Monte Carlo demonstrations of a sea state bias induced during altimeter waveform retracking

Walter H. F. Smith
NOAA Lab for Satellite Altimetry
Expected waveform power

$t'$, range (SSH)
w, width (SWH)
$S/N$, signal-to-noise ratio
Expected waveform error

![expected waveform graph](image-url)
Waveform realization

![Waveform realization diagram]

- $S$
- $W$
- $t'$
- $N$
Least-squares fit (MLE)

Minimize

$$\chi^2 = \sum_{\text{gates}} \left[ \frac{p_i - \text{model}(t_i, t'_i; S, N, w, \xi)}{e_i} \right]^2$$

With the error in the $i$th waveform gate sample weighted proportional to the sampled power

$$e_i = p_i / \sqrt{\text{#waveforms}}$$

This gives most of the weight to the pre-arrival part of the waveform.
χ² large.
SSH too high.

χ² small.
SSH too low.
\[ \frac{\partial (\text{Chi-square})}{\partial (\text{range})} \]

- \( d(\chi^2)/dx \) (cm\(^{-1}\))
- SSH too high.
- Large \( \chi^2 \) gradient
- SSH too low.
- Small \( \chi^2 \) gradient

Range error (m)
Monte Carlo Experiment

Generate a mean waveform with $t' = 0$ and all other parameters ($S$, $N$, $w$, $\xi$) fixed to known values.

Generate many random realizations of this waveform, adding random error proportional to power / $\sqrt{\#}$.

Retrack each realization, holding $S$, $N$, $w$, $\xi$ fixed to the correct values and solving for $t'_{est}$ that minimizes $\chi^2$.

The mean of all such $t'_{est}$ is the SSB for that particular choice of waveform parameters.

I repeat this experiment for different values of $w$ (SWH) and Signal-to-Noise-Ratio, $\text{SNR} = S/N$. 
Retracking-Induced SSB

SSB is consistently positive (range too large, sea level too low).

SSB increases with SWH, ~ linearly.

d(SSB)/d(SWH) decreases as SNR increases (decreases as $\sigma_0$ increases?).

Results shown are for Geosat.
SNR varies geographically
SNR varies with AGC...
AGC varies with $\sigma_0$
Instrumental features

Chi-square depends on instrumental features: # of gates before/after track point, noise characteristics before/after track point. [Waveforms from Wm Emery.]
Conclusions

Retracking introduces an apparent SSB.

It depends on SWH and may be of order 1% of SWH.

It depends on SNR, and thus is geographically correlated, and also correlated to $\sigma_0$.

It should also depend on the algorithm used to minimize the least-squares misfit, and on the weighting used.

It should depend on the error characteristics of each altimeter instrument.

Further Monte Carlo simulations should be done, using the actual retracking algorithm and waveform characteristics of each altimeter.