

# Global sea level rise uncertainty due to land motion and reference frame issues

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## Background

At present the uncertainty in altimeter drift estimate from the global tide gauge network is about 0.4 mm/yr. This is the dominant term in the error bar on global sea level rates from altimetry.

$$\eta_n^{adj} = o_n + (\lambda_n^{est} - \lambda_n^{true})(t - t_0) + \epsilon'_n$$

$$h_n = \Delta^{true} + o_n + \epsilon''_n$$

$$\Delta^{est} = \sum_n w_n (h_n - \eta_n^{adj})$$

$$\Delta^{est} - \Delta = \sum_n w_n \epsilon_n + (t - t_0) \sum_n w_n (\lambda_n^{est} - \lambda_n^{true})$$

$$\sigma_{\Delta}^2 = \sigma_{random}^2 + (t - t_0)^2 \sigma_{land}^2$$

The random error is minimized by choosing the  $w_n$  weights appropriately..

For a trend, the ‘formal’ error comes from the random term. There is, however, also a bias error, given by  $\sigma_{land}$ . The latter is dominant in this case.

$$\lambda_n^{est} - \lambda_n^{true} = w_{in}(\lambda_{in} - \lambda_n^{true}) + (1 - w_{in})(\lambda_{en} - \lambda_n^{true})$$

$$\lambda_{in} - \lambda_n^{true} = b_i + \epsilon_{in}$$

$$\lambda_{en} - \lambda_n^{true} = \epsilon_{en}$$

Given variances for the bias error and the random errors, we form an estimate for  $\sigma_{land}$  and minimize by choosing  $w_{in}$  appropriately.

$$\sigma_{land}^2 = \sigma_{bi}^2 \left( \sum_n w_n w_{in} \right)^2 + \sum_n w_n^2 w_{in}^2 \sigma_{in}^2 + \sum_n w_n^2 (1-w_{in})^2 \sigma_{en}^2$$

We then choose the  $w_{in}$  in order to minimize the land motion error. Letting the  $w_{in}$  go to zero (i.e., only using GPS rates) eliminates the bias error, but this is only optimal if the internal and external errors are comparable, which is presently not the case..

The result is the  $0.4\text{mm}/\text{yr}$  error estimate.

## The Problem

is that we have not taken into account bias errors in the external (i.e., GPS) land motion estimates due to uncertainties in the ITRF.

Specifically, we are concerned with the scale rate uncertainty ( $0.1\text{ppb}/\text{yr}$ , Zuheir et al.) and the z-translation error of order  $1\text{mm}/\text{yr} \sin(\theta)$ .

We need to modify the external land motion error estimate to

$$\lambda_{en} - \lambda_n^{true} = b_s + b_z \sin(\theta) + \epsilon_{en}$$

The important result is that the bias error estimate for the trend is now dominated by

$$\sigma_{land}^2 \sim \sigma_{bi}^2 \left( \sum_n w_n w_{in} \right)^2 + \sigma_{bs}^2 \left( \sum_n w_n (1 - w_{in}) \right)^2$$

$$\sigma_{land}^2 \sim \sigma_{bi}^2 \left( \sum_n w_n w_{in} \right)^2 + \sigma_{bs}^2 \left( \sum_n w_n (1 - w_{in}) \right)^2$$

The net result is that we now have an error floor given roughly by the minimum of  $\sigma_{bi}^2$  and  $\sigma_{bs}^2$ .

Further, if  $\sigma_{bs}^2$  is greater than  $\sigma_{bi}^2$ , then we actually need to *downweight* the GPS rates!

Possibly to the point of not using them at all!