Assimilation of Altimetry Data for Nonlinear Shallow Water Tides: Quarter-diurnal tides of the Northwest European Shelf, and the Atlantic Ocean

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Modeling of non-linear shallow water tides: time step (TS) shallow water equations (SWE):

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{f}\times\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} + \mathbf{F} + \mathrm{A_H}\nabla^2\mathbf{u} = -g\nabla(\eta - \eta_{EQ})$$

$$-\frac{\partial \eta}{\partial t} = \nabla\cdot H\mathbf{u} + \nabla\cdot\eta\mathbf{u}$$

Non-linear terms give rise to overtides (e.g., M_4) and compound tides (e.g., MS_4)

e.g., force at M_2 frequency, harmonically analyze at M_2 and M_4 frequencies

Linearized frequency domain (LFD) approach:

expand solution in multiples (and sums) of forcing frequencies,

$$\eta(x,t) = \sum_{n=-\infty}^{\infty} \tilde{\eta}_n(x)e^{in\omega t}$$

collect terms of common frequency, retaining only first order interactions→ linearized frequency domain equations for astronomically forced and non-linear constituents:

M_2

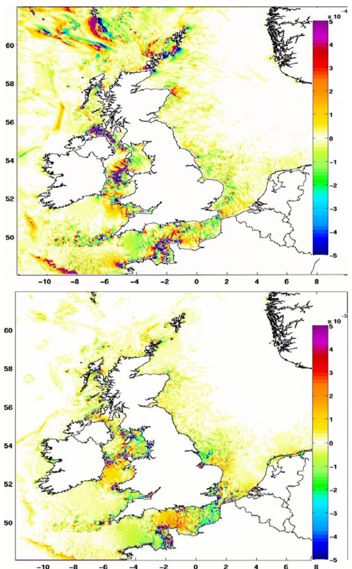
$$i\omega\tilde{\mathbf{u}}_1 + \mathbf{f}\times\tilde{\mathbf{u}}_1 + g\nabla\tilde{\eta}_1 + \tilde{\mathbf{F}}_1 + \mathrm{A_H}\nabla^2\tilde{\mathbf{u}}_1 = g\nabla\tilde{\eta}_{EQ}$$

$$\nabla\cdot H\tilde{\mathbf{u}}_1 + i\omega\tilde{\eta}_1 = 0 \qquad (1)$$

M_4

$$i2\omega\tilde{\mathbf{u}}_2 + \mathbf{f}\times\tilde{\mathbf{u}}_2 + g\nabla\tilde{\eta}_2 + \tilde{\mathbf{F}}_2 + \mathrm{A_H}\nabla^2\tilde{\mathbf{u}}_2 = -\frac{1}{2}\tilde{\mathbf{u}}_1\cdot\nabla\tilde{\mathbf{u}}_1$$

$$\nabla\cdot H\tilde{\mathbf{u}}_2 + i2\omega\tilde{\eta}_2 = -\frac{1}{2}\nabla\cdot\tilde{\eta}_1\tilde{\mathbf{u}}_1 \qquad (2)$$

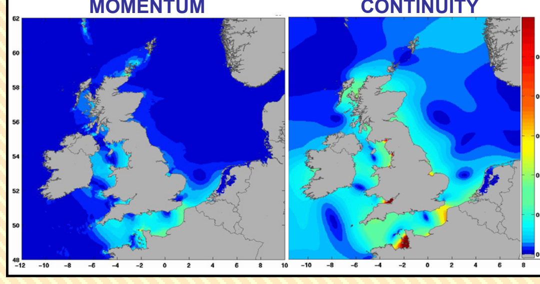


M_4 forcing momentum equation, derived from European Shelf 1/30° M_2 solution.

M_4 forcing continuity equation, derived from European Shelf 1/30° M_2 solution.

