Lamont-Doherty Earth Observatory of Columbia University

# POWER SPECTRAL PARAMETERIZATIONS OF ERROR AS A FUNCTION OF RESOLUTION IN GRIDDED ALTIMETRY MAPS


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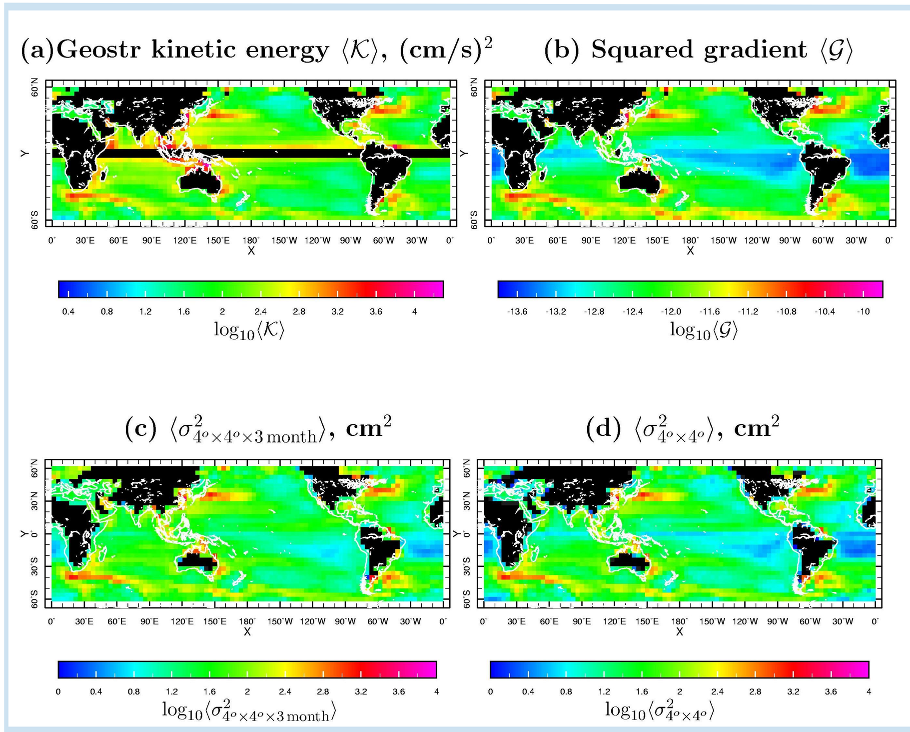
## 1. INTRODUCTION

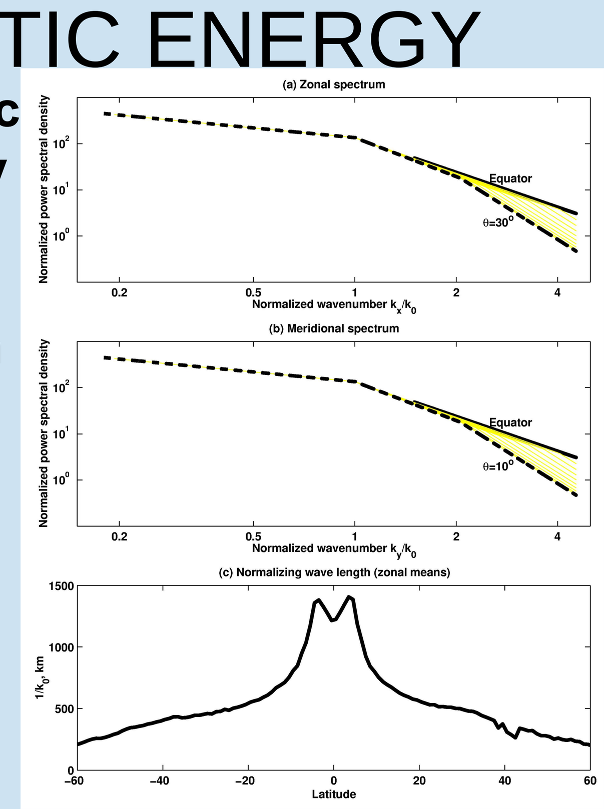
**Data assimilation procedures interpret observed data as if they could be expressed in terms of the averages over model grid box areas. In reality, however, observations are either point-wise values (in cases of in situ data) or averages over certain footprints (in cases of satellite data). Therefore the difference between observations and model values ought to reflect the influence of the small-scale variability of the observed physical field, because this variability is getting averaged differently by the model grid and by the observational system. This difference turns out to be a major contribution to the effective data error and needs to be taken into account in data assimilation procedures. Multi-satellite missions to date have resulted in satellite altimetry fields of unprecedented resolution which, in turn, make it possible for us to obtain detailed descriptions of small-scale and short-term variability of sea surface height. Data error models suitable for use in data assimilation procedures were developed. They are verified by comparing satellite altimetry analyses with in situ (tide gauge) data.**

