An Analytical Model of the Electromagnetic Bias Using the Physical Optics Scattering Theory

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EM Bias Background

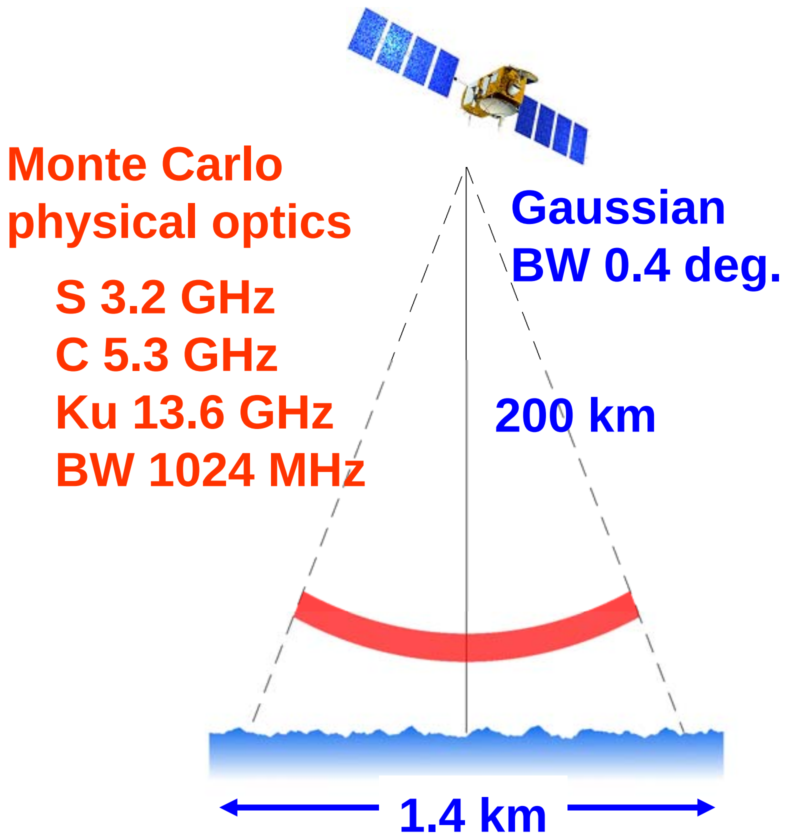
- The EM bias is one of the largest sources of errors in altimetry.
- The EM bias is caused by nonlinear behavior of sea waves, i.e. smooth and shallow wave troughs are stronger reflectors than wave crests.
- Theoretically, the EM bias is defined as $$\beta_{\text{EM}} = \frac{\langle z\sigma^0 \rangle}{\langle \sigma^0 \rangle}$$
  - **Jackson (1979)** used geometrical optics to describe the cross section as proportional to the height pdf of specular surface points:
  $$\beta_{\text{EM}} = -\frac{1}{8}\lambda_{12} H_{1/3}$$
  - **Srokosz (1986)** extended Jackson's results to 2-D sea surfaces.
  - **Elfouhaily et al (2000)** treated long and short waves separately by introducing a cutoff into the formulation.
  $$\beta_{\text{EM}} = -\frac{1}{8}\lambda_{12} H_{1/3} \left(\frac{S_L^2}{S_S^2 + S_L^2} \right)$$

Motivation

- An alternate approach based on a Monte Carlo simulation has been presented at the 2008 Nice meeting.
- Pulse returns can be obtained through coupled electromagnetic/fully nonlinear hydrodynamic Monte Carlo simulations, along with the corresponding sea surface profiles.
- The EM bias is obtained by comparing the pulse returns from linear and nonlinear sea surfaces.
- Short wave and long wave effects can be examined by varying the range of the length scales included in the surface profiles.
- However, a large number of realizations are needed for good convergence. An analytical EM bias model is developed to help verify and explain Monte Carlo results.

