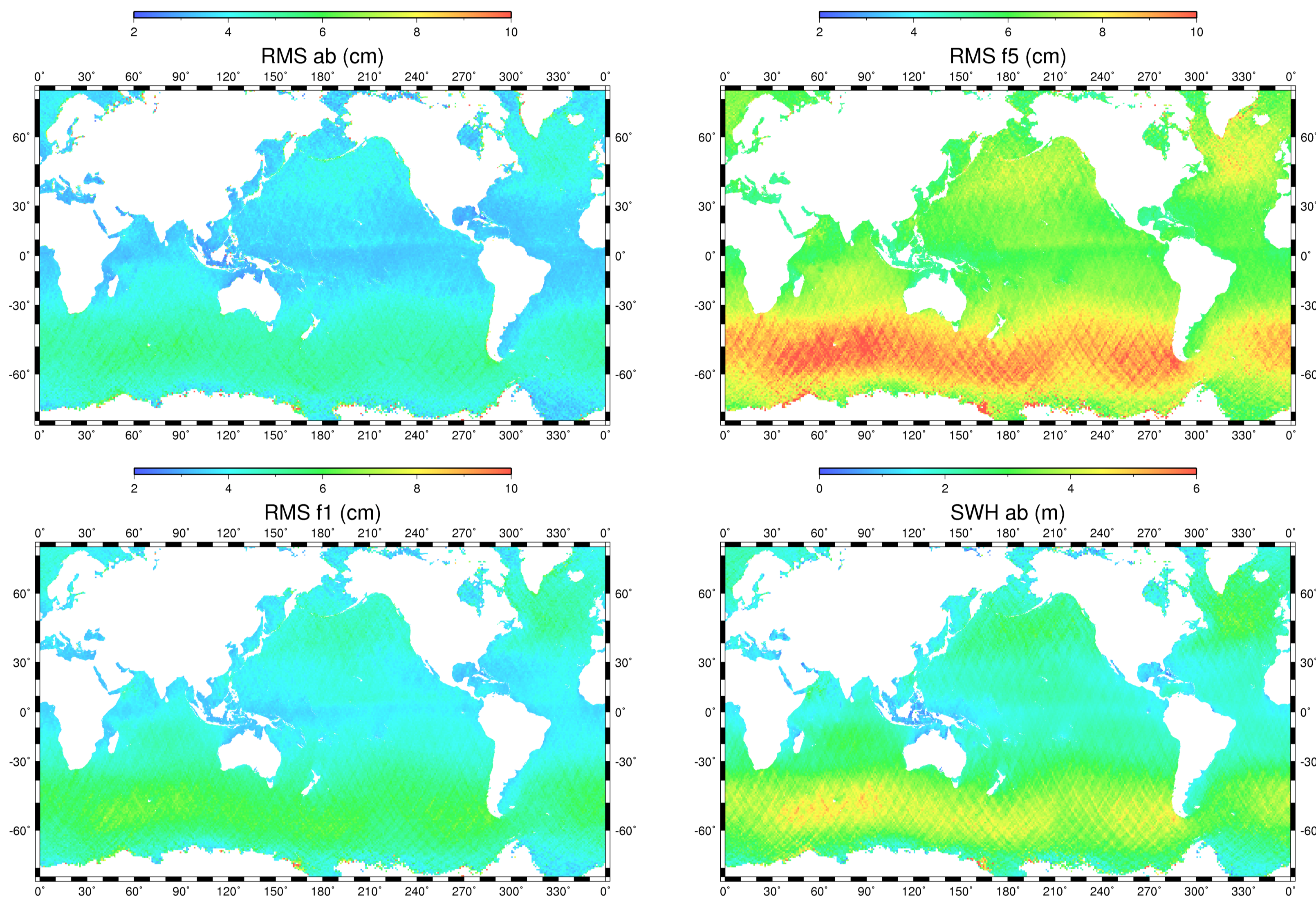




TARGET TRACKING, CORRELATED NOISE, AND PRECISION IN SATELLITE ALTIMETRY

Walter H. F. Smith, NOAA and David T. Sandwell, Scripps Institution of Oceanography

