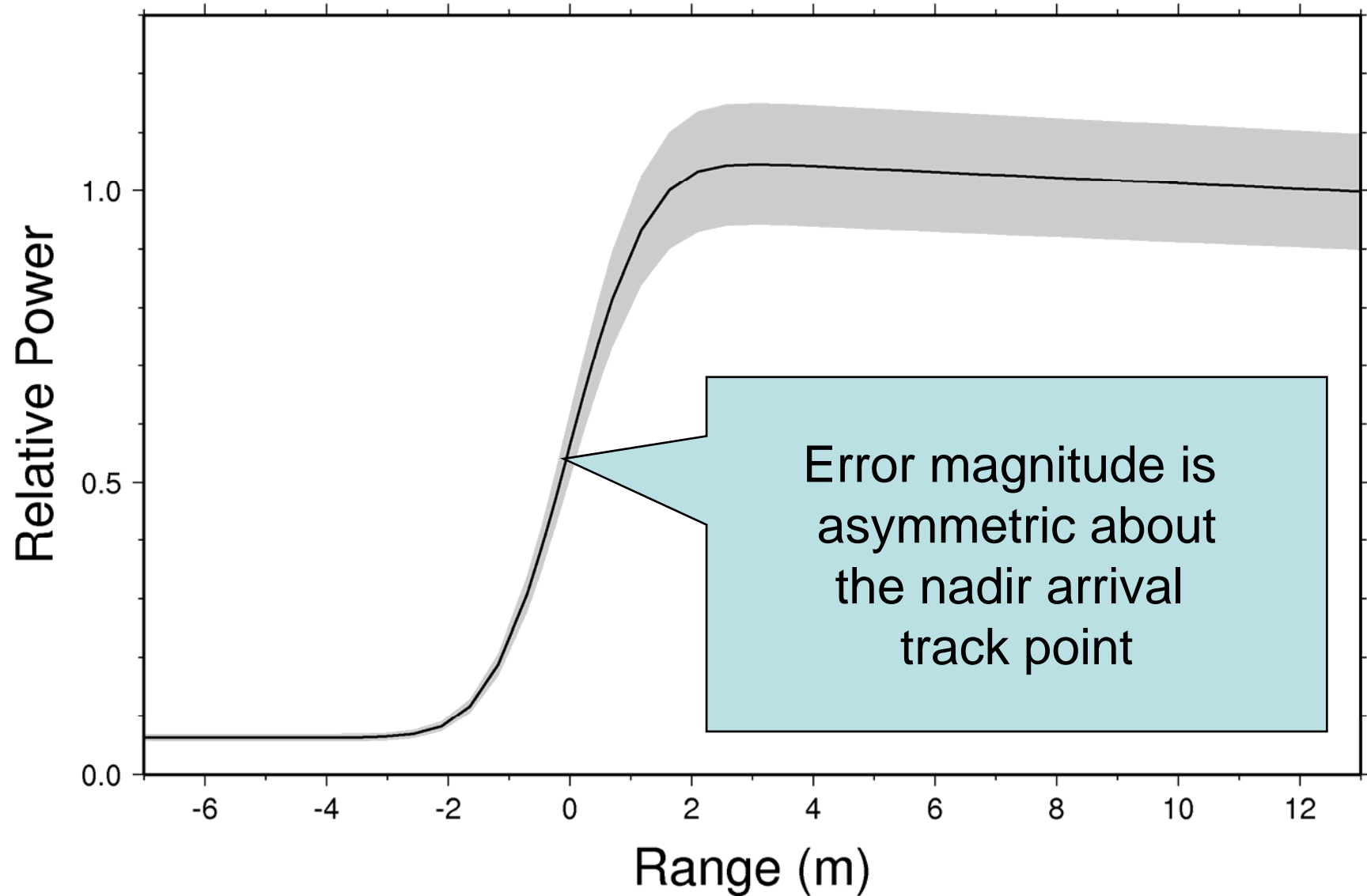


# Monte Carlo investigation of differences between MLE3 and MLE4 outputs from J1 and J2

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# Waveform error $\propto$ power



# Asymmetry in least-squares fit

Minimize

$$\chi^2 = \sum_{gates} \left[ \frac{p_i - \text{model}(t_i, t'_{est}, S, N, w, \xi)}{e_i} \right]^2$$

*Weighted* models set  $e_i$  proportional to  $p_i$ , giving most of the weight in  $\chi^2$  to the low power portion of the waveform, before the plateau is reached.

MLE3 and MLE4 are *unweighted*:  $e_i =$  constant. This gives most of the weight in  $\chi^2$  to the noisy plateau after the tracked arrival.

In both schemes,  $\chi^2$  is asymmetric around the desired arrival time.

