Theoretical EM Bias Model Using RMS Slope

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ABSTRACT

Using a simple first order hydrodynamic model and the physical optics scattering model, we derive a theoretical expression for the EM bias in terms of RMS wave slope. The theoretical model is validated using tower experiment truth data from the Gulf of Mexico (Arnold et al., JGR, vol. 100, pp. 969-980, 1995) and Bass Straights Australia (Melville and Felizardo, Topex/Poseidon/Jason-1 SWT Meeting, 1998) and compared to empirical least-squares fit models.

MODEL DERIVATION

SSB is defined by

$$\epsilon = \frac{\langle \eta\sigma^0(\eta)\rangle}{\langle \sigma^0(\eta)\rangle}.$$

for the sea state bias, where η is displacement from local mean surface height. Expanding this expression about $\eta = 0$ yields

$$\epsilon \simeq \langle \eta^2\rangle \frac{\sigma^{0\prime}(0)}{\sigma^0(0)}. \qquad (1)$$

which reduces the determination of EM bias to the variation of the local scattering with respect to deviation from the mean surface level.

HYDRODYNAMIC MODEL

We assume that the sea surface consists of long waves ($\lambda_{Sea} \gg 1$ m) and small waves ($\lambda_{Sea} \simeq \lambda_{EM}$). A simple first order hydrodynamic model [Melville and Felizardo, preprint, 1999] leads to a linear modulation of the small wave surface height standard deviation,

$$h_s(\eta) = h_s(0)\,[1 + S\eta/h_l]$$

where h_l is the long wave surface height standard deviation and S is RMS long wave surface slope.

SCATTERING MODEL

The physical optics approximation (PO) for a 1D surface with Gaussian surface height distribution leads to the nadir backscattering coefficient

$$\sigma^0 \simeq \frac{k_{EM}^2 L^2}{\pi}\int_{-1}^{1} du\,(1-|u|)e^{-\lambda[1-C(Lu)]}$$

where $C(x)$ is the sea surface height correlation function and $\lambda = 4k_{EM}^2 h_s^2$. With the linear short wave modulation model and the physical optics backscattering model, we can evaluate the bias using (1). This leads to

$$\epsilon = -\gamma S h_l$$

where

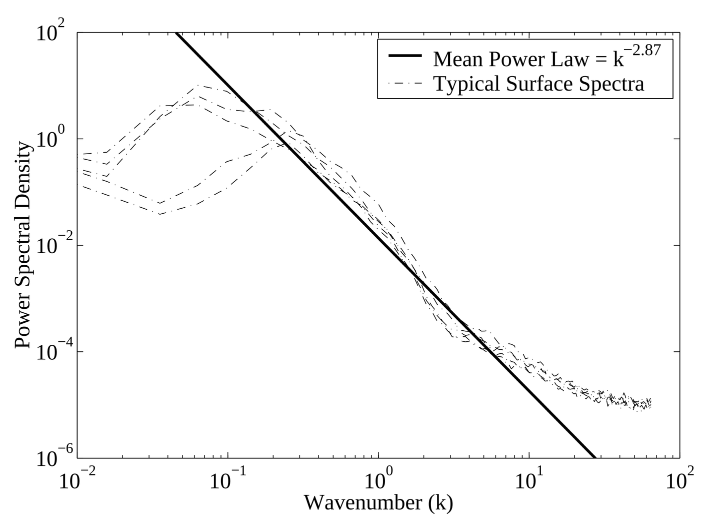
$$\gamma = \frac{2\lambda\int_{-1}^{1} du\,(1-|u|)[1-C(Lu)]e^{-\lambda[1-C(Lu)]}}{\int_{-1}^{1} du\,(1-|u|)e^{-\lambda[1-C(Lu)]}}.$$

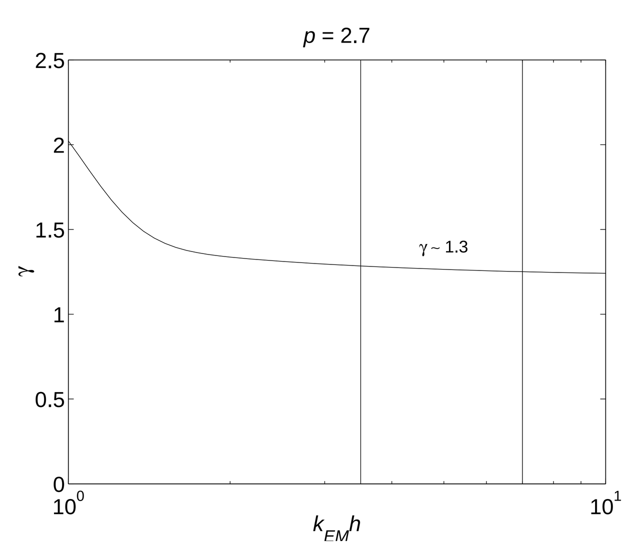
The EM bias can also be written as

$$\epsilon = -(\gamma/4)SH$$

where H is significant wave height.

