From TOPEX/POSEIDON to JASON Science Working Team Meeting - Arles, France - November 18-21, 2003

Accuracy Assessment of Along-track Jason-1 and TOPEX/POSEIDON Sea Surface Slope

Bob Leben and Brian Powell
Colorado Center for Astrodynamics Research
University of Colorado, Boulder

Abstract

Sea surface slope computed from along track Jason-1 and TOPEX/POSEIDON (T/P) altimeter data at ocean mesoscale wavelengths are compared to determine the 1 Hz instrument noise of the Poseidon-2 and Topex altimeters. This geophysical evaluation shows that the Ku band 1-Hz range noise for both instruments is better than 1.7 cm at 2 m significant wave heights (H1/3), exceeding error budget requirements for both missions. Furthermore, we show that the quality of these instruments allows optimal filtering of the 1 Hz along track sea surface height data for sea surface slopes that can be used to calculate cross track geostrophic velocities anomalies at the baroclinic Rossby radius of deformation to better than 5 cm/sec precision along 87.5% of the satellite ground track between 2 and 60 degrees absolute latitude over the deep abyssal ocean (depths greater than 1000 m). This level of precision will facilitate scientific studies of surface geostrophic velocity variability using data from the Jason-1 and T/P Tandem Mission.

Introduction

All radar altimeters are affected by internally generated white noise that affects the accuracy of sea surface slope estimates at the ocean mesoscale, which are used to calculate geostophic velocities associated with eddies, meanders, squirts and jets. In this poster, we use an optimal difference operator for computing sea surface slope to examine the performance of the Poseidon 2 and TOPEX altimeters in this regard. These filters were designed to calculate along track slope in the presence of white noise. Direct comparison of the filtered Jason-1 and T/P sea surface slope data collected during the verification mission phase permits geophysical evaluation of the on-orbit performance of the two altimeter systems.

Our approach to computing accurate sea surface slope is based on the derivation of finite-difference equations for the first derivative using a Taylor series expansion. In typical finite difference applications, the coefficients found by the Taylor series expansions around successive points cancel higher order terms in the series, which minimized truncations error in the derivative calculation to provide higher order accuracy. Rather than minimizing truncation error, we solve for a set of coefficients that minimize the white noise propagated through a calculation of along track slopes. This is the dominate error affecting the accuracy of sea surface slopes at mesoscale wavelengths

For brevity, we show the derivation for the optimal five-point central difference operator. The standard central difference operator for a five-point stencil (two points on either side fo the point of interest) is given by:

$$\frac{\Delta h}{\Delta t} = w_1\left(\frac{h_{i+1} - h_{i-1}}{2\Delta t}\right) + w_2\left(\frac{h_{i+2} - h_{i-2}}{4\Delta t}\right)$$

where, h is the height at a given location and Δt is the time difference between each point (in this case 1 second), and w is the coefficient for each difference A constraint that all coefficients must sum to one is placed on the system. We wish to solve these equations such that the white noise, σ_h, associated with the height measurements is minimized. Because the noise is uncorrelated, the residual noise in the estimate o f the derivative is equal the square root of the sum of the squares of the error in each measurement:

$$\sigma_v = \sqrt{\frac{w_1^2}{4\Delta t^2}\left(2\sigma_h^2\right) + \frac{w_2^2}{16\Delta t^2}\left(2\sigma_h^2\right)} = \frac{\sqrt{2}\sigma_h}{4\Delta t}\sqrt{4w_1^2 + w_2^2}$$