**Data used:** Multimission altimetry analyses (Cheney et al. 1994, Ducet et al. 2000 products, DUACS gridded products);
Tide gauge data from University of Hawaii;
Sea surface height from POCM 4C ¼ degree resolution model (Tokmakian and Challenor, 1999).

## 2. SMALL-SCALE VARIABILITY AND EDDY KINETIC ENERGY

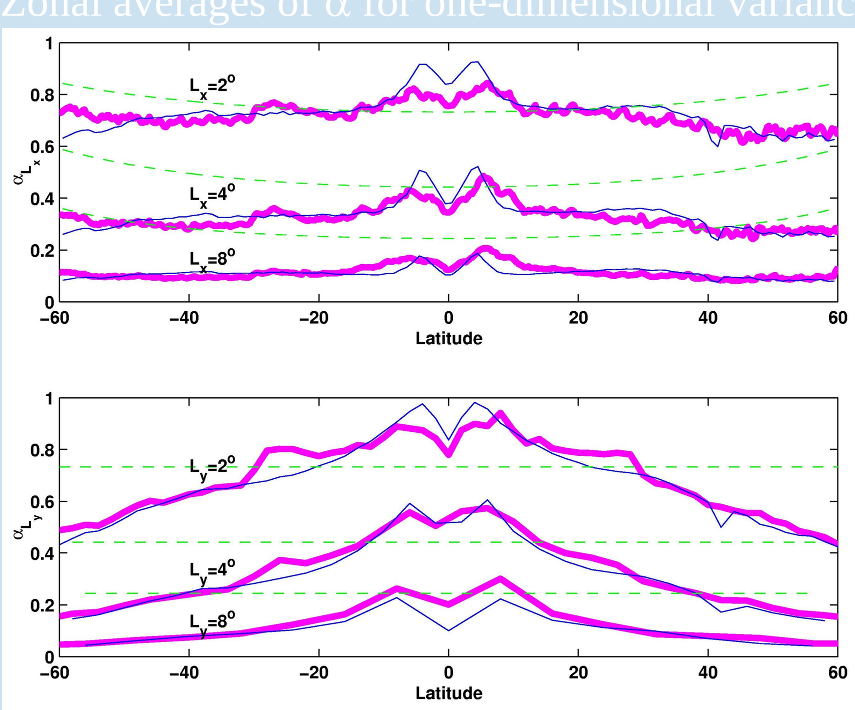



**Connection between surface geostrophic kinetic energy and small-scale variability (left panel) in sea surface height:**
**$\langle\sigma^2\rangle = C(f/g)^2 \langle K\rangle$,**
**where $C=\alpha (L_x^2+L_y^2)/6$, and $\alpha$ depends on the wavenumber power spectrum of the sea surface height. Parameter $\alpha$ shows how small differences in sea surface height scale to the $L_x X L_y$ box. Stammer et al. (1997) midlatitudinal and tropical spectral approximations are spliced together for the use in this work (right).**




