Optimal filtering of mean dynamic topography models obtained using GRACE geoid models.

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ABSTRACT
Least squares collocation is an estimation technique where discretely located observations of different kinds can be integrated. The technique allows a rigorous description of the full covariance associated with signal, the errors as well as the estimated quantities. In this presentation error covariances associated with the gravity field and the ocean dynamic topography are analysed and described. The multi-disciplinary project "Geoid and Ocean Circulation in the North Atlantic (GOCINA)" aims at enhancing the capacity in Earth observation using data from the European Space Agency missions ENVISAT and GOCE. Two examples of the technique used on GOCINA data are presented One example where the standard GOCE gravity field models are used and one example where GOCE data are used directly in least squares collocation to obtain an optimal product. In this study the techniques are applied to enhance the estimation of the Mean Dynamic Topography using the high resolution Mean Sea Surface KMS04 and the geoid model GGM02S from GRACE. Especially, for modeling marine geodetic quantities with incomplete global coverage the methods have its advantages compared to a regular expansion into spherical harmonic functions.

1 Background

Least squares collocation has been used widely for gravity field determination (e.g. Moritz, 1980). The method has mainly been used in regional computations since the computational effort in inverting the equation system is quite big and depends on the number of observations. However, the technique has it strengths because different discrete data types may be integrated and their full signal and error characteristics are taken rigorously into account. Tscherning (2004) has developed the technique further and applied it to GOCE data (both SST and SSG) and demonstrated its value.

In global analyses of the gravity field quantities are expanded into spherical harmonic functions. This is a well known and efficient method to condense and synthesize the information from a large set of observations. The technique has its drawbacks because it is difficult to model high resolution fields where higher degree and order functions are required. The number of equations depends on the number of coefficients which depend on the harmonic degree and order up to which the expansion is going. Hence, the computational effort in inverting the equation system becomes too big when a high resolution expansion is needed.

In analyses of the ocean tides and ocean mean dynamic topography the use of global expansions into spherical harmonic functions have demonstrated its disadvantages. The main problem occurs because the oceans only cover a subset of the sphere. Hereby, the nice properties of the spherical harmonic functions disappear. They are no longer orthogonal and even for low degree expansions the equation systems become singular.

The main focus in this study is to test the least squares collocation and its representation of the quantities using representers on the determination of the mean dynamic ocean topography.

2 Least Squares Collocation and Representers

The least squares and minimum norm estimation technique called least squares collocation is used in this study. That is a method where an estimate of a quantity such as the geoid is obtained using the following expression

$$x = C_x^T \left(\mathrm{C} + \mathrm{D}\right)^{-1} \mathrm{y} \tag{1}$$

where C and D are covariance matrices associated with the signal and the errors of the observations *y*. *x* is the estimated quantity. The errors and error covariances were are using

$$\hat{c}_{x'x} = c_{x'x} - C_x^T \left(\mathrm{C}+\mathrm{D}\right)^{-1} \mathrm{C}_{x'} \tag{2}$$

where *cxx/* is the *a-priori* (signal) covariance between *x* and *x/* (see e.g. Moritz, 1980).

By splitting up Eq.(1) and defining the vector **b** as the representers expressed as

$$\mathrm{b} = \left(\mathrm{C} + \mathrm{D}\right)^{-1} \mathrm{y} \tag{3}$$

we obtain

$$x = C_x^T \mathrm{b} = \sum_{i=1}^{N} b_i K(P,Q_i) \tag{4}$$

where the estimated quantity *x* in a point *P* now has been expressed by a sum of a series of the coefficients, or representers *bi*, multiplied by the reproducing kernel *K(P,Qi)* associating the estimate with the observations. The reproducing kernel is an expression for the covariance function.