The precision of altimetric height measurements is limited by random noise arising from the radar's sampling of the ocean surface and the data processing employed both within the altimeter instrument in real time and, when retracking is performed, also after the fact on the ground. This noise dominates the height error budget at scales of 20 km or less along a height profile and so limits the accuracy of the along-track sea surface slope signal needed to resolve geostrophic flow velocities and marine gravity anomalies. When sea surface heights are reported by the instrument's on-board tracker (such as in the Geosat and Topex data sets) the random error process is serially correlated. Correlation arises within the instrument, which must employ a radar target following scheme, called an alpha-beta tracker, in order to furnish useful measurements. The tracker has a resonance that amplifies noise at frequencies below 1.07 Hz. It also introduces a group delay of about 0.25 seconds, so that local extrema in height are shifted down-track of their true position. Because the errors are correlated, averaging N successive samples does not reduce the variance by $1/N$. If the data are later reprocessed on the ground, the retracking process may mitigate the serial correlation and group delay but, if conventional maximum likelihood retracking is used, the apparent noise level will be amplified by a factor of 1.5 or so, and will be strongly coupled to significant wave height (SWH). By employing a two-step constrained retracking [Sandwell and Smith (2005) *Geophysical Journal International*, **163**, 79–89] one can reduce the noise level considerably, making it nearly independent of SWH, and nearly approaching the theoretical precision limit. We have reduced the noise in Geosat to under 8 mm in a 1-second average at 2 m SWH. We believe similar retracking can improve the precision of all altimeters far beyond what is available in current data products.



Random Noise Level in Geosat data at 10-Hz rate

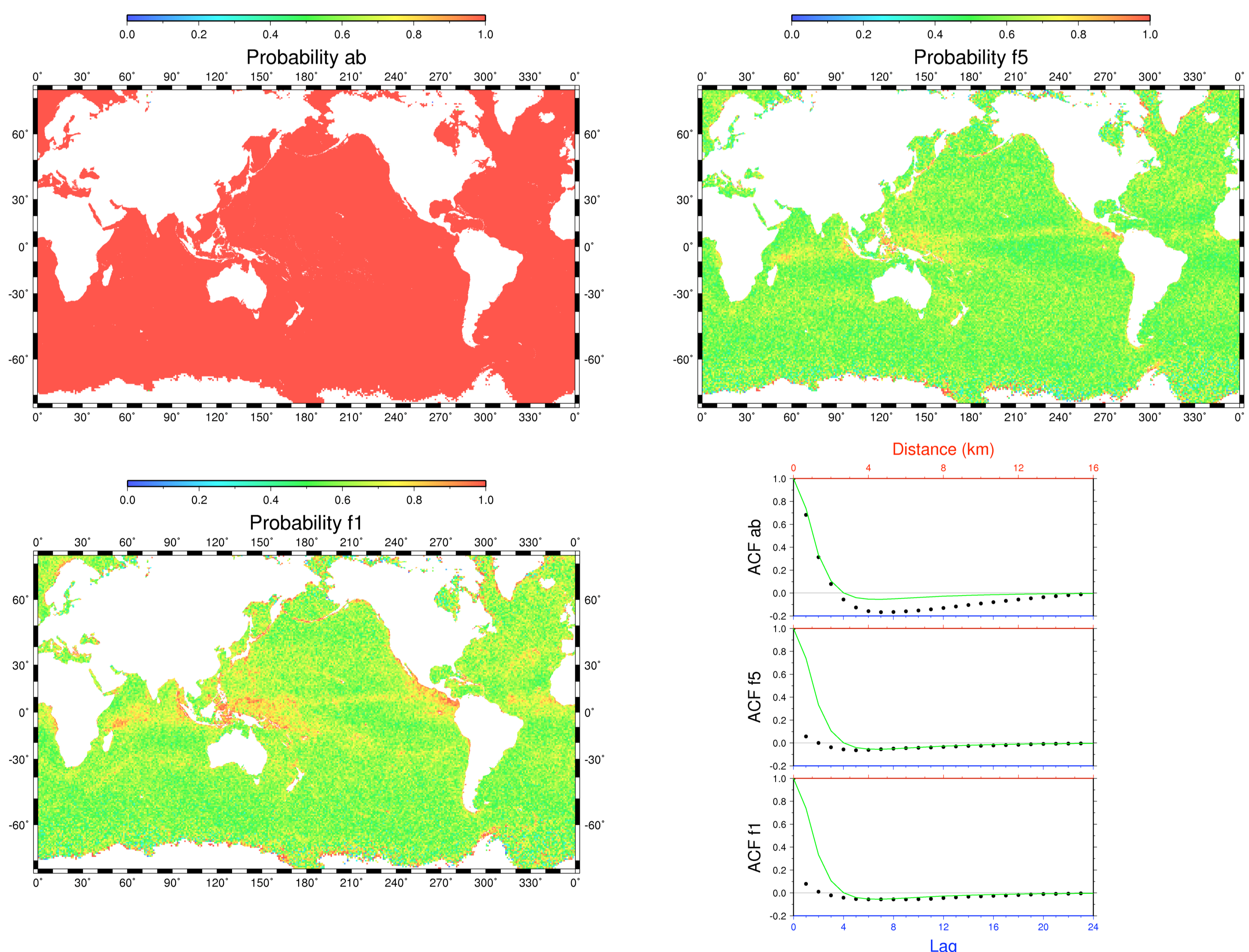
Top left: from on-board alpha-beta tracker.

