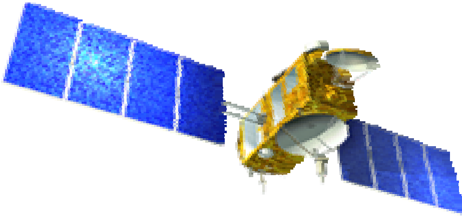
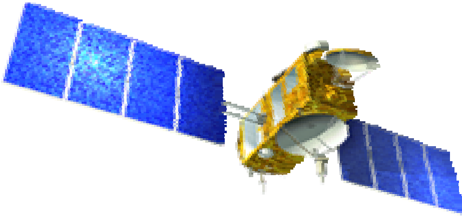


New Retracking Algorithm Using a Second Order Waveforms Model

Pierre THIBAUT, Laiba AMAROUCHE and Ouan Zan ZANIFE



INTRODUCTION

Objective :

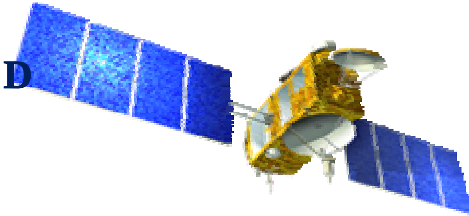
The ground segment has to take into account problems of mispointing of the Jason-1 platform

The aim of this study is to propose a new formulation of the echo model to be able to estimate the altimetric parameters even when the platform is not well pointed

Summary of the presentation :

- ⇨ **Analytical formulation of the second order model**
- ⇨ **Use of this model to estimate the altimetric parameters in the case of high mispointing angles on real Poseidon-2 data**
  - ⇨ **During the first STR incident**
  - ⇨ **On a cycle of data**
- ⇨ **Conclusions**

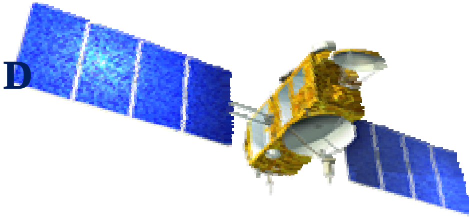
ANALYTICAL FORMULATION OF THE SECOND ORDER MODEL



The altimetric waveform model can be written as the convolution of three terms :

- The radar response to a sea surface without waves and an infinitely short pulse : **FSSR**
- The probability density function of the wave heights : **PDF**
- The point target response of the radar : **PTR**

$$S(t) = FSSR(t)*PTR(t)*PDF(t)$$



ANALYTICAL FORMULATION OF THE SECOND ORDER MODEL

RADAR RESPONSE TO A SEA SURFACE WITHOUT WAVES AND AN ININITELY SHORT PULSE

Before simplification, the FSSR is written as follows (Brown 77) :

$$FSSR(t) = A\exp(-\delta t)U(t)\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(k+1/2)}{\sqrt{\pi}\,\Gamma(k+1)}\left[\frac{\gamma\beta t^{1/2}}{8\cos^2\xi}\right]^k I_k(\beta t^{1/2})$$

With :

$$\delta = \frac{4}{\gamma}\frac{c}{h}\cos(2\xi)$$

$$\beta = \frac{4}{\gamma}\left[\frac{c}{h}\right]^{1/2}\sin(2\xi)$$

$$A = A_0\exp\left[-\frac{4}{\gamma}\sin^2\xi\right]$$

The I_k functions are the Bessel 's modified functions. Brown has only kept the order 0 Bessel 's functions. Upper order terms are negligeable (for ξ < 1°) because :

$$\frac{\gamma\beta t^{1/2}}{8\cos^2\xi} << 1$$

FSSR can be written as follows :

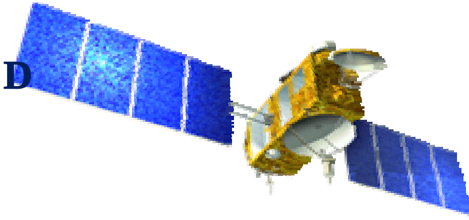
$$FSSR(t) = A\exp(-\delta t)I_0(\beta t^{1/2})U(t)$$

$$I_0(z) = \sum_{n=0}^{\infty}\left(\frac{z^2}{4}\right)^n\left(\frac{1}{n!}\right)^2$$

With : $z = \beta t^{1/2}$

- **<u>1st Order :</u> good for ξ < 0.4° for gate 90**
  0.3° for gate 128
- **<u>2nd Order :</u> good for ξ < 0.7° for gate 90**
  0.6° for gate 128

ANALYTICAL FORMULATION OF THE SECOND ORDER MODEL



1st ORDER MODELIZATION

Using 0th and 1st order development of the Bessel 's functions and writting the result on an exponential form :

$$I_0(\beta t^{1/2}) \approx 1 + \frac{\beta^2 t}{4} \approx e^{\frac{\beta^2 t}{4}}$$

The waveform can be written simply as follows :

$$W(t) = A\exp(-v)\left[1 + erf(u)\right]$$

With :

$$u = \frac{t - \tau - \alpha\sigma_c^2}{\sqrt{2}\sigma_c} \qquad v = \alpha\left(t - \tau - \frac{\alpha}{2}\sigma_c^2\right) \qquad \alpha = \delta - \frac{\beta^2}{4}$$

This exponential formulation is true for mispointing angles lower than 0.25°

2nd ORDER MODELIZATION

$$I_0(\beta t^{1/2}) \approx 2e^{\frac{\beta^2 t}{8}} - 1$$

The altimetric echo model can thus be written as follows, after convolution of the 3 terms :

$$W(t) = A\exp(-v_1)\left[1 + erf(u_1)\right] - \frac{A}{2}\exp(-v_2)\left[1 + erf(u_2)\right]$$

With :

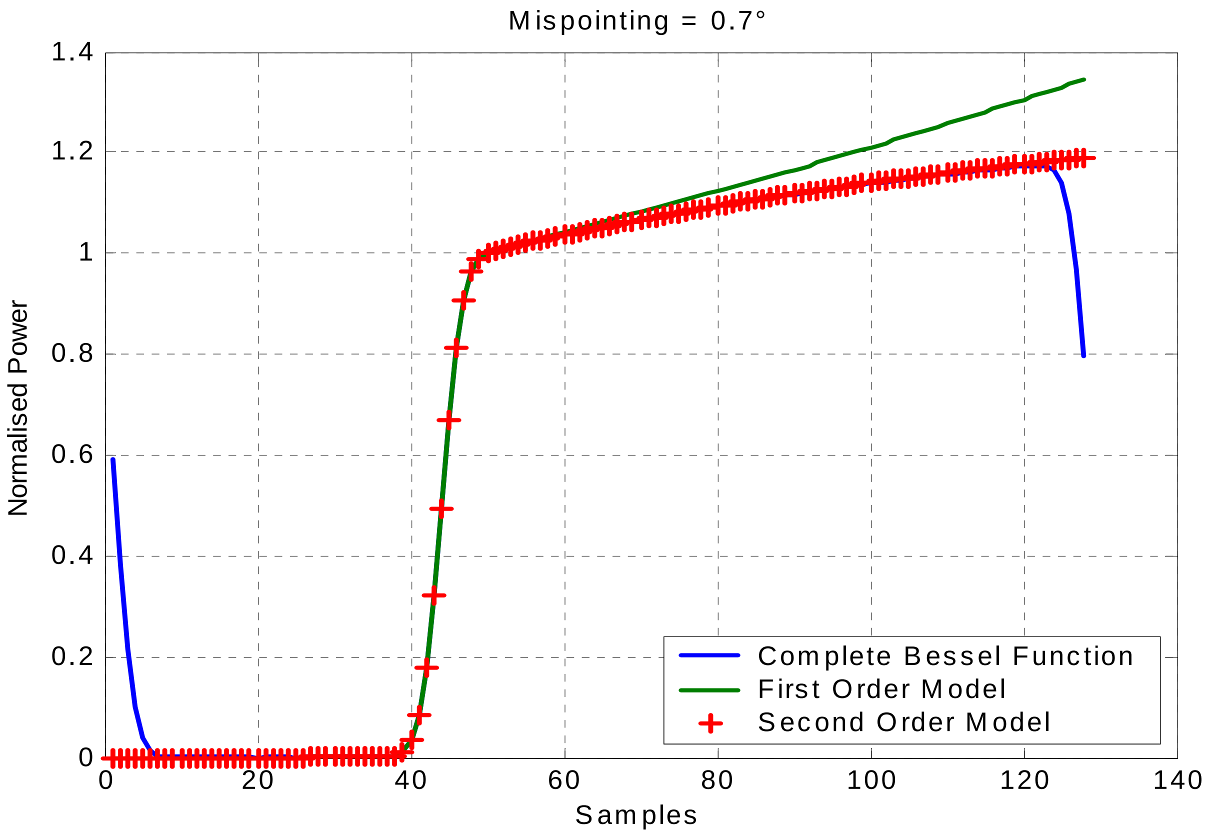
$$u_1 = \frac{t - \tau - \alpha_1\sigma_c^2}{\sqrt{2}\sigma_c} \qquad v_1 = \alpha_1\left(t - \tau - \frac{\alpha_1}{2}\sigma_c^2\right) \qquad \alpha_1 = \delta - \frac{\beta^2}{8}$$

$$u_2 = \frac{t - \tau - \alpha_2\sigma_c^2}{\sqrt{2}\sigma_c} \qquad v_2 = \alpha_2\left(t - \tau - \frac{\alpha_2}{2}\sigma_c^2\right) \qquad \alpha_2 = \delta$$