Non-liner constituent solutions calculated with linear constituent forcing

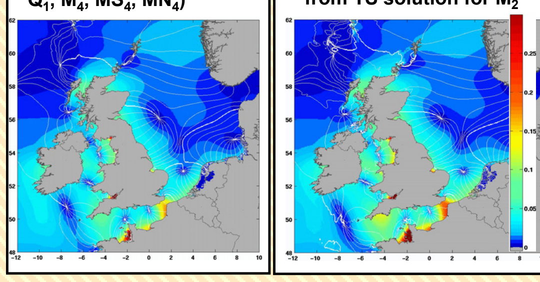
Amplitude of M_4 obtained by direct factorization (DF) of linearized SWE with non-zero M2 forcing from one of the equations only:



Continuity term dominates, but contribution of momentum forcing is also important.

Time stepping (TS) of NL SWE (constituents: M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_4, MS_4, MN_4)

Direct Factorization (DF) Solution of linearized SWE; forcing derived from TS solution for M_2



Solutions very similar → a feasible M_4 solution can be obtained by DF, using M_2 forcing.

Data Assimilation: frequency domain approach based on linearized, coupled systems of (1)-(2)

Write equations (1)-(2) as coupled system

$$\begin{bmatrix}\mathbf{A}_{11} & 0\\ \mathbf{N}_{21} & \mathbf{A}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\upsilon}_1\\ \boldsymbol{\upsilon}_2\end{bmatrix} = \begin{bmatrix}\mathbf{f}_1\\ 0\end{bmatrix} + \begin{bmatrix}\delta\mathbf{f}_1\\ \delta\mathbf{f}_2\end{bmatrix}$$

$$\mathbf{N}(\boldsymbol{\upsilon}) = \mathbf{f} + \delta\mathbf{f}$$

Forcing for M_{4C}; SWE for M_2, M_4; Astron. forcing (M2); Dynamical errors

data for both constituents

$$\mathbf{d}_i = \mathbf{L}_i\boldsymbol{\upsilon}_i + \boldsymbol{\varepsilon}_i \qquad i = 1,2$$

Minimize penalty functional for coupled (weakly non-linear) problem

$$J[\boldsymbol{\upsilon}] = (\mathbf{N}(\boldsymbol{\upsilon}) - \mathbf{f})^\dagger \mathbf{C}_\mathbf{f}^{-1}(\mathbf{N}(\boldsymbol{\upsilon}) - \mathbf{f}) + (\mathbf{d} - \mathbf{L})^\dagger \boldsymbol{\Sigma}_\mathbf{d}^{-1}(\mathbf{d} - \mathbf{L})$$

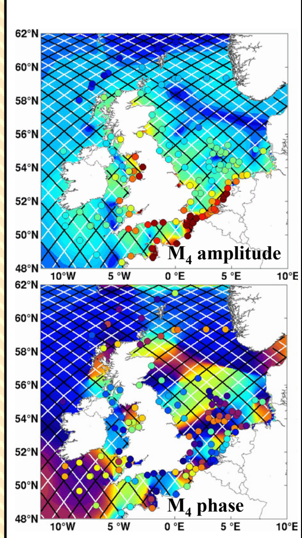
Coupled problem can be solved (approximately) with a two step procedure:

(1) solve linear inverse problem for M_2 (using only M_2 data); use inverse solution to compute forcing for M_4

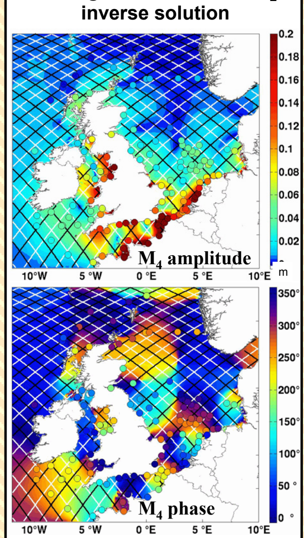
(2) Use this to compute "prior" for M_4, solve linear inverse problem (with appropriate covariance) using M_4 data