## 3. FORMALISM VALIDATION

Zonal averages of $\alpha$ for one-dimensional variances



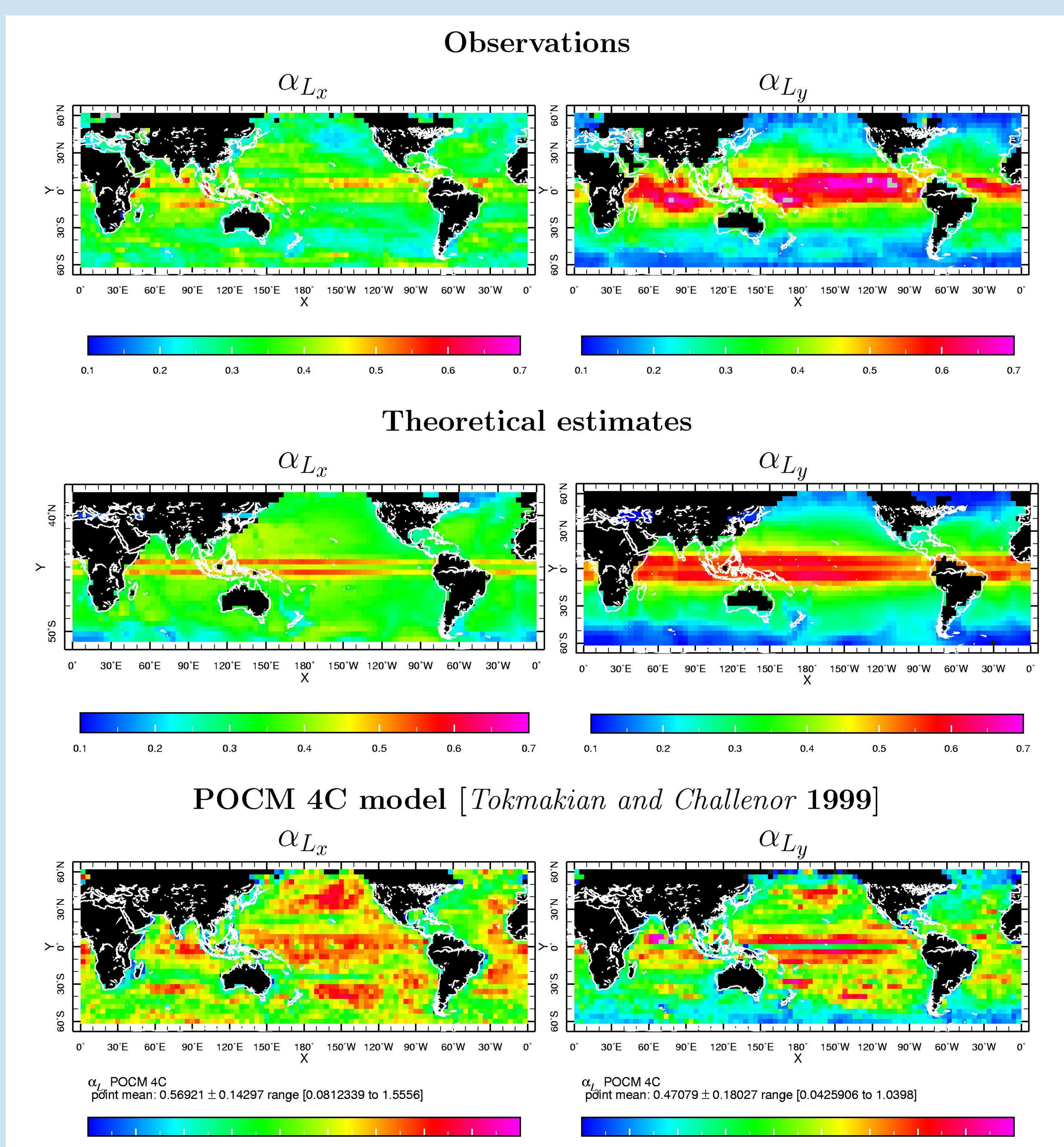

Observations (magenta)