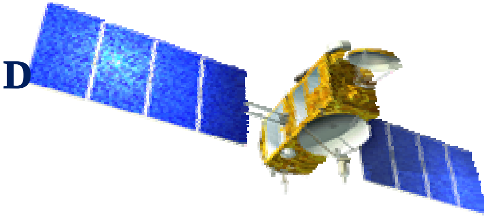
$$\delta = \frac{4}{\gamma}\frac{c}{h}\cos(2\xi) \qquad \beta = \frac{4}{\gamma}\left[\frac{c}{h}\right]^{1/2}\sin(2\xi)$$

ANALYTICAL FORMULATION OF THE SECOND ORDER MODEL

Comparison of the new formulation with the expression used in the Poseidon-2 ground retracking and with the function obtained from the three terms convolution (with a complete Bessel 's function)



ANALYTICAL FORMULATION OF THE SECOND ORDER MODEL



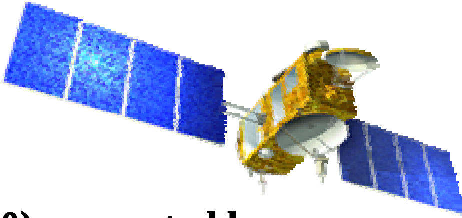
Drawback of a second order model :
impossibility to derive the mispointing angle from the slope of the trailing edge

$$\xi^2 = \frac{1}{2}\,\frac{1+\frac{Slope}{\alpha}}{1+\frac{2}{\gamma}}$$

<u>Several solutions were investigated :</u>

- **Use of a MLE2 (ξ^2 and σ_0) on the trailing edge of the echo : problems of stability**
- **Use of a correction table to derive the real mispointing angle from the on board mispointing angle**
- **Use of a MLE 4 parameters (Range, SWH, σ_0 and $\xi 2$)**

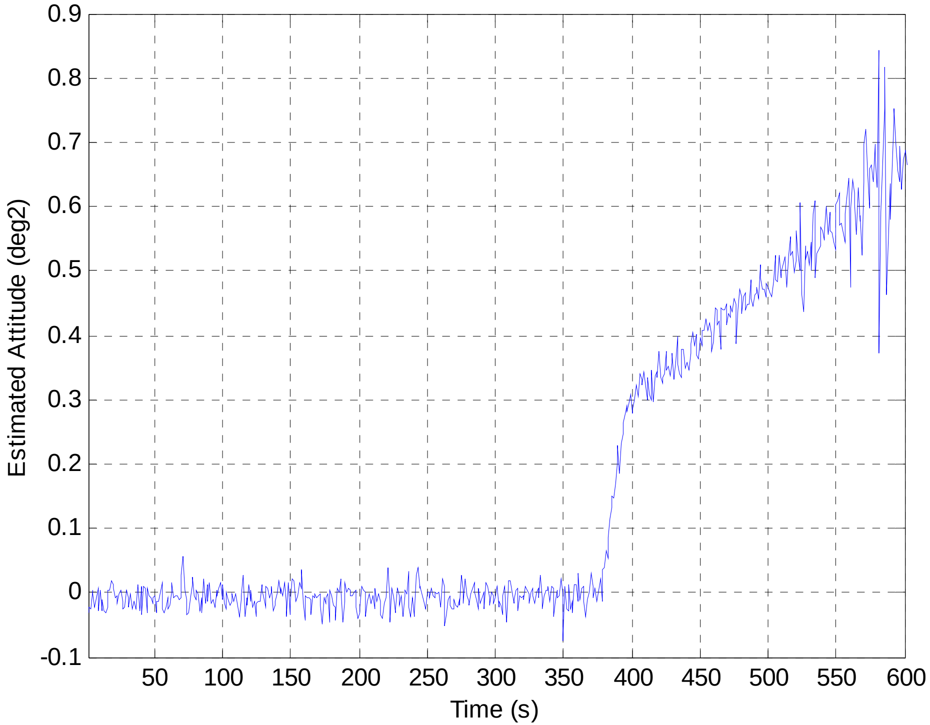
COMPARISON OF RESULTS ON REAL DATA



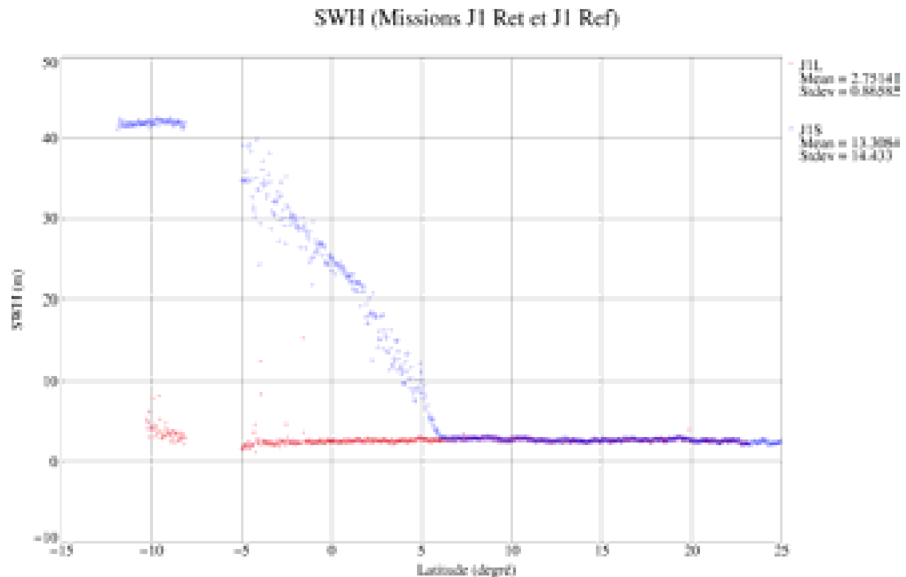
The comparison is done on the altimetric parameters (SWH,SSH and σ0) computed by :

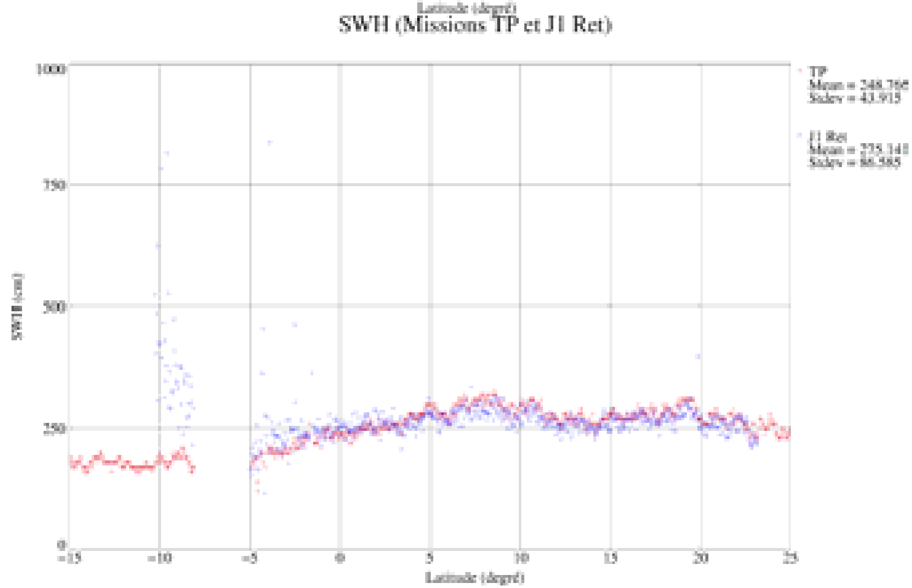
- CMA retracking algorithm
- MLE4 based on a second order model (SWH,range,σ0 and mispointing angle ξ^2)
- Topex data

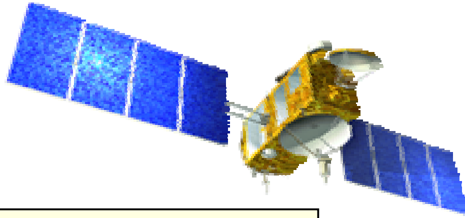
04/04/2002
First Star Tracker Incident
Cycle 008