M4 solutions for the Northwest European Shelf: 1/30° grid

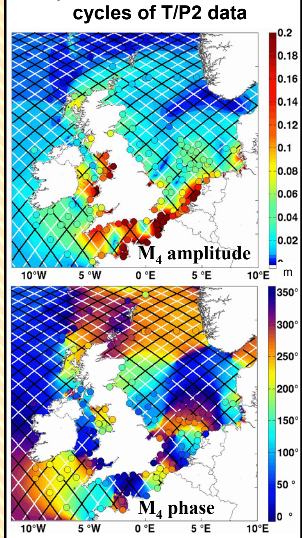
Time-stepping of NL SWE



Linearized SWE with M_4 forcing calculated from M_2 inverse solution



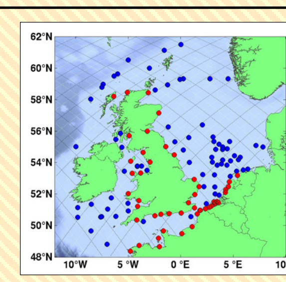
M_4 inverse solution: 531 cycles of T/P+J1 and 114 cycles of T/P2 data



Comparison to validation tide gauges

76 pelagic gauges

51 coastal gauges



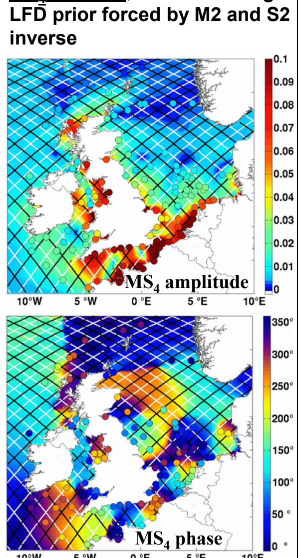
	M4		MS4		MN4	
	OW76	C51	OW76	C51	OW76	C51
RMS signal	4.71	11.42	2.91	7.49	1.77	4.04
(1) Time step solution	5.04	11.93	3.15	7.27	1.87	4.43
(2) LFD: M_4 prior, forcing from M_2 prior	3.26	7.10	1.94	4.29	1.24	2.52
(3) LFD: M_4 prior, forcing from M_2 inverse	2.17	4.52	1.44	2.91	0.94	1.84
(4) LFD: M_4 inverse, no prior	1.21	2.62	1.15	3.41	0.82	1.60
(5) LFD: M_4 inverse, prior from case 2	0.89	2.67	1.14	2.65	0.71	1.25
(6) LFD: M_4 inverse, prior from case 3	0.91	2.49	1.05	2.27	0.67	1.13

- Using M_2 inverse to compute forcing dramatically improves M4 prior
- Fitting M_4 data improves solutions further
- Best results (especially for coastal gauges) obtained with two stage inversion approach

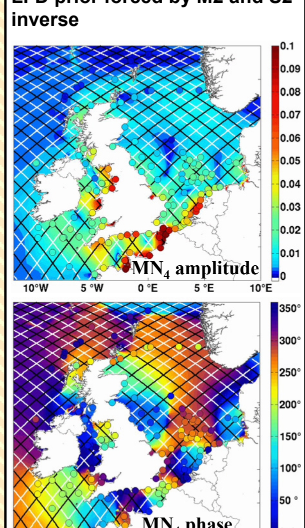
Compound Tides:

- interaction between two different constituents
- use similar approach—forcing for prior in first stage is computed from inverse solutions for two astronomical constituents
- similar improvements in agreement with validation tide gauges

MS_4 solution, obtained using LFD prior forced by M2 and S2 inverse



MN_4 solution, obtained using LFD prior forced by M2 and S2 inverse



Atlantic Ocean

M_4 prior for the Atlantic Ocean 1/12° resolution, obtained by DF of SWE with forcing from M_2 inverse.

M_4 inverse (InvTPT2): 531 cycles of Topex/Poseidon+ Jason and 114 cycles of Topex2 (in shallow water <1000m) data assimilated (each 4th site along track).