Theoretical values for wavenumber spectral approximations

Stammer (1997) – blue

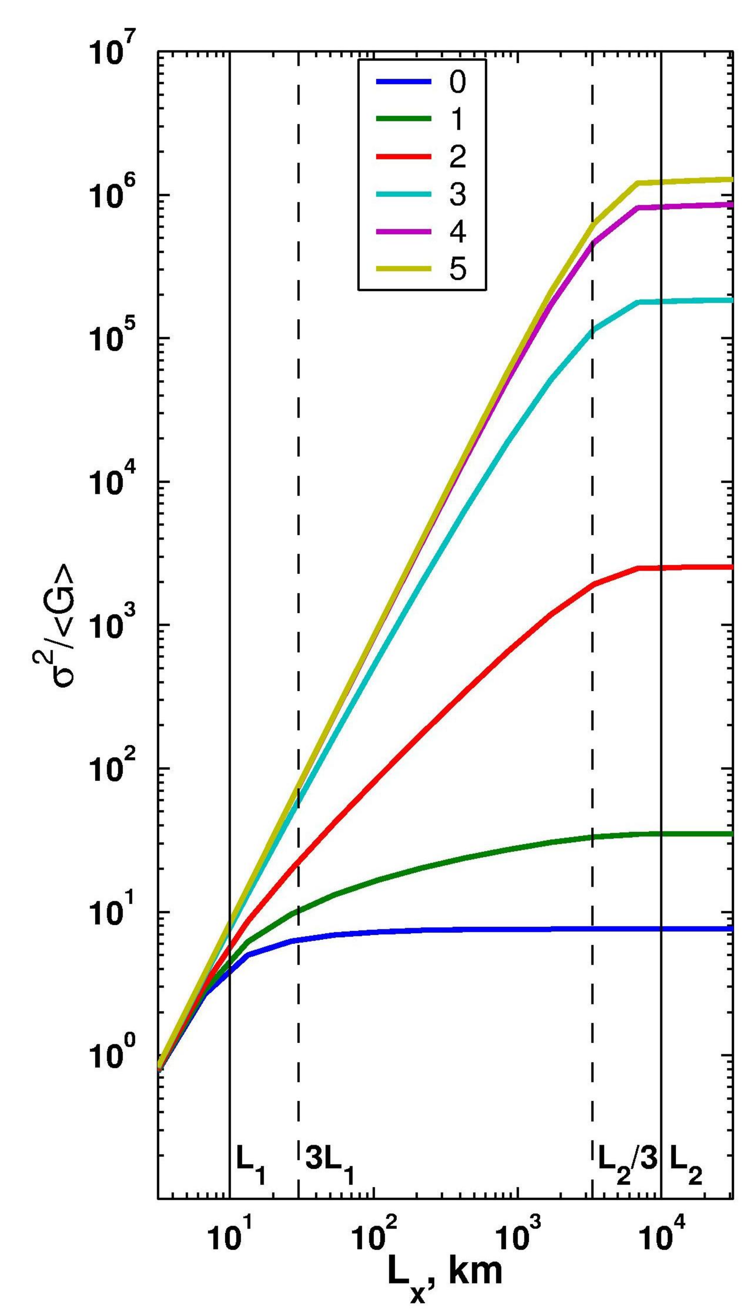
Zang and Wunsch (2001) - green

### Spatial patterns of one-dimensional coefficients




## 4. SCALING FOR POWER-LAW SPECTRAL FORMS:

$$P(k)=A/k^n$$




**As n varies from 0 to infinity scaling of the ratio between variability under scale L and squared sea surface height gradient changes between $L^0$ and $L^2$. Scaling power stays at 2 for n values larger than 4.**