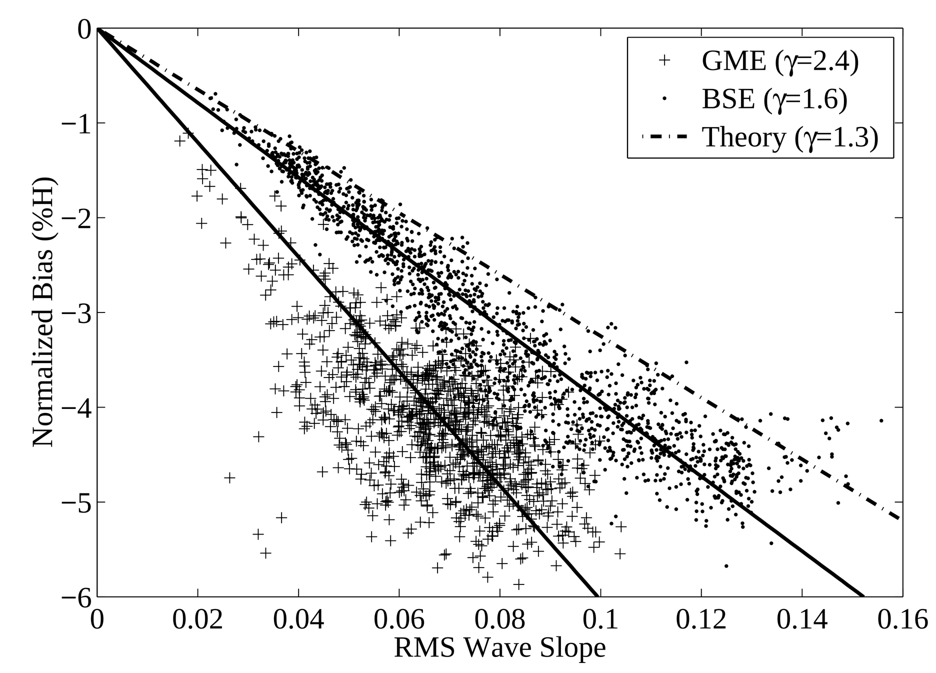
Determination of γ parameter. Fitting k^{-p} power law surface height PSD to experimentally measured surface height time series gives exponent p = -2.7. Numerical integration gives a value for γ of 1.3 for the range of small wave surface heights observed in the GME tower experiment.





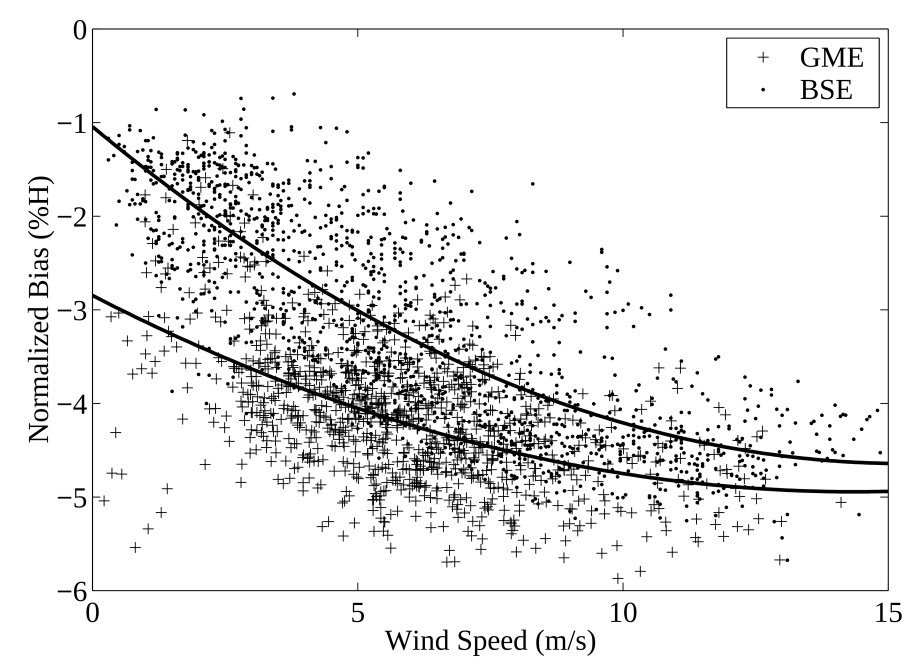
RMS SLOPE - THEORY

Normalized bias scatter plots with theory lines for theoretical γ value and best fit γ values for the GME and BSE tower experiment.