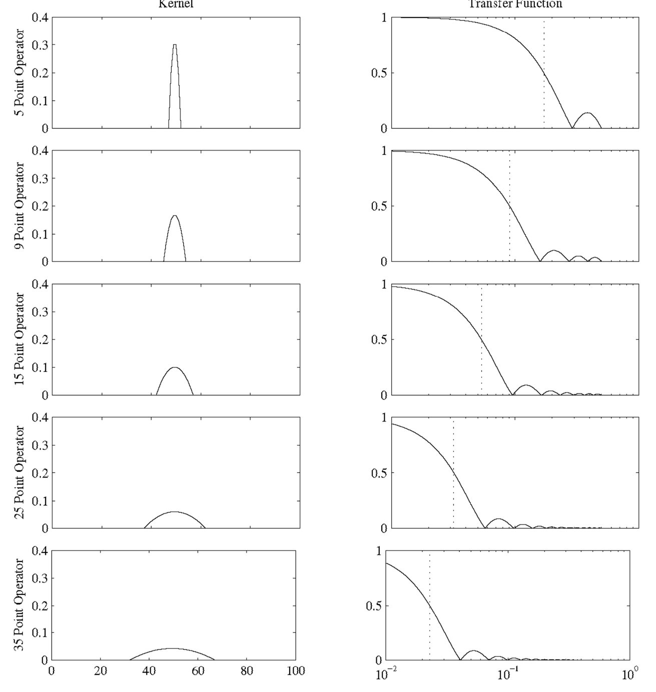
Using the method of Lagrange Multipliers with the given constraint, a solution of the coefficients, w_i, can be found that minimizes this noise. In the five-point case, the coefficients are found to be: $w_1=1/5$ and $w_2=4/5$. Substituting the coefficients back into the equation gives the noise in the velocities (slopes) as a function of the noise in the heights. For the five-point case, $\sigma_v=0.3162(\mathrm{sec}^{-1})\sigma_h$. Thus, with estimates of σ_v, we can convert to the equivalent 1-Hz height noise, σ_h, or *vice versa*, by the scale factor given. These are tabulated in Table 1. Topex slope noise in mm/sec (calculated assuming 1.7 cm rms height noise) can be converted to geostrophic velocities by multiplying by g/(f L), where L is the spacing between points.

Table 1: Optimal difference filter characteristics.

Points (T)	Noise in δv (factor of δh)	Half-Power Frequency (Hz)	T/P Operator Size (km)	TOPEX Alt. Noise ($mm\ s^{-1}$)	T/P Half-Power Wavelength (km)
3	0.7071	0.3334	11.5062	12.0208	17.2558
4	0.4472	0.2234	17.2593	7.6026	25.7525
5	0.3162	0.1709	23.0124	5.3759	33.6635
6	0.2390	0.1392	28.7655	4.0638	41.3297
7	0.1890	0.1178	34.5186	3.2127	48.8379
8	0.1543	0.1022	40.2717	2.6232	56.2926
9	0.1291	0.0903	46.0248	2.1947	63.7110
10	0.1101	0.0810	51.7779	1.8716	71.0259
11	0.0953	0.0734	57.5310	1.6209	78.3801
12	0.0836	0.0671	63.2841	1.4216	85.7392
13	0.0741	0.0619	69.0372	1.2601	92.9418
14	0.0663	0.0574	74.7903	1.1271	100.2282
15	0.0598	0.0535	80.5434	1.0159	107.5346
16	0.0542	0.0501	86.2965	0.9220	114.8323
17	0.0495	0.0471	92.0496	0.8416	122.1465
18	0.0454	0.0445	97.8027	0.7723	129.2831
19	0.0419	0.0421	103.5558	0.7121	136.6532
20	0.0388	0.0400	109.3089	0.6592	143.8275
21	0.0360	0.0381	115.0620	0.6126	151.0000

An interesting feature of the optimal differencing technique is that an equivalent smoothing kernal for the along tack height data can be derived from the optimal difference filter. This smoothing kernal, if applied to along track height before differencing consecutive points to calculate along tack slope, gives the same results as the optimal difference filter. Using this result, we can calculate the corresponding gain as a function of frequency and the half-power point for filters of varying width. This allows selection of the appropriate window size to resolve the oceanic mesoscale, a spatial scale that varies significantly with latitude.

Figure 1: Spectral characteristics of the optimal derivative filter.are shown below. On the left is the equivalent filter weightings on the 1-Hz height measurements. On the right is the filter gain with the corressponding half-power point of the filter shown by the vertical dotted line.