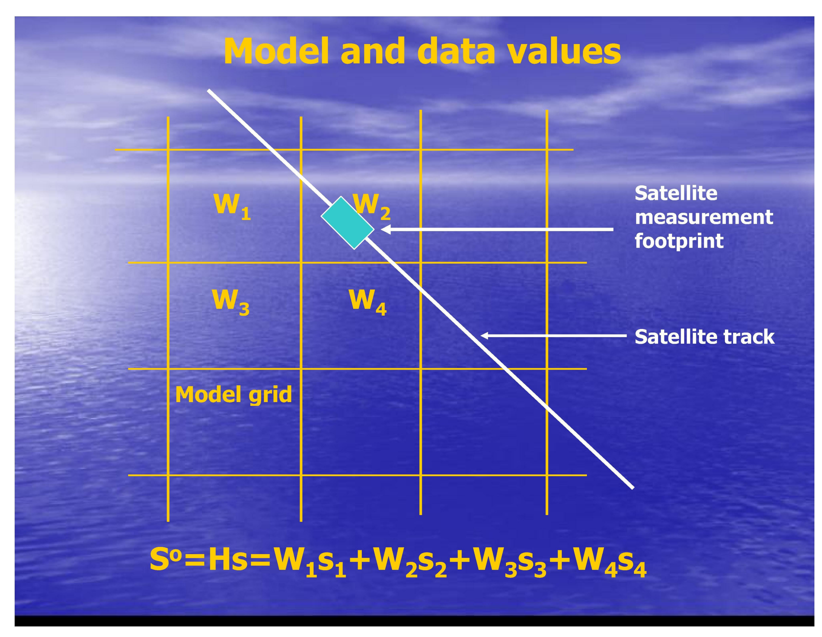
## 5. Definition of Representation Error

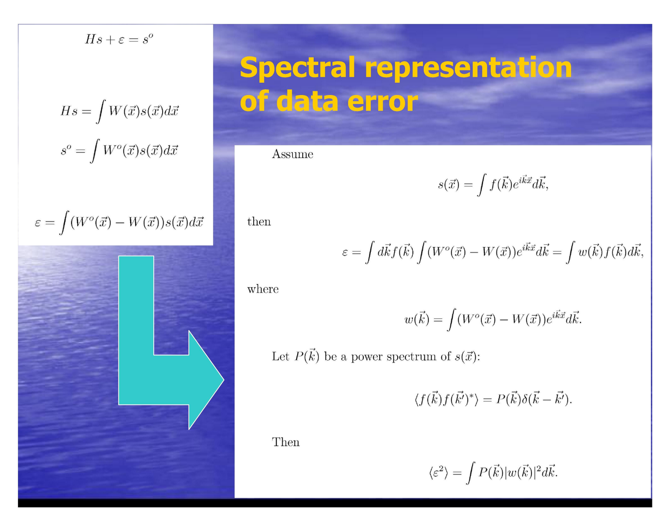
The goal of this talk is to address data error modeling for data assimilation purposes, to reflect the difference in averaging of physical field by the model grid and observing systems.

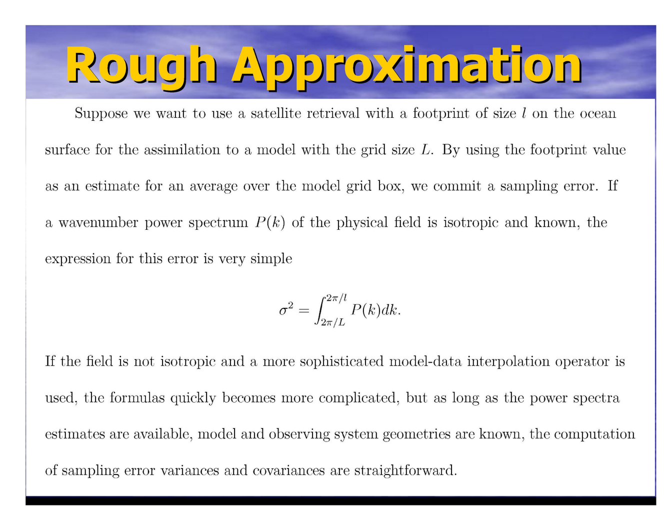
Consider a typical situation in the ocean modeling:
Grid resolution ï 30km x 60km,
Sea surface height altimetry data ï 7km footprint,
Sea surface temperature ï 1-4-25km averages, depending on the product.
In situ observations ï local.

What is the error of the data with regards to the model grid values? It needs to be specified for the assimilation procedures.