COMPARISON OF RESULTS ON REAL DATA





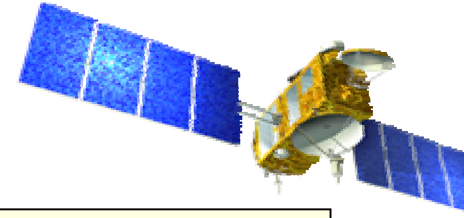


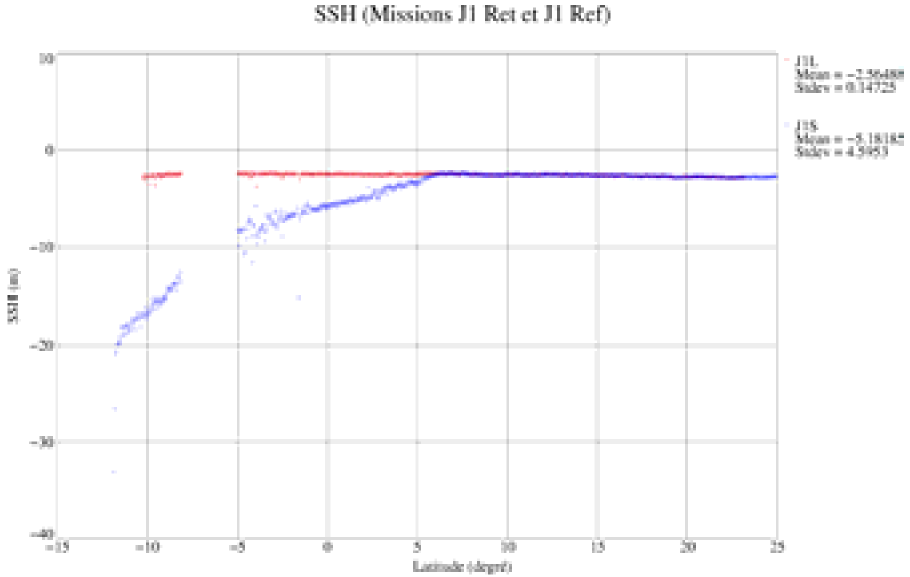
04/04/2002
First Star Tracker Incident

Significant Wave Height
MLE4 New Model - CMA IGDR

Significant Wave Height
TOPEX GDR - MLE4 New Model

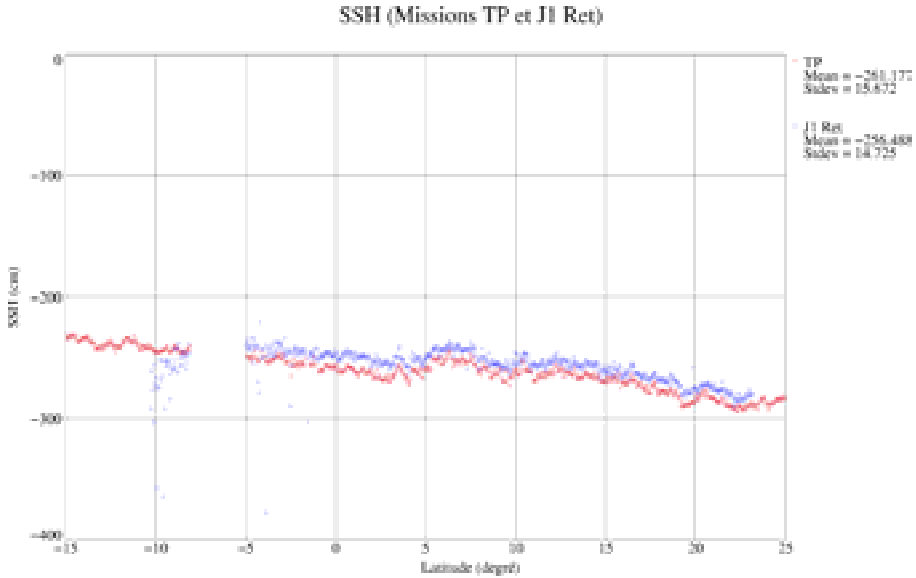
COMPARISON OF RESULTS ON REAL DATA





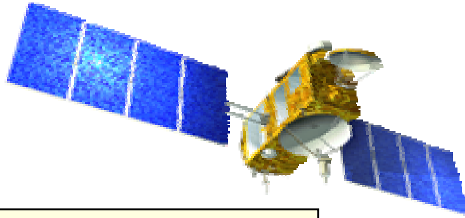
04/04/2002
First Star Tracker Incident

Sea Surface Height
MLE4 New Model - CMA IGDR

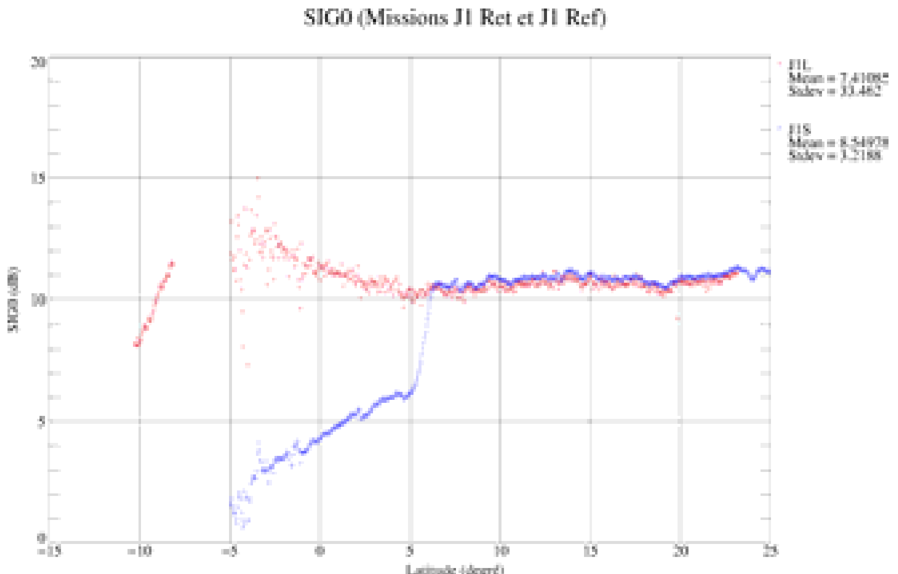


Sea Surface Height
TOPEX GDR - MLE4 New Model

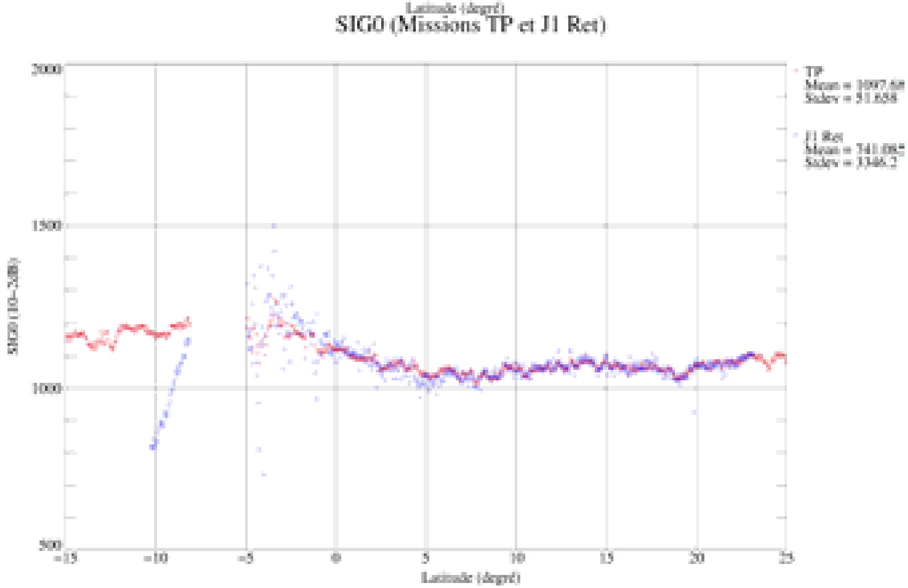
COMPARISON OF RESULTS ON REAL DATA



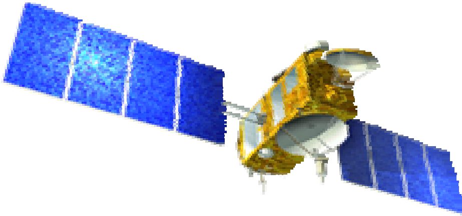
04/04/2002
First Star Tracker Incident



Sigma 0
MLE4 New Model - CMA IGDR



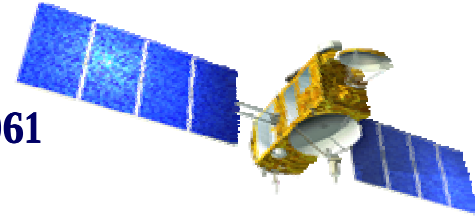
Sigma 0
TOPEX GDR - MLE4 New Model



GENERAL CALVAL RESULTS ON CYCLE 061
(using Doris ionospheric correction) (1/2)

	MLE-3	MLE-4
Number of passes	253	253
Number of data	674968	674968
Number of missing data	153717	153717
Number of valid data	498594	496470
Number of edited data	176374	178498
Percentage of edited data	26.13 %	26.44 %

SSH Crossover Analysis	9.08 cm	8.86 cm
EM Bias (% SWH)	-3.55	-3.57
Time-Tag Bias	0.263 ms	0.256 ms



GENERAL CALVAL RESULTS ON CYCLE 061

(using Doris ionospheric correction) (2/2)