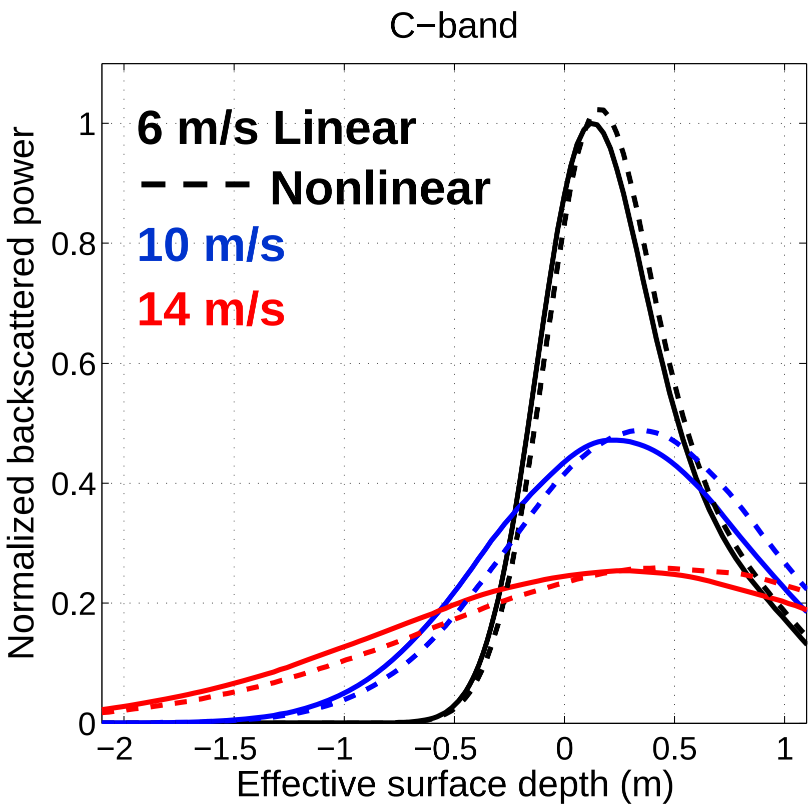
Monte Carlo Simulation

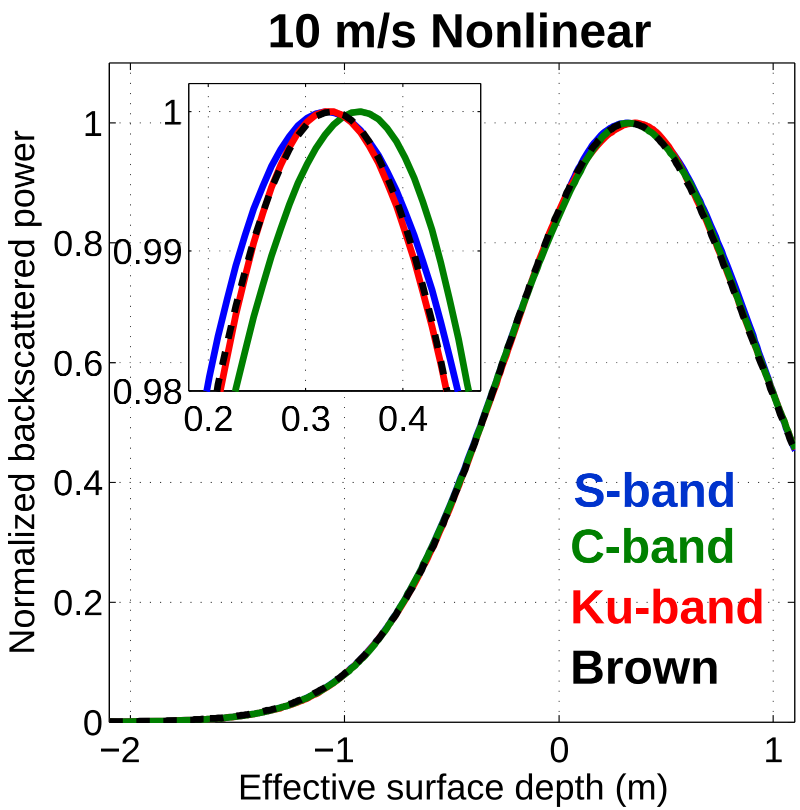
1. Generate a set of linear and nonlinear sea surfaces.
2. Compute near-normal incidence backscattering over a range of frequencies.
3. Transform backscattered fields versus frequency into the time domain.
4. Average over realizations.
5. Estimate sea surface height and electromagnetic bias from pulse returns.