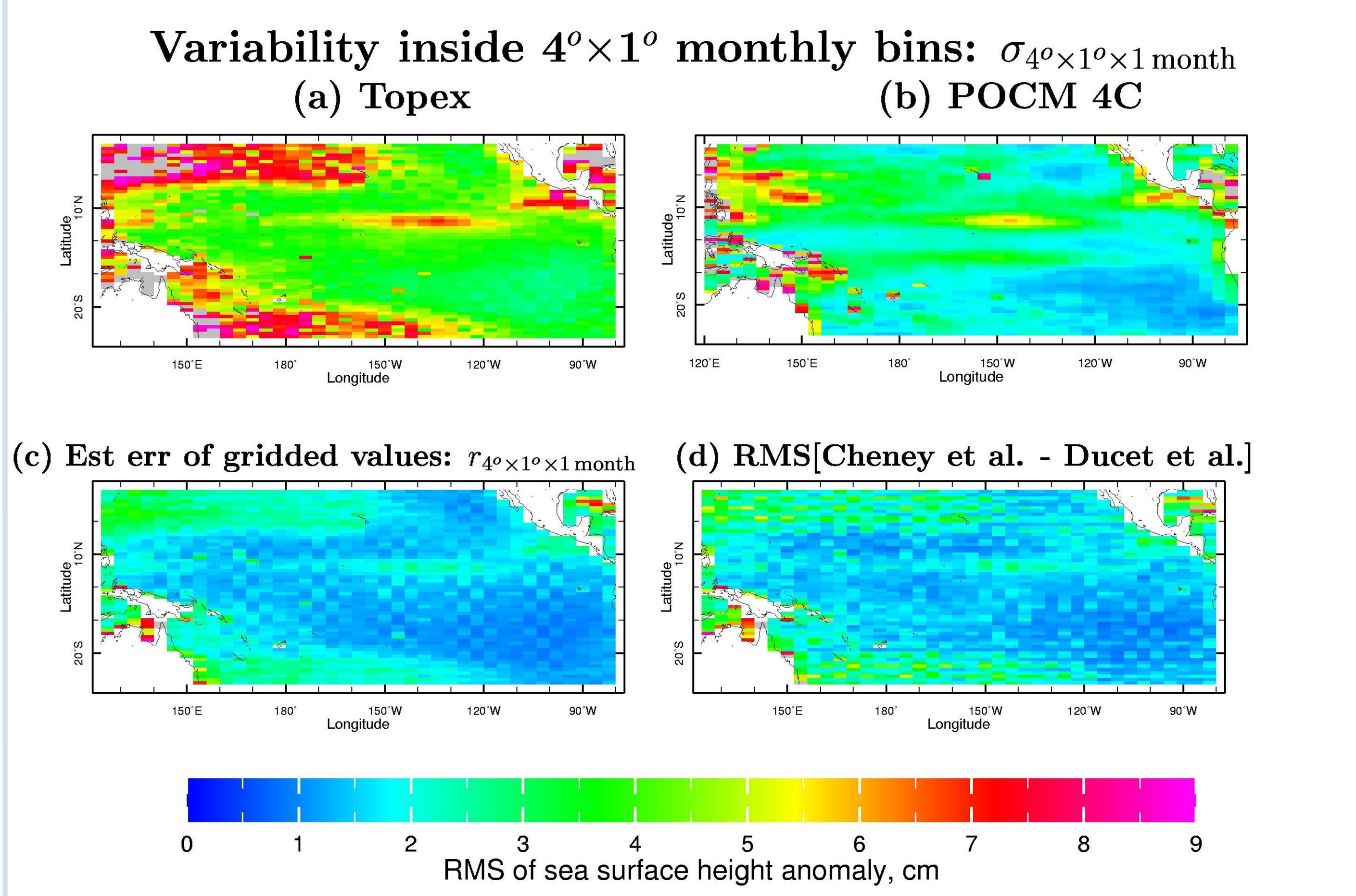
In addition to measurement error of the data, we need to take into account the error due to the difference in averaging of the physical field by the model and by different types of the observing systems.