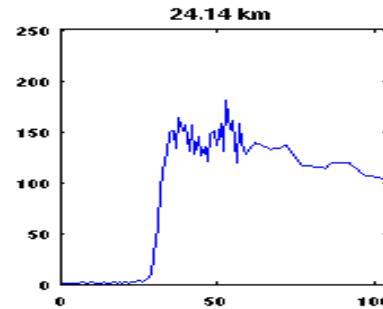
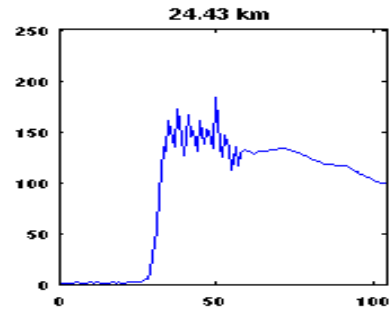
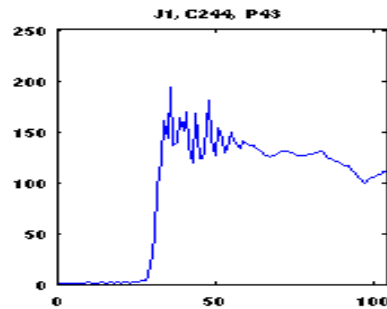
# Asymmetric driving forces

MLE3 and MLE4 seek the waveform model parameters by iterative refinement of an initial guess. The refinement is driven by Gauss-Newton steps solved with a QR algorithm.

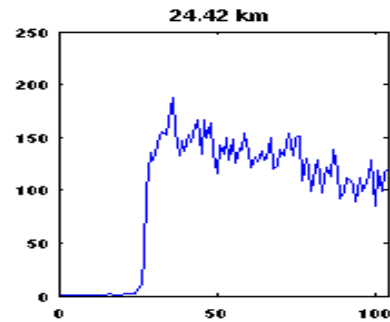
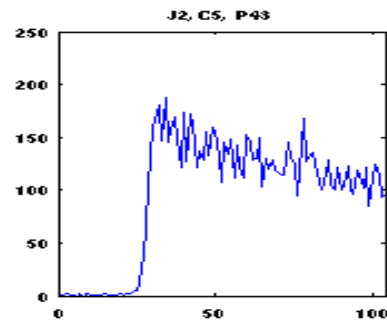
The "driving force" that changes the model parameters at each step is  $\nabla\chi^2$ .

Since  $\chi^2$  is asymmetric around the desired range solution, the driving forces are asymmetric. This causes random errors in the waveform to give *biased* random errors in the fitted parameters. These can induce an apparent SSB, as shown previously at OSTST 2008 in Nice.