Top right: from conventional maximum-likelihood retracking.

Bottom left: from two-step retracking.

Bottom right: Significant Wave Height.

The noise level is weakly correlated with SWH in the a-b tracked and two-step retracked data, but strongly correlated with SWH when the data are retracked by conventional methods. Conventional retracking couples a random error in SWH into a random error in range. This was demonstrated with Monte Carlo experiments by Smith & Sandwell [*Geophysical Journal International*, **163**, 79–89, 2005].



Correlation in 10-Hz Noise.

When the on-board tracker's heights are used (Geosat, Topex) the noise process is serially correlated; averaging N points does not reduce the variance by $1/N$. Retracking whitens the noise. The lower right panel shows the autocorrelation functions for noise samples from the on-board tracker ("ab"), conventional ("f5") and two-step ("f1") retracking. The green curve is predicted theoretically from the equations of the on-board alpha-beta tracker, accounting for the spectral folding in going from 20-Hz (tracker update frequency) to 10-Hz (Geosat data output frequency).

The maps show the probability that the first-lag autocorrelation is significantly different from zero. The two-step retracking (lower left map) elevates the first-lag autocorrelation slightly above where it is for conventional retracking. Both retrackings yield white noise over most of the ocean most of the time.

Relationship between SWH and height precision.

The noise levels shown at right are for individual 10-Hz samples, and should be about a factor of 3.2 higher than the precisions of 1-Hz samples, which are more commonly quoted. The global mean SWH is around 2 m. Dark gray areas show the most frequently occurring data. Conventional retracking (middle) appears noisier than on-board tracked data (top), though this is expected due to the smoothing nature of the on-board tracker. Two-step retracking (bottom) has lower noise level and noise increasing less rapidly with wave height. Dotted line: expected noise level if noise variance equals point target response variance plus wave height variance divided by number of waveforms averaged. Dashed line: two-step retracking has achieved a much lower noise level, corresponding to 8 mm in a 1-Hz average at 2 m SWH. Dotted line: noise level achieved in Monte Carlo simulations of retracking searching for a zero of $d(\chi^2)/d(\text{range})$. Two-step retracker used a Fibonacci search set to stop when range uncertainty reached 1 cm. It appears we can do better.