Ku band

RMS_Range_MLE4 - RMS_Range_MLE3 = 0.6 cm

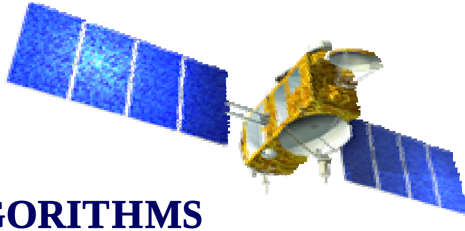
SWH_MLE4 - SWH_MLE3 = -3.8 cm

Sig0_MLE4 - Sig0_MLE3 = 0.05 dB

ξ_MLE4 - ξ_MLE3 = 0.095 deg

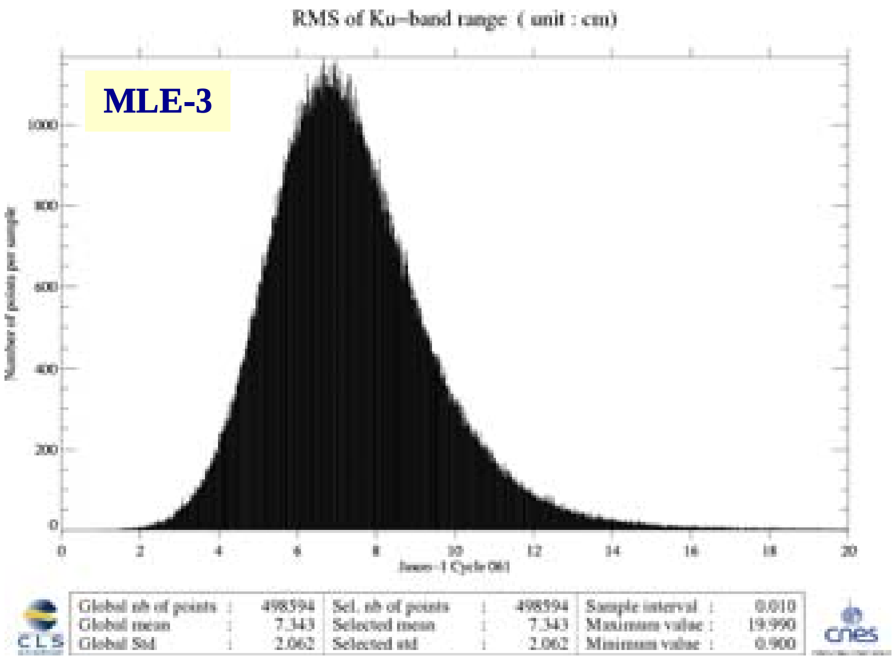
Number of valid points per second is equivalent

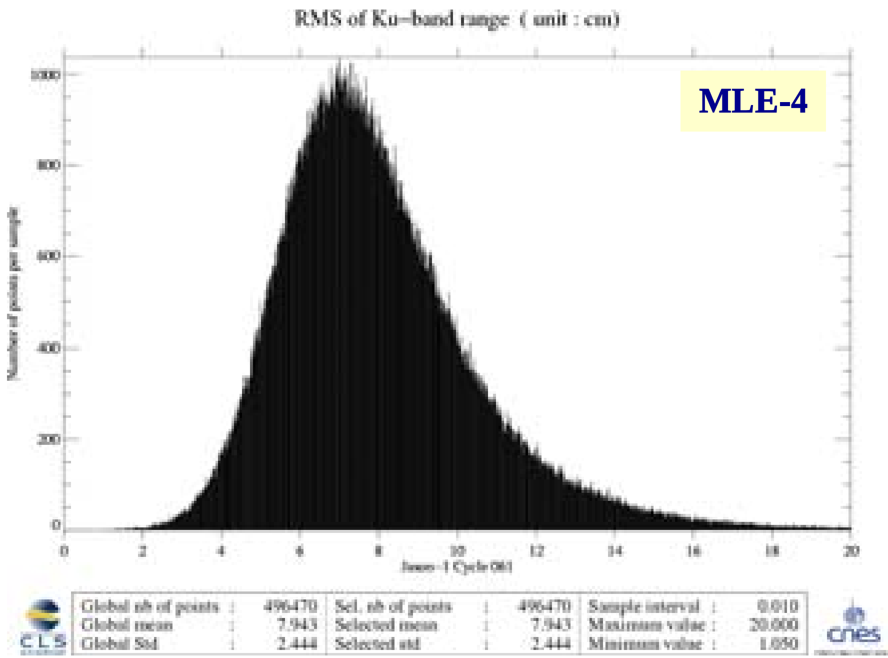
No evolution of the MQE parameter



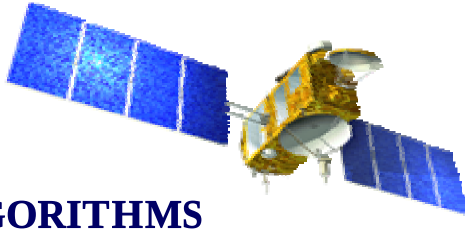
COMPARISON OF MLE-3 and MLE-4 RETRACKING ALGORITHMS (cycle 61)

RMS of Ku-band Range



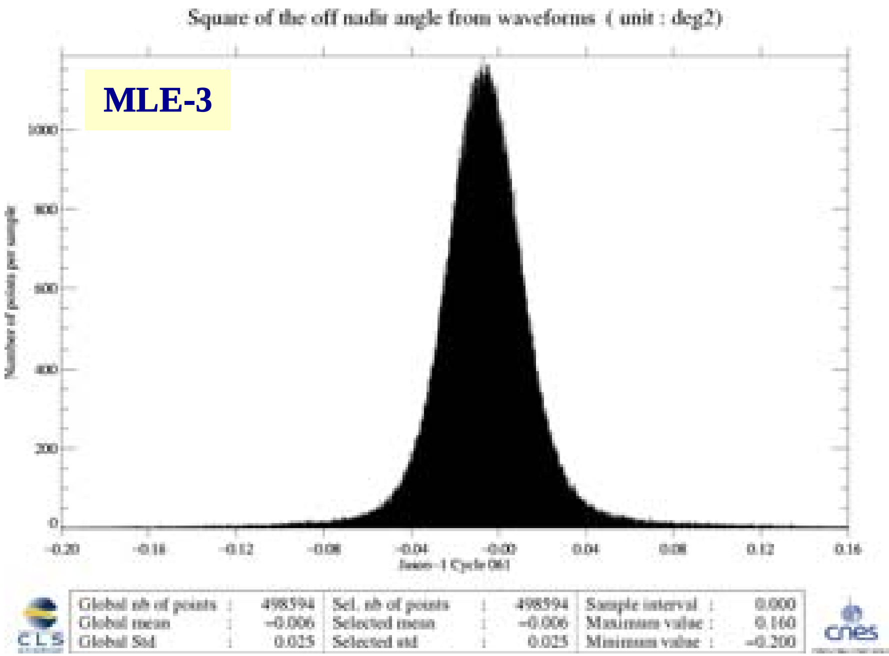


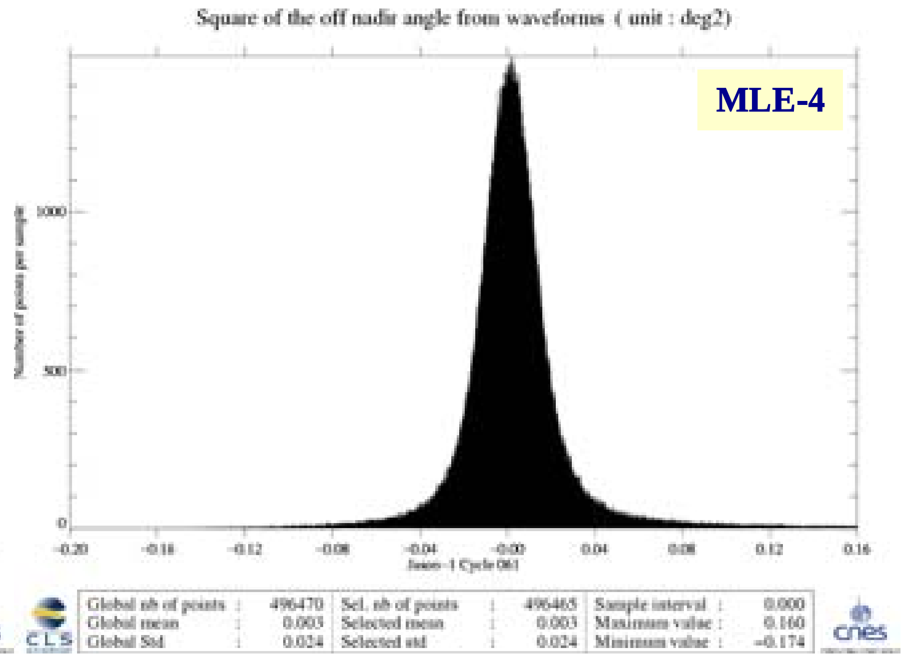
RMS_MLE4 - RMS_MLE3 = 0.6 cm



COMPARISON OF MLE-3 and MLE-4 RETRACKING ALGORITHMS (cycle 61)

