Monte Carlo investigation of differences between MLE3 and MLE4 outputs from J1 and J2

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Waveform error $\propto$ power

Error magnitude is asymmetric about the nadir arrival track point.
Asymmetry in least-squares fit

Minimize

\[ \chi^2 = \sum_{\text{gates}} \left[ \frac{p_i - \text{model}(t_i, t'_\text{est}, S, N, w, \xi)}{e_i} \right]^2 \]

*Weighted* models set \( e_i \) proportional to \( p_i \), giving most of the weight in \( \chi^2 \) to the low power portion of the waveform, before the plateau is reached.

MLE3 and MLE4 are *unweighted*: \( e_i = \) constant. This gives most of the weight in \( \chi^2 \) to the noisy plateau after the tracked arrival.

In both schemes, \( \chi^2 \) is asymmetric around the desired arrival time.
Asymmetric driving forces

MLE3 and MLE4 seek the waveform model parameters by iterative refinement of an initial guess. The refinement is driven by Gauss-Newton steps solved with a QR algorithm.

The "driving force" that changes the model parameters at each step is $\nabla \chi^2$.

Since $\chi^2$ is asymmetric around the desired range solution, the driving forces are asymmetric. This causes random errors in the waveform to give biased random errors in the fitted parameters. These can induce an apparent SSB, as shown previously at OSTST 2008 in Nice.
J1 and J2 noise differences

The J2 waveform is uncompressed and noisy throughout. The J1 waveform compression scheme reduces and correlates the waveform noise.
What we hope to learn, 1

The J1 and J2 $\chi^2$ should be different, because of the compression on J1 not applied to J2, so that a random realization of a waveform should give different retracker output if processed like J1 and like J2. Can this explain the J1-J2 range biases, and how they vary with other tracker parameters?
What we hope to learn, 2

The $\chi^2$ driving force also changes as the number of gate samples before and after the nadir arrival changes, as happens cyclically in J2's Diode-DEM tracking mode. This should introduce tracker errors with a cyclic form. We want to model what happens in this case.

Figure from P. Thibaut, OSTST Nice 2008
Apologies and Regrets

Gauss-Newton iteration for non-linear least squares, as in MLE3 and MLE4, is very sensitive to coding errors in calculating $\nabla \chi^2$. I have been unable to get my code sufficiently debugged and tested in time for this meeting.

All I can do at this time is suggest that there should be interesting J1-J2 bias differences and SSB differences, and there should be cyclic errors in Diode-DEM mode. But I cannot demonstrate any concrete results.

I realize this is extremely frustrating to all.

I apologize for this state of things, and I thank you for your patience and encouragement.