow). In these boxes there is a greater than 95% probability that we should reject the null hypothesis, which assumes equivalent PDFs.

Some of the PDF differences are clearly due to a single noisy track and will be easily corrected as the data product is refined.

Wind Forcing and PDFs



Sample wind PDFs along the dateline are typically exponential. These are derived from RSS QuikSCAT Level2B swath winds.

Wind PDFs are shaped differently than ocean velocity PDFs. What accounts for these differences?

Consider the system:

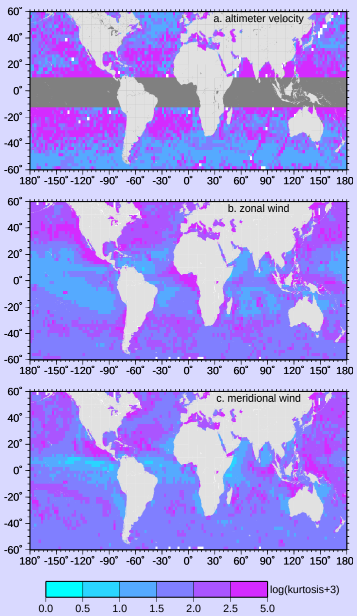
$$\frac{\partial u}{\partial t} = -bu + \tau^x.$$

In this system, ocean velocities (u) are accelerated and decelerated by the wind (τ^x). Linear drag dissipates wind energy with a time scale T, where $b = 1/T$.

If b is small, then the ocean has a very long memory, and ocean velocities will represent a summation of many independent wind forcings. Since we sum many independent samples, the central limit theorem says that u should have a Gaussian distribution, regardless of the PDF of τ^x.

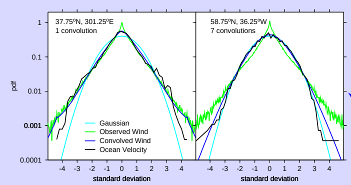
If b is large, then the ocean has very little memory, and ocean velocities will directly reflect wind velocities. In this case we expect that u should have the same PDF as τ^x.

The real ocean must lie somewhere in between these extremes. Can we use the differences between the PDFs of u and τ^x to infer the effective dissipation time scale b for the ocean?

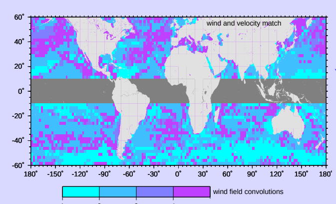


Log of kurtosis for (a) ocean surface velocities, (b) zonal wind, (c) meridional wind shows differences between fields. Altimeter fields have the highest kurtosis (indicating least Gaussian behavior) at low latitudes, where outliers are most common. Wind pdfs have highest kurtosis at high latitudes, where winds are strongest.

Kurtosis estimates are influenced by variance, because neither data type is strictly Gaussian. For example, if the data distribution were exponential, then kurtosis would be a poor diagnostic, but the 4th moment divided by the 2nd moment to the 5/3 power would be constant. The data are not uniformly represented with an exponential relationship.



An ocean velocity PDF resembles a wind PDF that has been convolved with itself. (Left) In the Gulf Stream, where velocity PDFs are nearly exponential, the ocean "remembers" roughly two random kicks from the wind. (Right) Elsewhere, where velocity PDFs are more nearly Gaussian, ocean velocities resemble a summation of many random forcings.



The number of summed random wind samples required to minimize differences between the wind PDF and the ocean velocity PDF varies from one in energetic regions to greater than 4 in central gyres and high latitude regions.

References

Gille, S. T. and S. G. Llewellyn Smith, 2000. Velocity-variance probability distribution functions from altimeter measurements. *J. Phys. Oceanogr.*, **30**, 125–136.

Yale, M. M., D. T. Sandwell, and W. H. F. Smith, 1995. Comparison of along-track resolution of stacked Geosat, ERS 1 and TOPEX satellite altimeters. *J. Geophys. Res.*, **100**, 15,117–15,127.

Wind data: QuikSCAT RSS Compact Level 2B Swath Winds (www.coaps.fsu.edu).
Altimeter data: Jason IGDR and TOPEX GDR. (podaac.jpl.nasa.gov)

For further information and preprints see: http://www-mae.ucsd.edu/~sgille/