Mispointing Angle

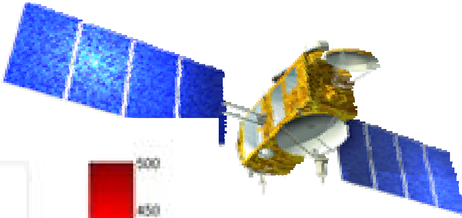




ξ^2_MLE4 - ξ^2_MLE3 = 0.009 deg^2

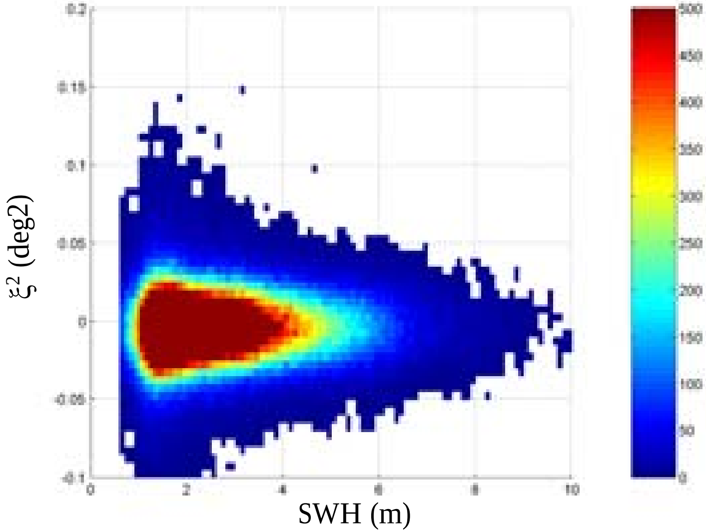
ξ_MLE4 - ξ_MLE3 = 0.095 deg

Cycle061

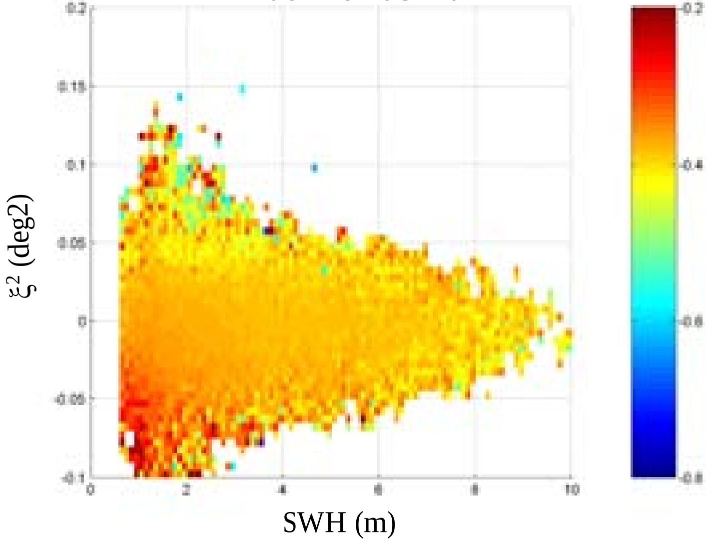


MLE-3

Nb of data per box

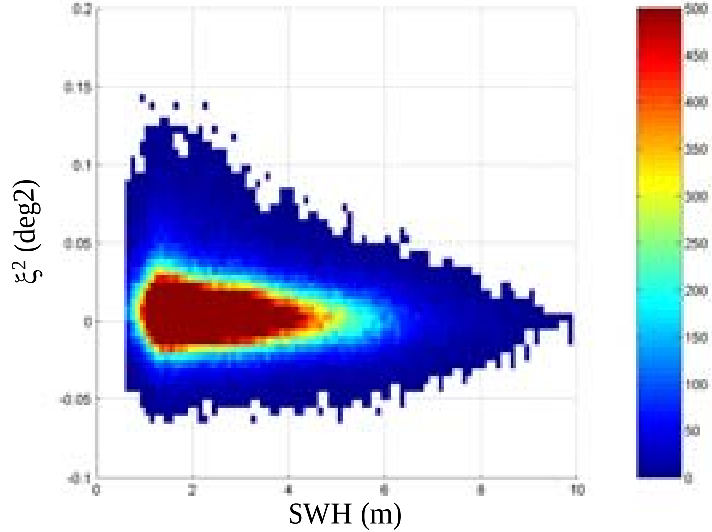


Track Point Shift

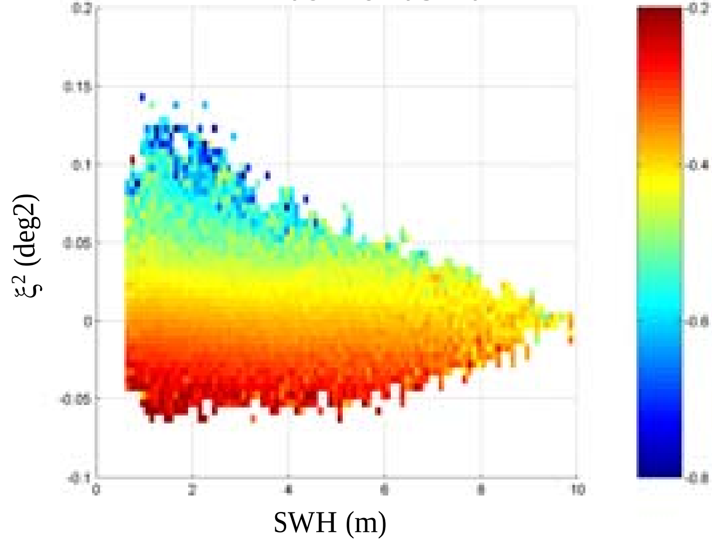


MLE-4

Nb of data per box

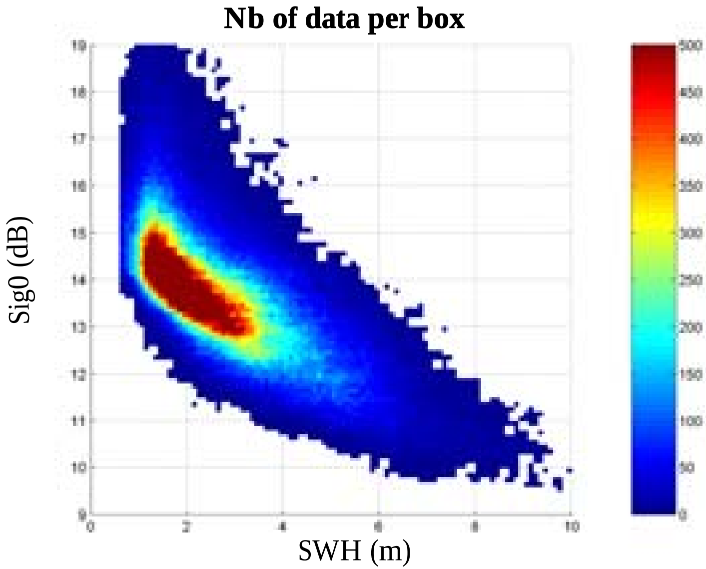


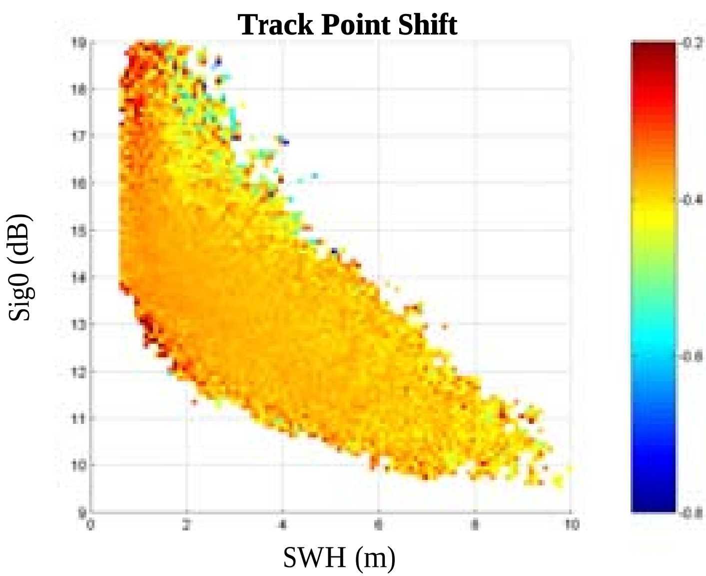
Track Point Shift