Noise Calculations

Assuming that the measurements collected are a combination of signal variance and measurement noise variance, the sample variances for the two satellites may be written as:

$$\sigma^2(Topex) = \sigma^2(signal) + \sigma^2(T_{noise})$$

$$\sigma^2(Jason) = \sigma^2(signal) + \sigma^2(J_{noise})$$

where $\sigma(T_{noise})$ and $\sigma(J_{noise})$ are the respective noise levels in the Topex and Poseidon-2 instruments. Since the oceanographic signals are nearly identical in the coincident measurements the equations above are equivalent to:

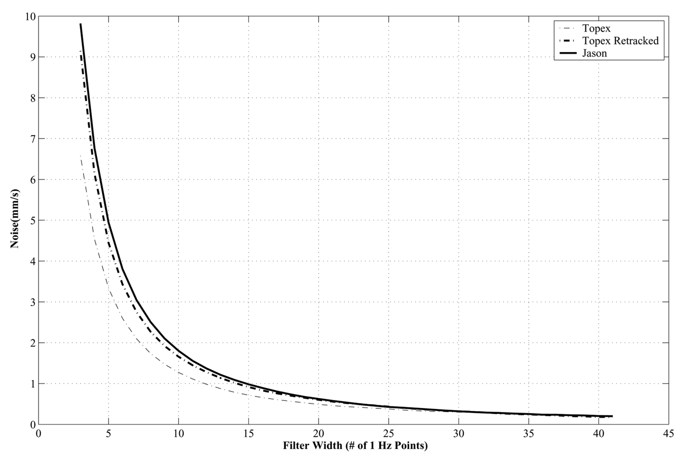
$$\sigma^2(Topex) - \sigma^2(Jason) = \sigma^2(T_{noise}) - \sigma^2(J_{noise})$$

We can also take the expected value of the square of the differences of the two conincident measurements, which gives:

$$\sigma^2(Topex - Jason) = \sigma^2(T_{noise}) + \sigma^2(J_{noise})$$

Using the last two equations, we solve for $\sigma(T_{noise})$ and $\sigma(J_{noise})$, the noise in the respective measurements. The variances that we use for these error estimates are computed from the along track slope. The slope noise can then be converted to equivalent 1-Hz noise using the theoretical characteristics of the filter .

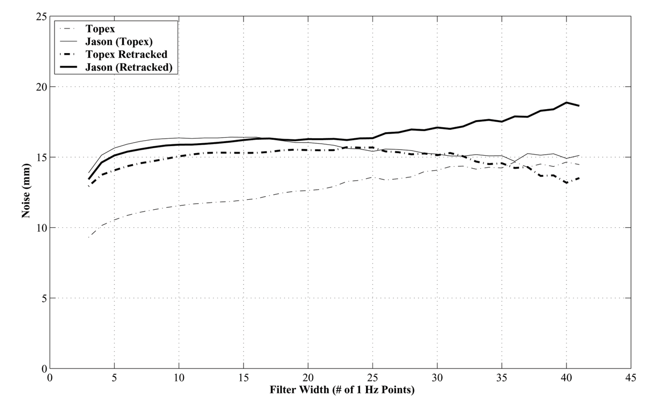
Figure 2: Using a single cycle of T/P (355) and Jason-1 (12) data, the alongtrack slope noise for varying filter lengths are shown at the right. The reduction in T/P retracked and Jason noise is consistent with the error estimates provided by the optimal filter indicating that the noise is white. The original T/P data shows some artifacts of the additional smoothing of the data records over 3-second frames.



Equivalent 1-Hz SSH Noise

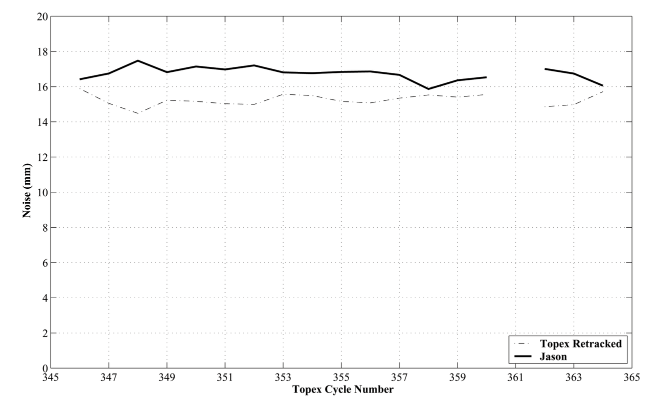
We estimate the equivalent 1-Hz noise corresponding the altimeter instruments range precision from the slope error estimated above. The scaling factors to convert between the two are derived for each filter window and tabulated in Table 1.