- 1-D perfectly conducting surface
- 1.4 km long sampled into 128K points
- Pierson-Moskowitz spectrum
- Creamer (1989) improved linear representation

Pulse Returns

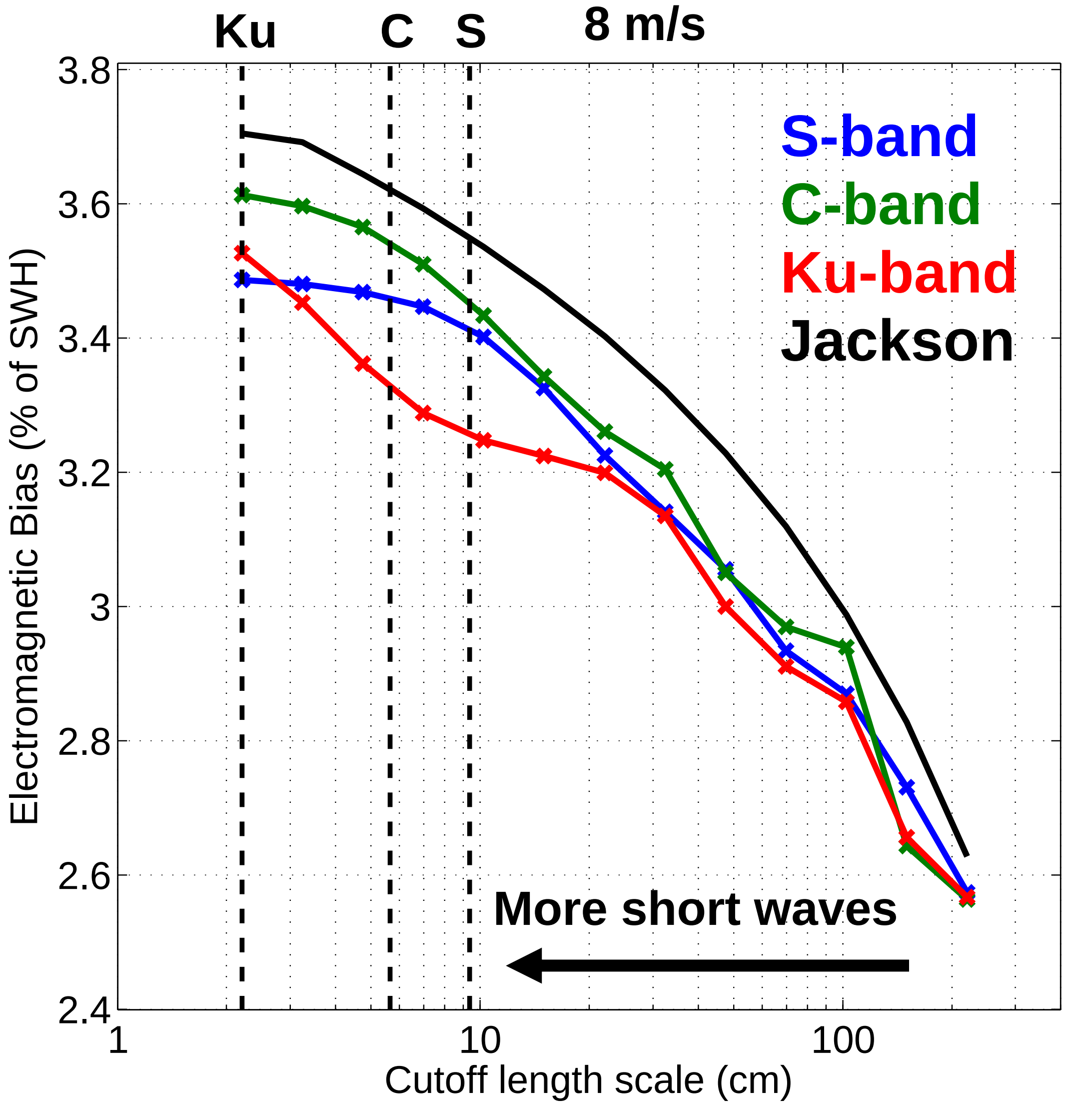




- The EM bias is computed as

$$\beta_{\mathrm{EM}} = \frac{c}{2}\left(\frac{\int tP_L(t)dt}{\int P_L(t)dt} - \frac{\int tP_{NL}(t)dt}{\int P_{NL}(t)dt}\right)$$

Short Wave Effects on the EM Bias



- A Monte Carlo method has been used to compute the EM bias as a function of wind speed, radar frequency, and short wave content.
- Results are reasonable, but hard to interpret. An analytical model is needed in order to provide physical insight into the EM bias mechanism.