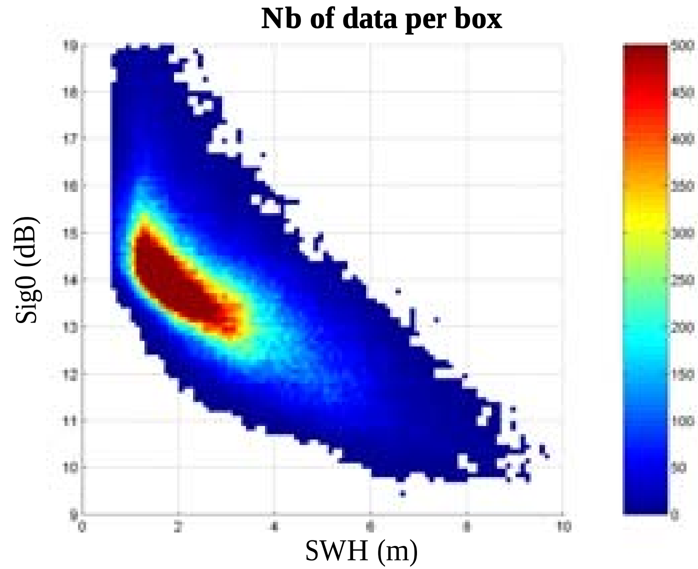
Cycle061

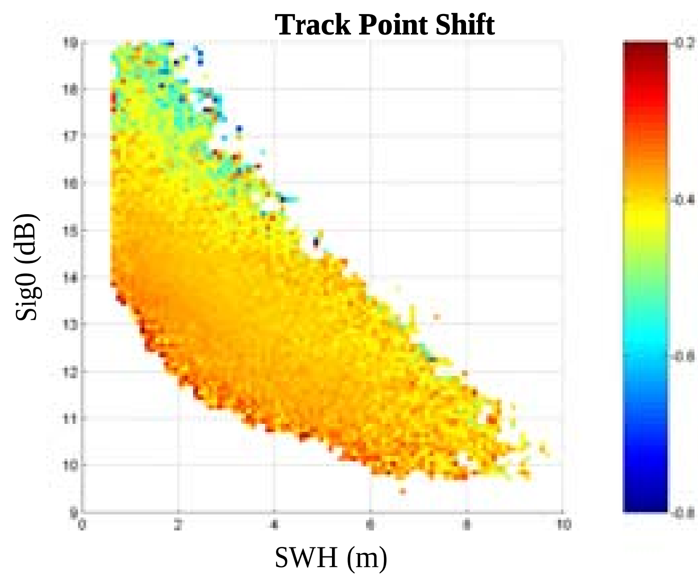
MLE-3

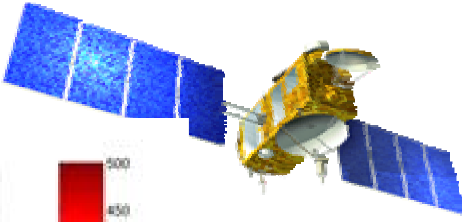




MLE-4

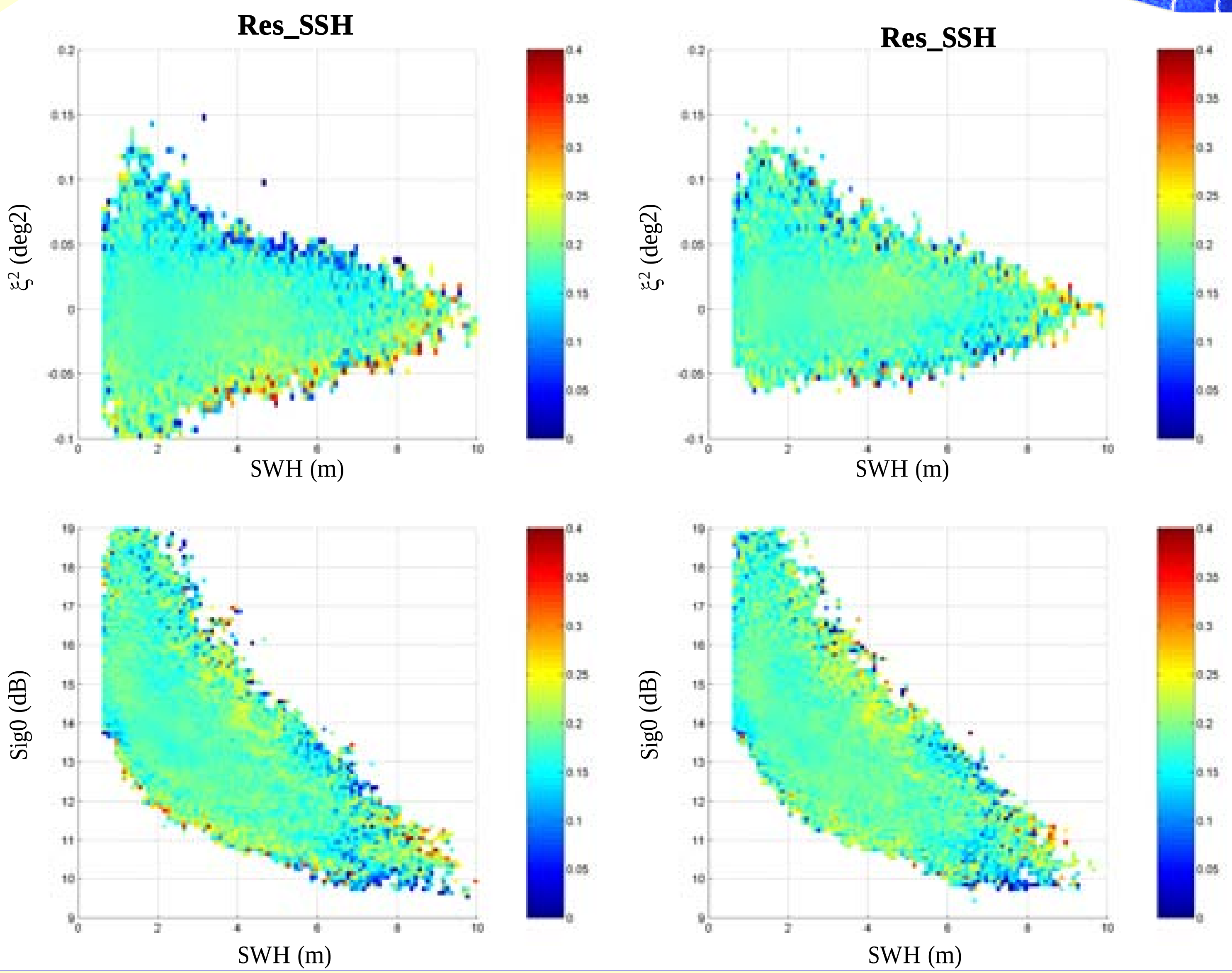


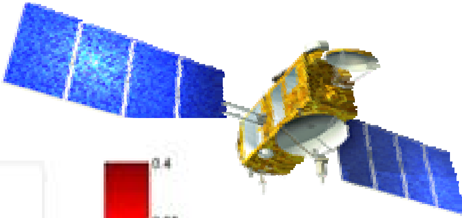


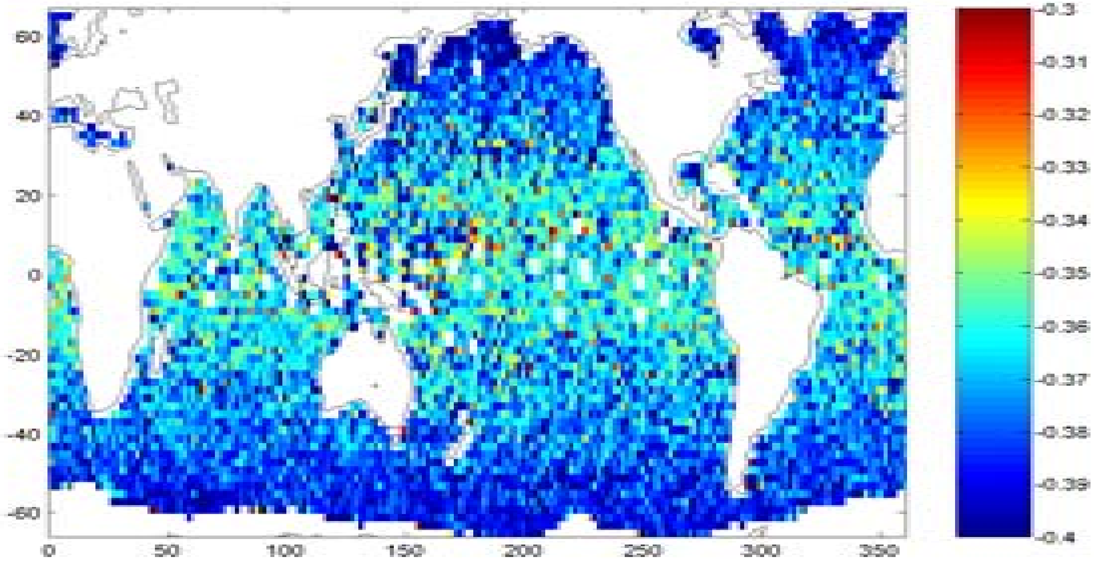


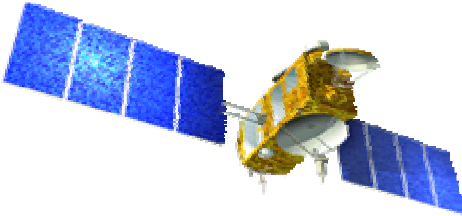
Cycle061

MLE-3 | MLE-4



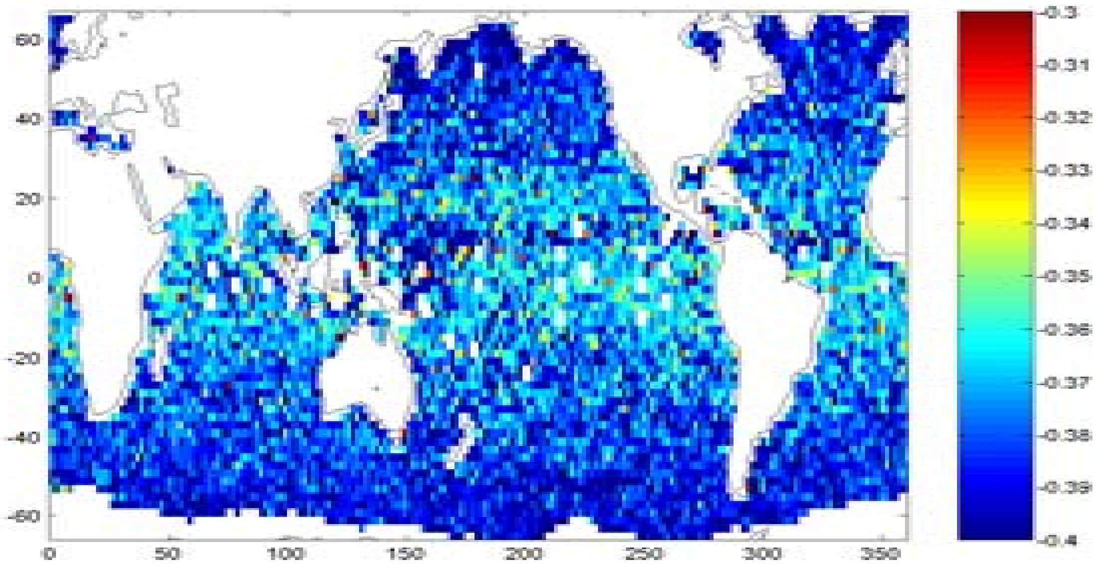






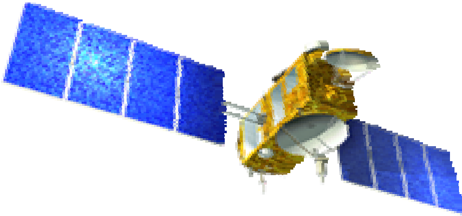
Track Point Shift

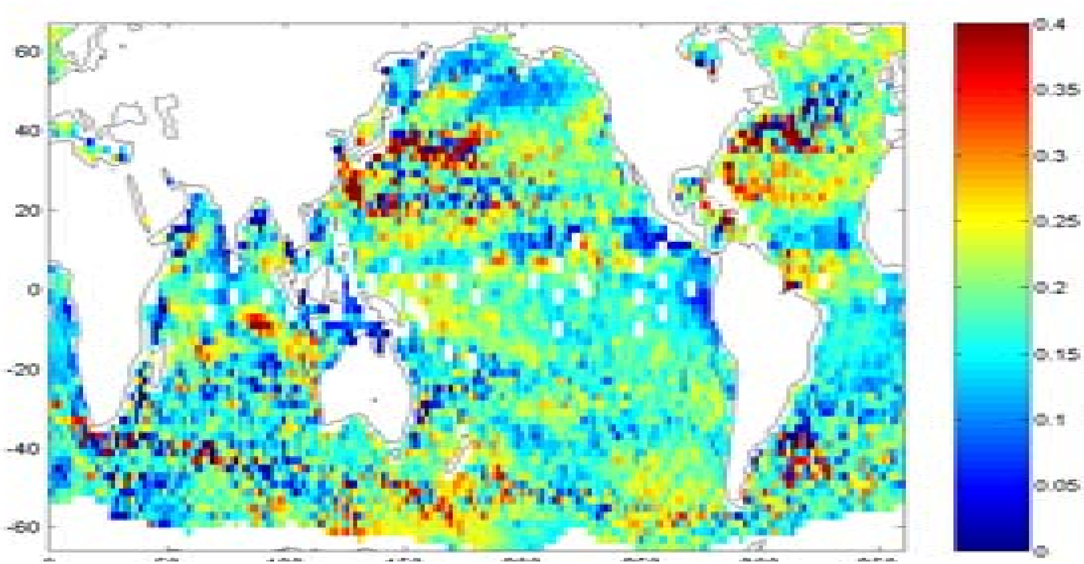
MLE-3