Figure 3: We show estimates of equivalent 1-Hz SSH noise from Jason cycle 12 and T/P cycle 355 collocated with T/P measured H1/3 values between 1.5m and 2.5m. In the idealized case of only white noise, these estimates should match spectral estimates using high-rate data and be relatively constant across window widths.



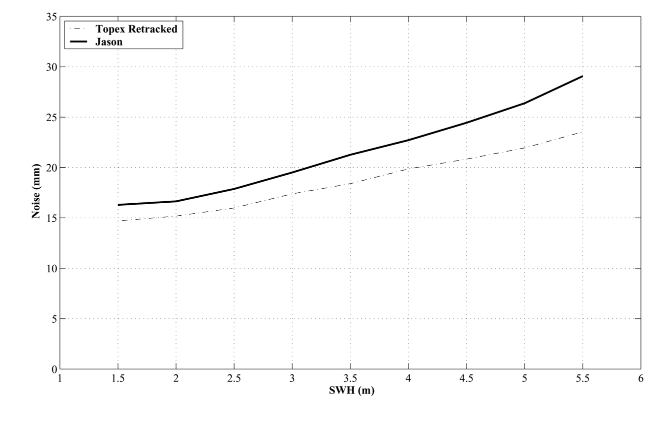
This is the case for the Poseidon-2 and retracked Topex measurements over the midrange of filter widths of 10 through 30 points. The original Topex GDR data, however, gave results that varied substantially over the entire range of filter widths examined, which is consistent with the "coloring" of the range noise associated with problems in the on-board tracking algorithm and additional processing over 3-second frames in ground processing. Next, we examine the performance on a cycle by cycle

Figure 4: Using the 15-point optimal filter we computed the equivalent 1-Hz noise over Topex cycles 346-360, 362-364 and the corresponding Jason-1 cycles. The mean and standard deviation is 1.52+/- 0.09 cm for T/P and 1.67+/- 0.08 cm for Jason-1. These results are consistent with spectral analysis of the high-rate data from each satellite.



Only about one third of all Jason and T/P altimeter data are collected over ocean points where H1/3 is between 1.5 and 2.5m. Increasing H1/3 increases the noise floor on the altimeter range through the effect of the random wave field on the returned power and the tracking algorithms.

Figure 5: The equivalent 1-Hz noise floor as a function of H1/3 for the two instruments was calculated from Jason cycle 12 (T/P cycle 255) as a function of measured Topex GDR H1/3 values using filtered data based on the 15-point filter. The trends and specific values are once again in good agreement with spectral estimates reported for the Poseidon-2 and Topex instruments



Discussion

A characteristic length scale associated with meoscale circulation is the baroclinic Rossby radius of deformation. The pronounced decrease of this scale with increasing latitude requires significant range precision from a satellite altimeter for study of mesoscale surface currents because of the commensurate increase in the noise floor associated with derivatives across these scales. To examine the impact of the altimeter's range precision on estimates of cross-track geostrophic velocity, we converted the budgeted Ku band range precision (1.7 cm at 2 m H1/3) to equivalent velocity error for the optimal filter a half-power points corresponding to length scales equal to twice the Rossby radius of deformation. We find the cross-track velocity error estimates to be less than 5 cm/sec along 87.5% of the satellite ground tracks between 2 and 60 degrees absolute latitude over the deep abyssal ocean where the dynamics and scales of motion considered are dominant. This is a lower bound on the attainable accuracy since the absolute accuracy will depend on corrections to the range measurement and their associated errors, which are not considered in this study. The level of precision exhibited by these state-of -the-art altimeter instruments, however, will facilitate scientific studies of surface geostrophic currents using data from the Jason-1 and T/P Tandem Mission.