Analytical EM Bias Model

- The analytical model utilizes the same formulation as used in the computation of the pulse return in the previous Monte Carlo simulation.

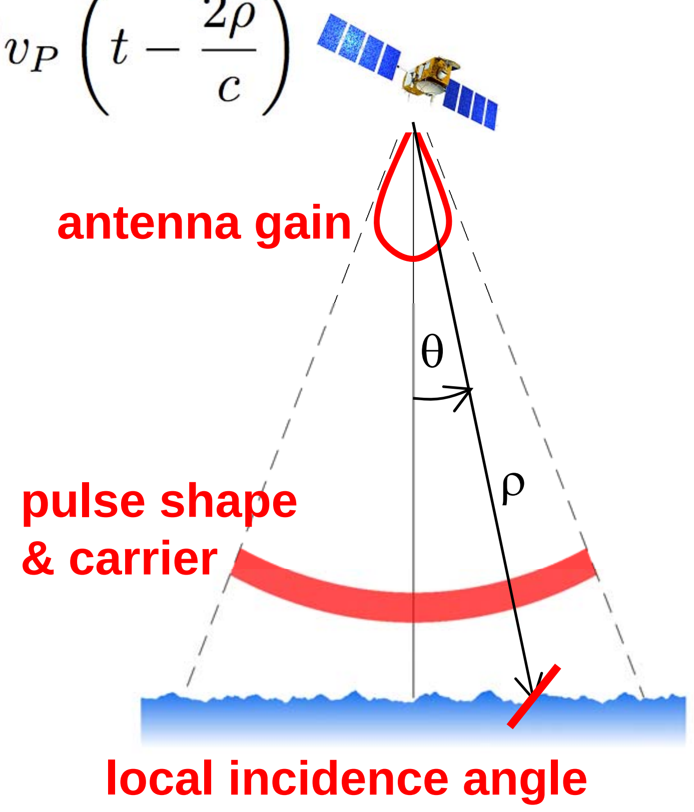
- Under the physical optics scattering theory, the voltage received by the antenna is given by

$$v(t) = \frac{2\eta}{I_T} \int_S dS \ \cos\theta_l \ \frac{G(\theta)}{2\pi\rho} \ e^{-i\omega_0\left(t-\frac{2\rho}{c}\right)} v_P\left(t - \frac{2\rho}{c}\right)$$

- The **average** power pulse return is proportional to

$$\langle v^*(t) v(t) \rangle$$

- This requires the joint PDF of two points on the surface profile.