# J1 and J2 noise differences



J1 C244  
P43

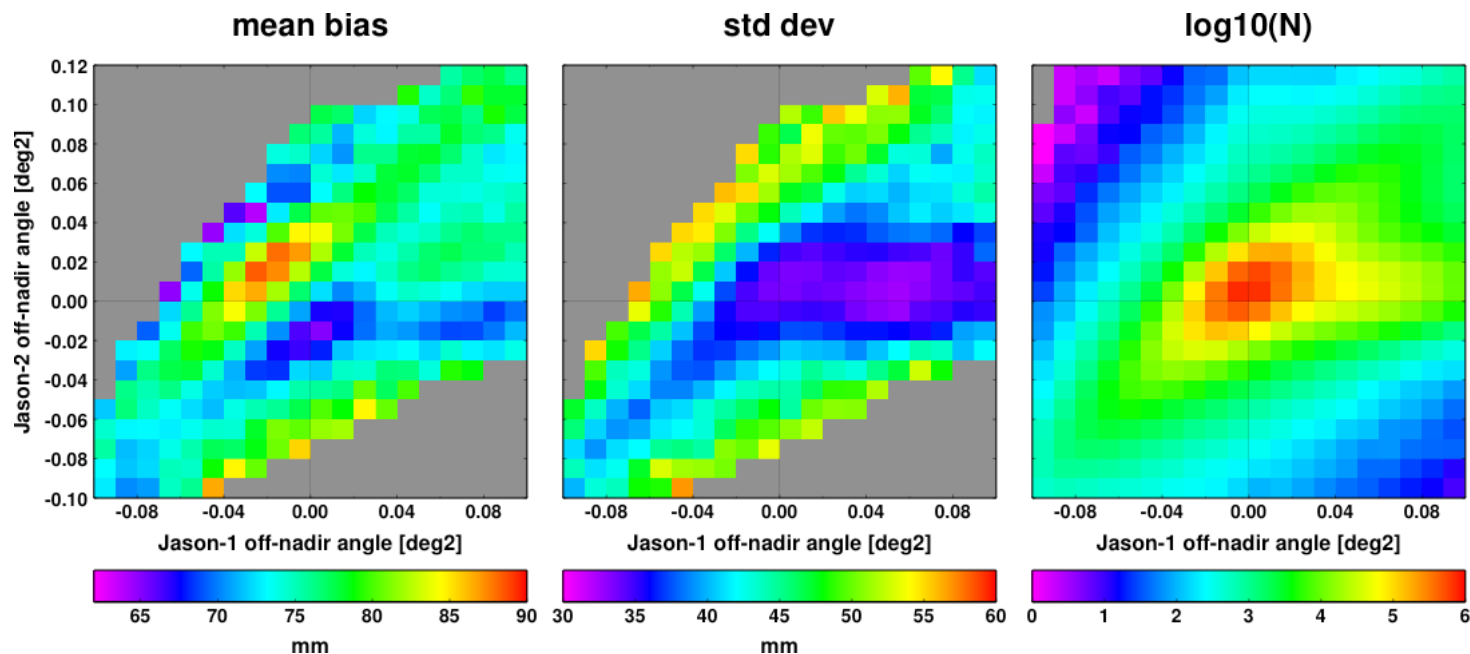


J2 C005  
P43

The J2 waveform is uncompressed and noisy throughout. The J1 waveform compression scheme reduces and correlates the waveform noise.

# What we hope to learn, 1

The J1 and J2  $\chi^2$  should be different, because of the compression on J1 not applied to J2, so that a random realization of a waveform should give different retracker output if processed like J1 and like J2. Can this explain the J1-J2 range biases, and how they vary with other tracker parameters?



# What we hope to learn, 2

The  $\chi^2$  driving force also changes as the number of gate samples before and after the nadir arrival changes, as happens cyclically in J2's Diode-DEM tracking mode. This should introduce tracker errors with a cyclic form. We want to model what happens in this case.

**Jason-2  
Diode/DEM  
Cycle 007**

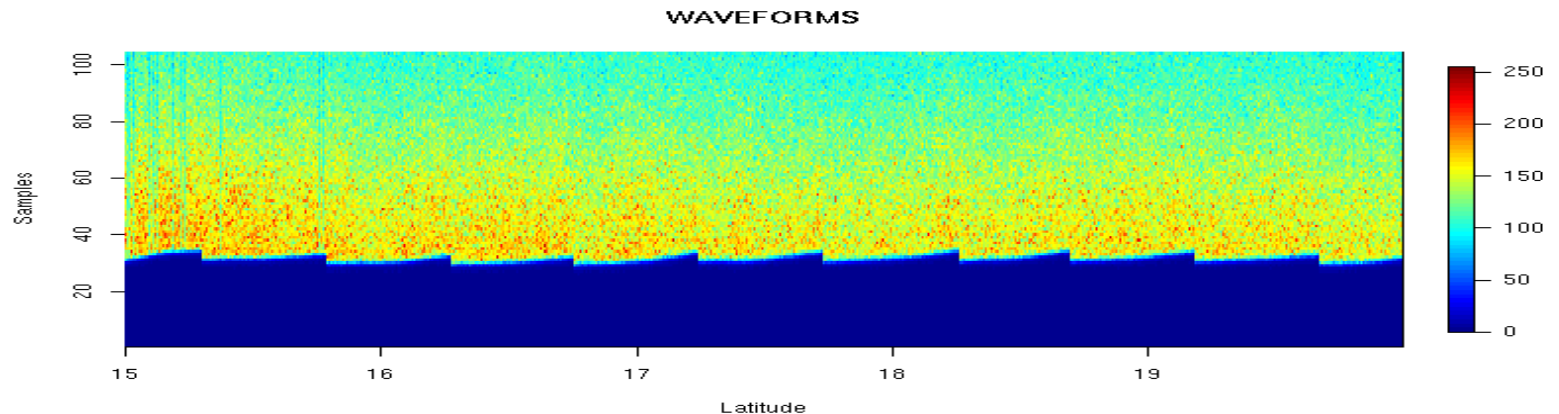


Figure from P. Thibaut, OSTST Nice 2008

# Apologies and Regrets

Gauss-Newton iteration for non-linear least squares, as in MLE3 and MLE4, is very sensitive to coding errors in calculating  $\nabla\chi^2$ . I have been unable to get my code sufficiently debugged and tested in time for this meeting.

All I can do at this time is suggest that there should be interesting J1-J2 bias differences and SSB differences, and there should be cyclic errors in Diode-DEM mode. But I cannot demonstrate any concrete results.

I realize this is extremely frustrating to all.

I apologize for this state of things, and I thank you for your patience and encouragement.