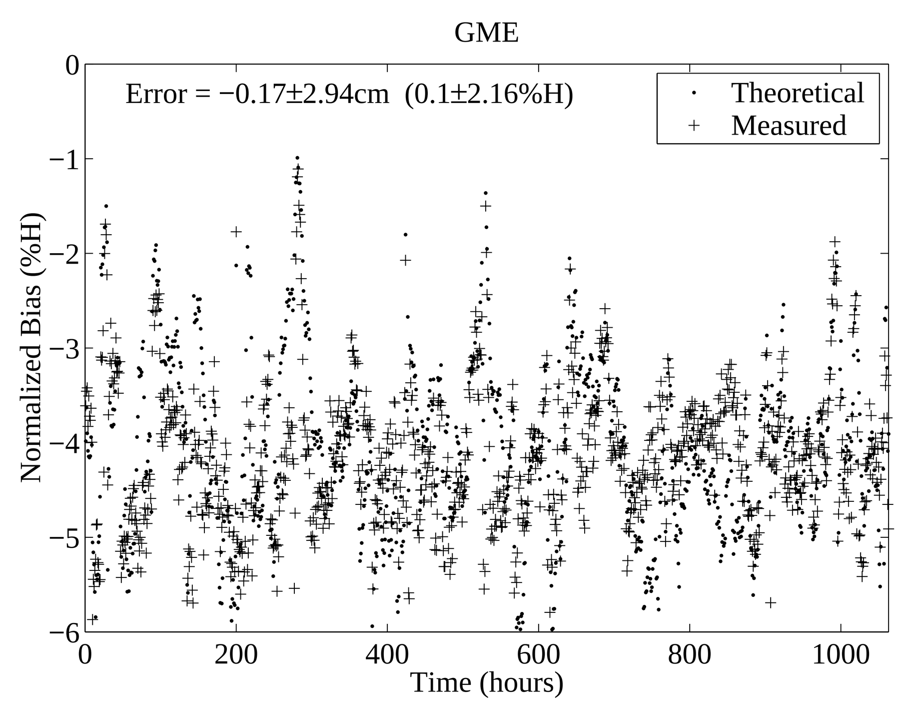
WIND SPEED -2nd ORDER FIT

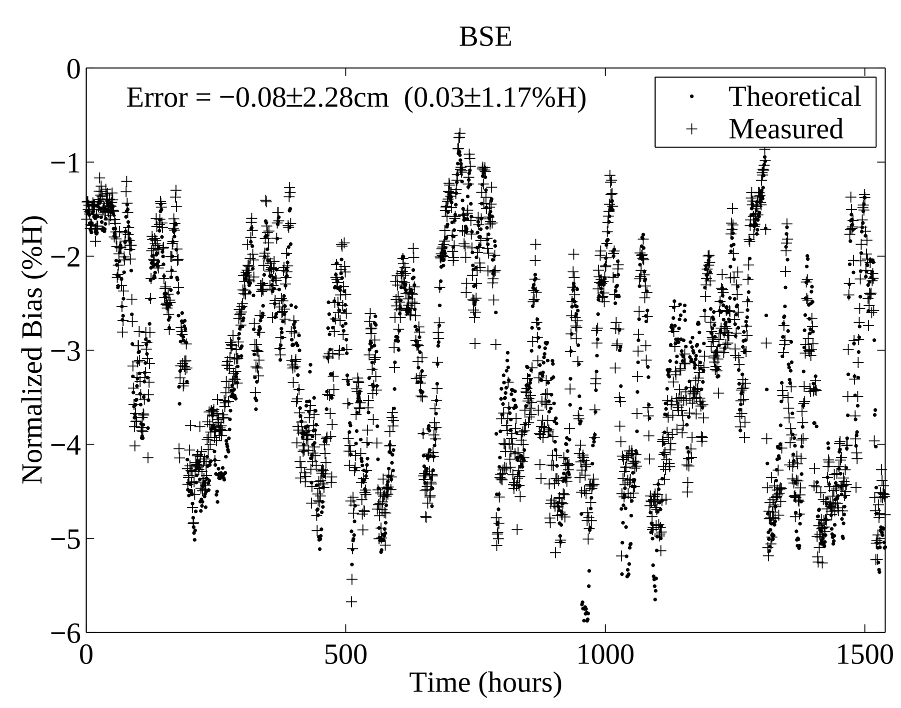
Normalized bias versus wind speed for comparison. Note increased scatter and reduced linearity as compared to normalized bias versus RMS slope. The mean $\pm$ 3σ error is GME: 0 ± 2.34cm ($0 \pm 1.86\%H$), BSE: 0 ± 3.06cm ($0 \pm 2.04\%H$).