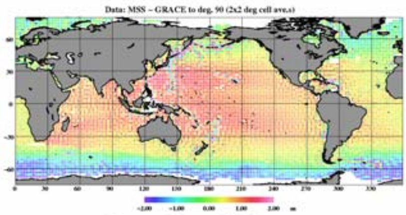
Hence, the estimates are expressed as a linear combination of a set of base functions which in this case are associated with the observations and not with some set of functions such as spherical harmonic functions. However, the spherical harmonic functions are fully included in the modeling of the reproducing kernel, as it is described in the next section.

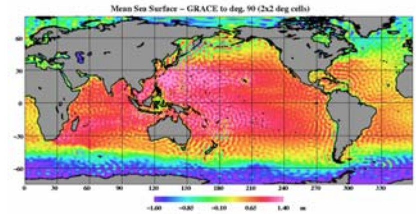
3 Geoid covariance modeling

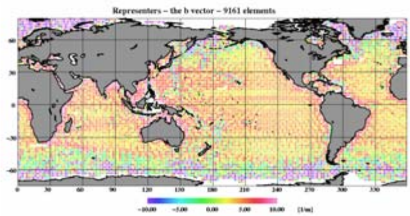
Using spherical harmonic functions signal and error covariances associated with the gravity field between points P and Q may be expressed as a sum of Legendre's polynomials multiplied by degree variances. That is

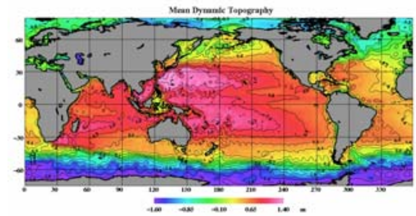
$$K(P,Q) = \sum_{i=2}^{\infty} \sigma_i^{TT} \mathrm{P}_i(\cos\psi) \tag{5}$$

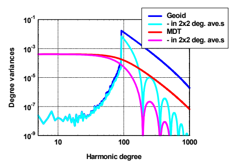
where are degree variances associated with the anomalous gravity potential field and *ψ* is the spherical distance between P and Q. Expressions associated with geoid heights and gravity anomalies are obtained by applying the respective functionals on *K*(P,Q), e.g. *CNN*=*LN*(*LN*(*K*(P,Q))) (more on collocation by Sansò, 1986, Tscherning, 1986).

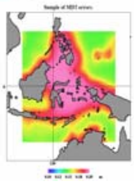


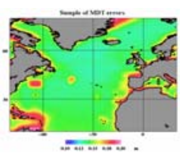


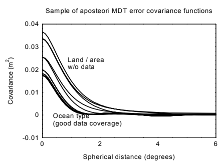












The determination of the degree variances is essential to obtain reliable and useful signal and error covariance functions. For the gravity field it has been accepted that the degree variances tend to zero somewhat faster than *i-3* and that the Tscherning-Rapp model (Tscherning & Rapp, 1974) may be used as a reliable model. When a spherical harmonic expansion of the gravity field up degree and order *N* has been used as a reference model and, hereby, been subtracted from the quantities, then the associated error degree variances should enter the expression, eq. (5), up to harmonic degree *N*. That is

$$\sigma_i^{TT} = \begin{cases} \varepsilon_i & i = 2,\ldots,N \\ \dfrac{A}{(i-1)(i-2)(i+4)} \left(\dfrac{R_B^2}{R^2}\right)^{i+1} & i = N+1,\ldots \end{cases} \tag{6}$$

where *A* = 1544850 m4/s4, *RB* = *R* – 6.823 km were found in an adjustment so that agreement with empirical covariance values calculated from marine gravity data was obtained. The error degree variances, ε*i*, are associated with the errors of the reference model from GRACE. The degree variances are shown in Figure 1.

4 Mean Dynamic Topography Covariance Modeling

To get reliable results of simulations and tests carried out using least squares methods it is important that both the signal and the error characteristics have been taken into account. In least squares collocation that means that the covariance function models should agree with empirically determined characteristics such as the variance and correlation length. In analysis of errors formally estimated using eq.(7), it is very important that those quantities are reliable. That is also the case when MDT errors are analysed. Hence, a model describing the magnitude and the spectral characteristics of the MDT is needed.