Track Point Shift

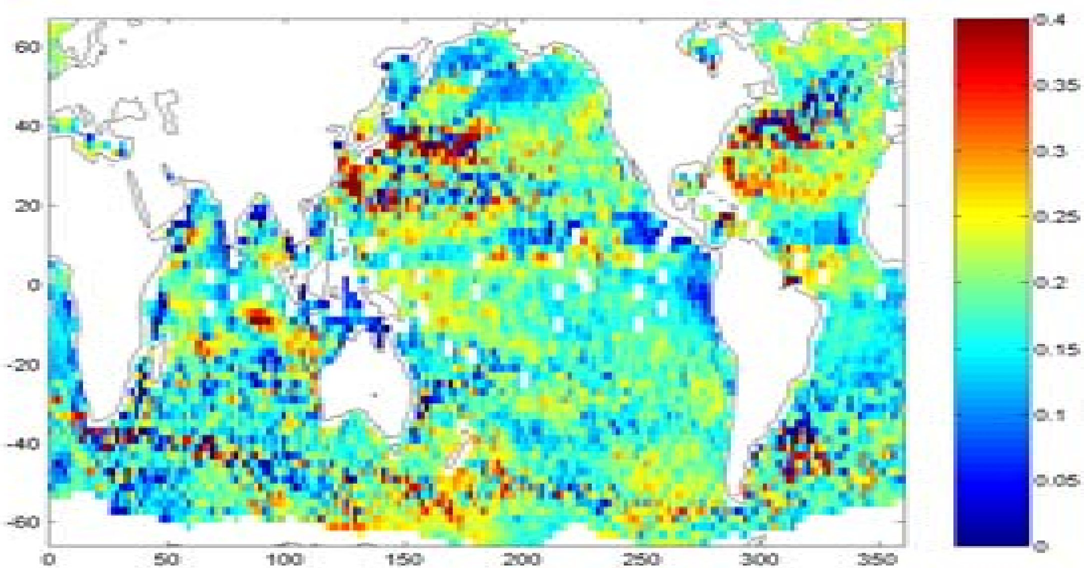
MLE-4





SSH-Res

MLE-3

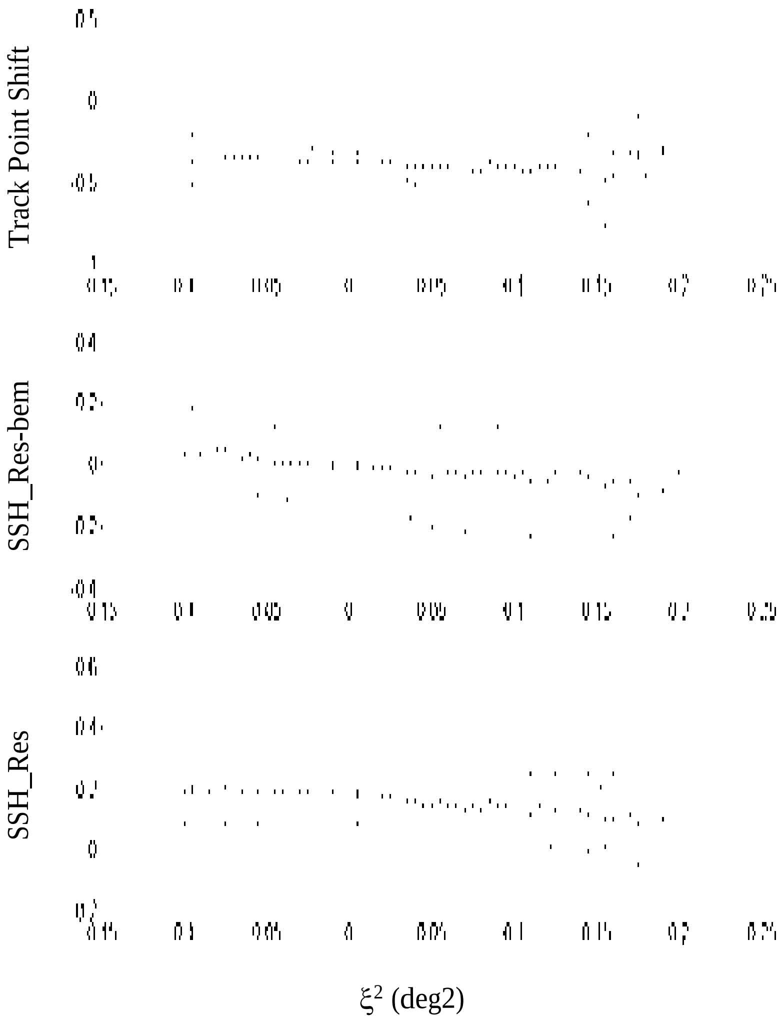


SSH-Res

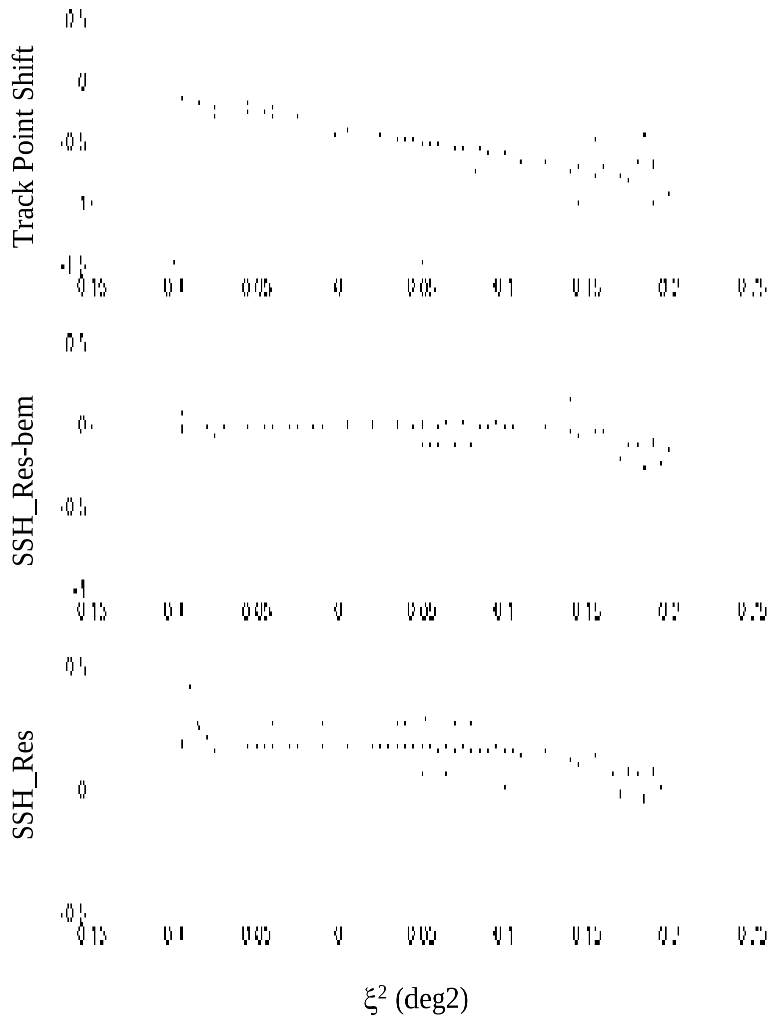
MLE-4

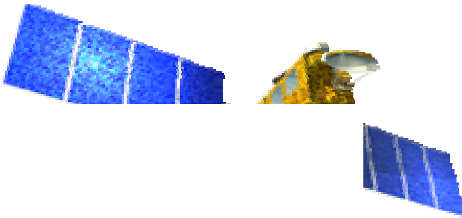
Cycle061

MLE-3

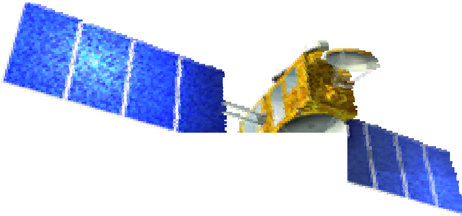


MLE-4

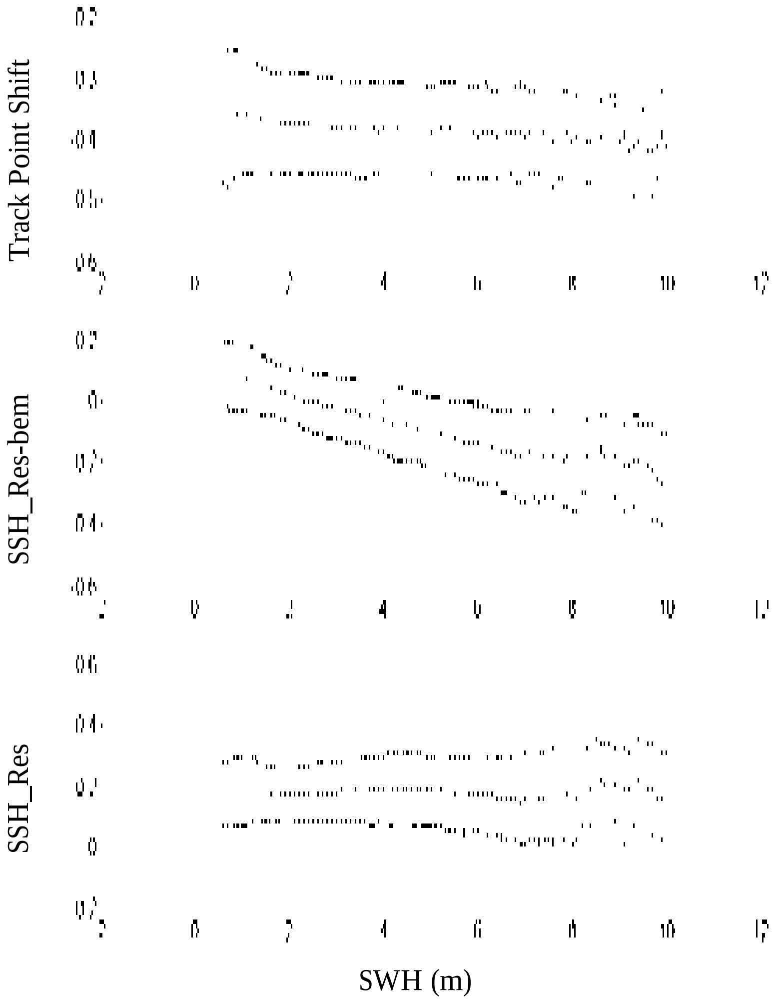




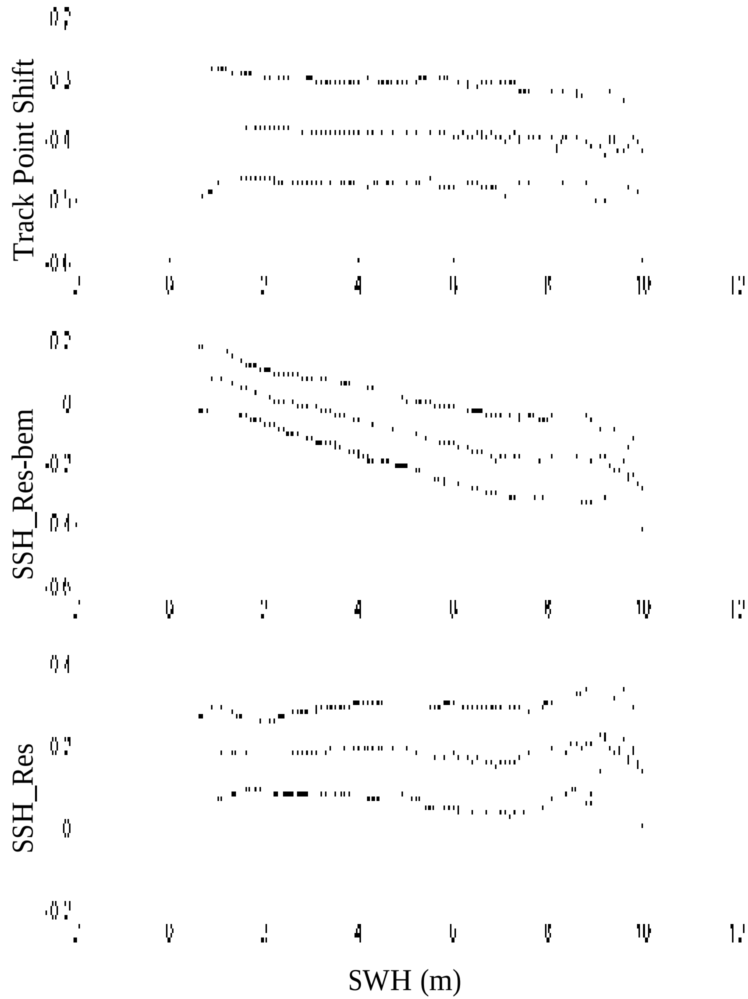
Cycle061

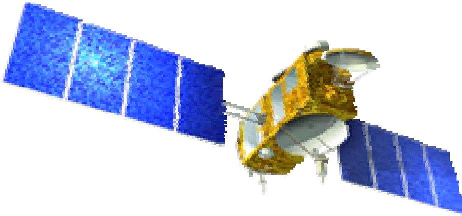


MLE-3



MLE-4





GENERAL CONCLUSIONS

- **A compact formulation has been proposed. It 's very close to the first order model formulation.**

- **It is « easy » to implement this formulation in the ground processing chains.**

- **This model is valid up to 0.7°. (instead of 0.3° for 1st order model)**

- **This formulation has been used to estimate altimetric parameters (range, SWH, PUI et ξ^2) for high mispointing angles values. Results obtained with this new formulation have been analysed and are good and coherent with those obtained by simulation**