TIMES SERIES - RMS SLOPE - THEORY

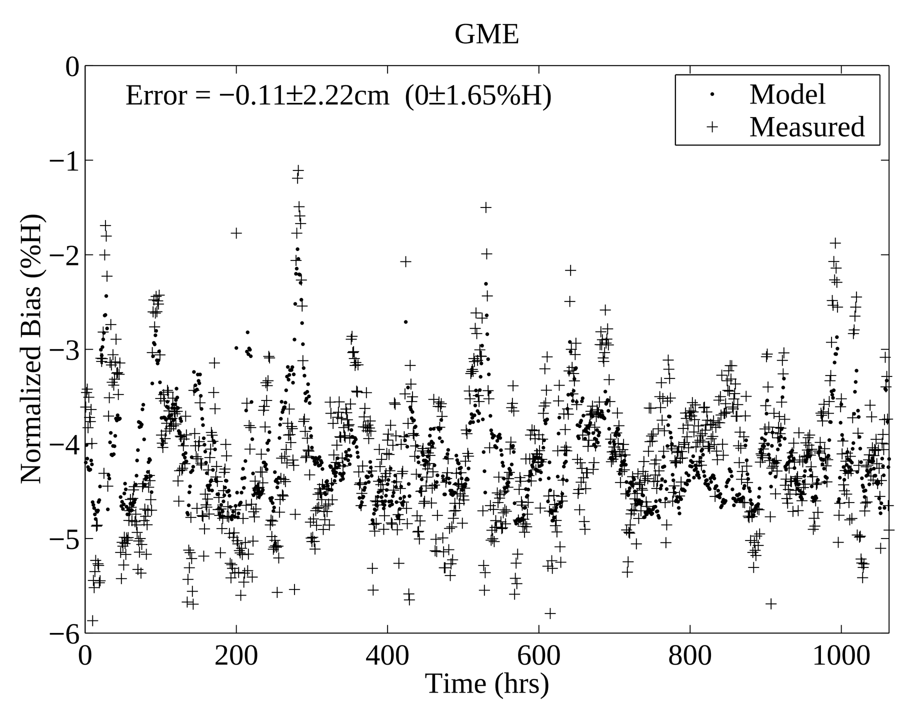
Normalized bias time series overlaid with theoretically predicted bias. Mean $\pm$ three standard deviations of model error (truth bias - theoretical bias) are annotated on plots.

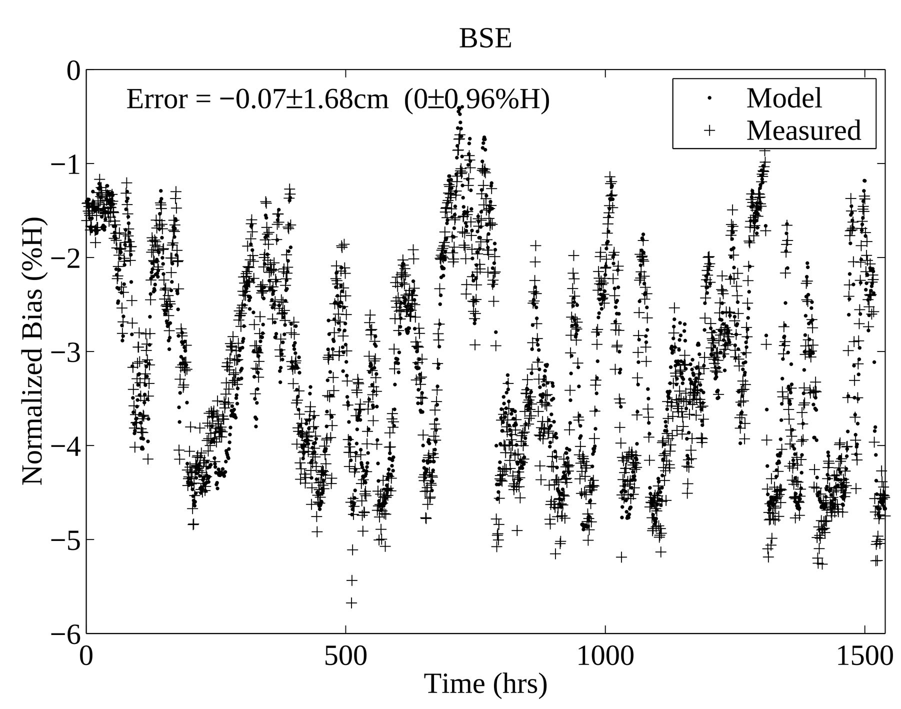




TIMES SERIES - RMS SLOPE - 2nd ORDER FIT

For comparison, time series and error values are given below for empirical 2nd order best fit of RMS slope to experimental normalized EM bias values





Presented at Jason-1 SWT June, 2002