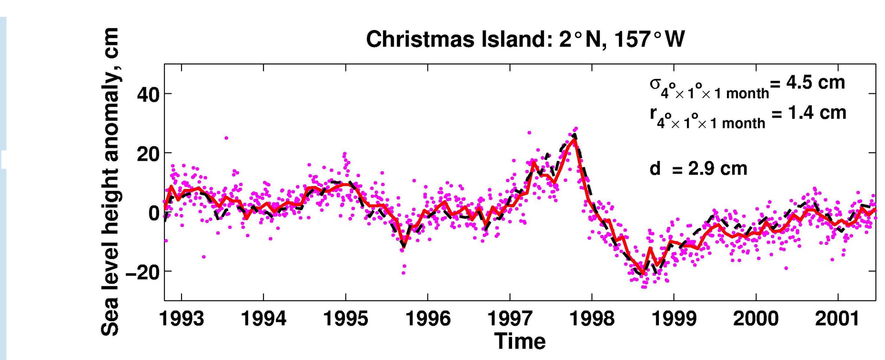


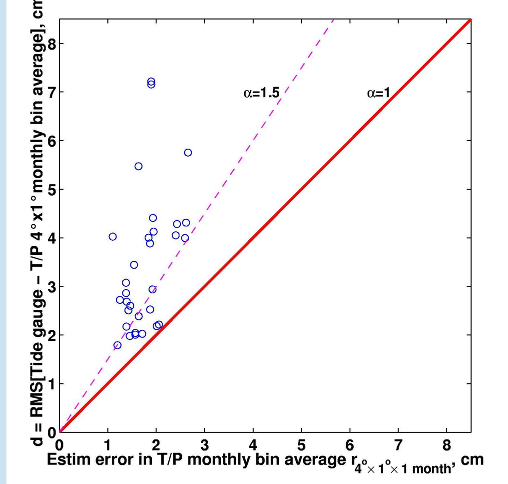



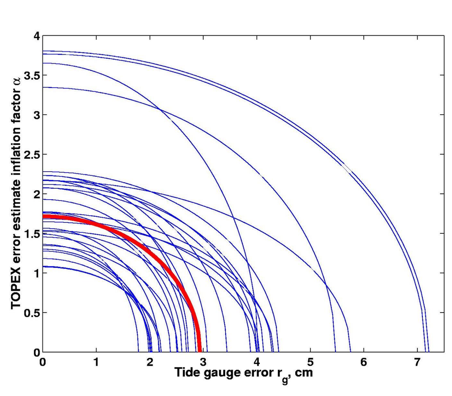




### Small-scale variability and gridded altimetry error





## 6. ERROR MODELS FOR ALTIMETRY AND TIDE GAUGES

**Validation of T/P error estimates by comparison with the tide gauge records, October 1992 -- March 2001. The top panel compares monthly tide gauge sea level height anomalies at Christmas Island (dashes) with altimetric measurements from the corresponding gridbox (centered at 2N and 158W) of *Cheney et al.* [1994] T/P product. Dots show values from individual altimetry passes, and the solid line shows their monthly averages for this gridbox. Temporal RMS values of the intrabox variability sigma inside the gridbox, the sampling error estimate r for the gridbox mean, and the RMS difference between the gridbox and tide gauge monthly means d are indicated as well. In the lower left panel, circles are differencesbetween 31 tide gauges and T/P bins. Differences would fall along the solid line if the only errors were the ``optimistic'' estimate of T/P errors. The dashed line inflates these optimistic estimates by a factor of 1.5. In the lower right panel, thin lines show constraints on the inflation factor alpha and tide gauge error r for individual tide gauges. The thick line shows the median constraint.**

## 7. RESULTS

1. Effective data error depend on the model resolution (and averaging intrinsic in individual observations). The richness of satellite data allows us to specify and use for error modeling spectral representations of assimilated fields. Error due to averaging difference may exceed the nominal measurement error.
2. We have derived a connection between eddy kinetic energy and small-scale sea surface height variability; a coefficient in this connection characterizes the sea surface height wavenumber spectrum; its values for a high-resolution model are drastically different from those computed for altimetry fields.
3. We analyzed the ratios of temporal and spatial contributions to the small-scale sea surface height variability. This ratio has proved useful for modeling effective error in tide gauge data.

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