A kernel function associated with the MDT, may be expressed in a similar manner as the gravity fields as

$$C_{\zeta\zeta} = \sum_{i=1}^{\infty} \sigma_i^{\zeta\zeta} \mathrm{P}_i(\cos\psi) \tag{7}$$

where the degree variance in this expression are associated with the MDT, naturally.

The degree variance model was constructed using 3rd degree Butterworth filters combined with an exponential factor (e.g. Knudsen, 1991). Hence, the spectrum of the MDT is assumed to have similar properties as the geoid spectrum; same type of smoothness and infinite. That is

$$\sigma_i^{ss} = b \cdot \left(\frac{k_2^3}{k_2^3 + i^3} - \frac{k_1^3}{k_1^3 + i^3}\right) \cdot s^{i+1} \tag{8}$$

where *b*, *k*1, *k*2, and *s* are determined so that the variance and the correlation length agree with empirically derived characteristics. This resulted in the model where *b* = 6.3 10-4 m2, *k1* = 1, *k2* = 90, *s* = ((*R*-5000.0)2/*R*2)2. The variance and correlation length are (0.20 m)2 and 1.3° respectively. The variance and correlation length of the current components are (0.16 m/s)2 and 0.22° respectively.

4 Results

The least squares collocation and its representation of the quantities using representers is tested on the determination of the mean dynamic topography. This is done using a set of observations that has been derived using the mean sea surface KMS04 and a geoid computed from the GMM02S coefficients up to harmonic degree and order 90. The differences forming a so-called residual mean sea surface, or an estimate of the dynamic topography, were averaged in cells of 2 by 2 degrees covering the global oceans (see Figure). This averaging eliminate most of the higher degree geoid signal without damaging the dynamic topography below harmonic degree 90 (see Figure 1).

Then the b-vector (eq. 3) was computed using proper covariance functions associated with the residual geoid and the mean dynamic topography both averaged in 2 by 2 degrees cells (see Figure).

Based on the representers in the b-vector and the same covariance functions the observations were reproduced in an estimation of the sum of the residual geoid and the topography averaged in 2 by 2 degree cells using eq. 4. The RMS values of the observations, the estimated values and the residuals were 0.91 m, 0.87 m, and 0.13 m respectively. The grid is shown in Figure. Note the dominating geoid signal at wavelengths of about 4 degrees (roughly corresponding to harmonic degree 90). Note also the underlying well known features of the topography.

The using a proper cross covariance function the full mean dynamic topography was estimated using eq. 4 (see Figure). Note how well the geoid residuals have been eliminated. The estimated topography has been optimally filtered and give full resolution in a compromise with smoothness within the signal to noise relations in a rigorous manner balancing the least squares and the least norm criteria.

5 Perspectives

The impact of the GOCE satellite mission on the recovery of the gravity field has previously been analysed for two simulated cases by Knudsen and Tscherning (2005). In the first case the GOCE Level 2 product is used where the gravity field is approximated by spherical harmonic coefficients up to degree and order 200. In the second case synthetic Level 1B GOCE data are used directly in a gravity field determination using least squares collocation. In case two the full spectrum geoid error was improved from 31 cm to 15 cm and the resolution was doubled. The results are important for the future users of GOCE that need the extra accuracy.

For the estimation and representation of the Mean Dynamic Topography the results from this study demonstrate that the collocation approach with its representers perform very well. Since only about half of the sphere is covered the number of averaged observations corresponds to the number of harmonic coefficients that provide the same resolution. Hence, the computational efforts are similar. Furthermore, the full signal and error characteristics are taken into account. This result in an optimal filtering of the data where aliasing caused by be truncation is avoided.

Finally, the major advantage is that full spectrum error covariances associated with the estimated topography are rigorously obtained. This is of crucial importance for the assimilation of altimetry into ocean circulation models.

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PS: Please note that this is a demo. The data coverage in the Gulf Stream area, Indonesia, Caribbean, and the Arctic should have been checked more carefully. No time for re-runs – sorry.

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