Analytical EM Bias Model (2)

- For linear surfaces, the joint PDF is Gaussian with the rms height and **the correlation function** as the parameters.
- For "weakly" nonlinear surfaces, the PDF is expressed in terms of an Edgeworth series and requires the third order statistics of the sea surface called **"the reduced bicorrelation function."**
- Note that only the zeroth and first moments of the pulse returns are needed for the EM Bias. Use the far-field approximation and assume a Gaussian form for the pulse shape and the antenna beamwidth.
- The final expression for the EM bias becomes

$$\beta_{\mathrm{EM}} = \int_{-\infty}^{\infty} dx\ A(x)\left[S_{\Sigma}(x) - S(0)\right] \exp\left\{ -\frac{4k_0^2\sigma^2(1-C(x))}{1+2\left(\frac{\sigma}{c\alpha}\right)^2(1-C(x))} \right\}$$

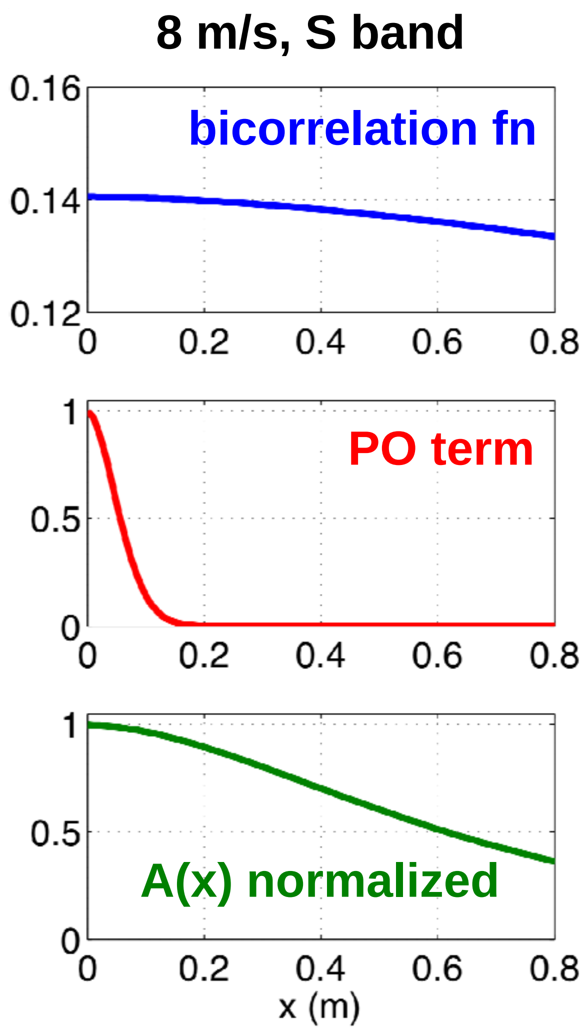
leftover term, slowly varying

the reduced bicorrelation fn

$$S(x) = \frac{1}{\sigma^3}\left\langle f^2(x_0) f(x_0+x)\right\rangle$$

similar to standard PO term

Analytical EM Bias Model (3)



- Most of the contribution comes from near the origin.
- An asymptotic evaluation of the integral yields

$$\beta_{\mathrm{EM}} = \frac{1}{2}A(0)S''(0)\int_{-\infty}^{\infty} x^2 e^{-2k_0^2\sigma^2 C''(0)x^2}\,dx$$

$$= -\frac{\sigma}{4}\frac{S''(0)}{C''(0)} = -\frac{1}{8}\lambda_{12}H_{1/3}$$

Jackson's EM bias !!

- Need a model for the reduced bicorrelation function (e.g. Longuet-Higgins, Creamer, etc.)
- Further examination of the integral will shed light on how the EM bias changes with the radar frequency and the short wave content of the sea surface.

Conclusions

- This talk presents an analytical model for the EM bias using the physical optics scattering theory.
- The resulting EM involves an integral that contains the correlation function and the reduced bicorrelation function of the sea surface.
- It can be shown that asymptotic evaluation of the integral yields Jackson's EM bias in terms of the height-slope skewness of the sea surface.
- Further analysis of the EM bias integral must be performed to provide additional insight into properties of the EM bias. Several models of the bicorrelation function will be examined.