Singular Value Decomposition applied on Altimeter Waveforms

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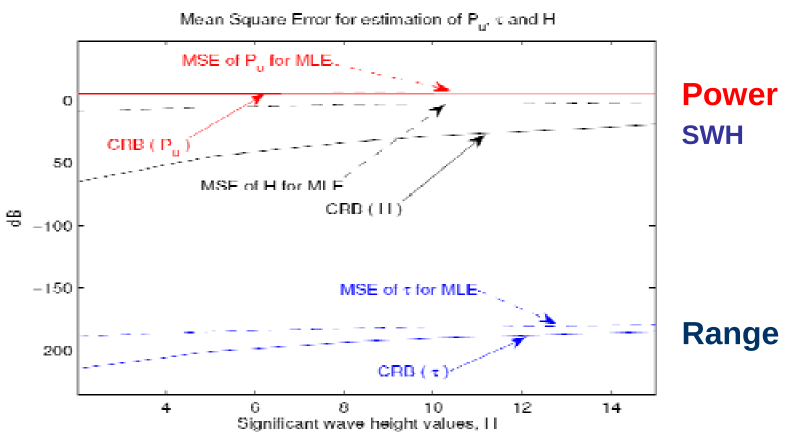
Context of the study

- The purpose of the SLOOP project (funded by CNES) is all in all, to improve the altimeter open ocean products.
  Concerning the retracking of the WFs, we proposed to investigate various retracking strategies and to compare them with classical ones ➔ MLE, LSE, MLE2passes, NN, MAP, … and SVD filtering
  The idea is to analyse their respective performances and to select the most attractive one

- Altimeter parameter estimation is based on the theoretical Hayne's model

Cramer Rao Bound and MLE MSE of the Hayne's model as a function of SWH



➔ Some space for improving the estimation of the epoch and the SWH

- The idea is also to try to reduce the noise level of the estimations without introducing artificial along-track spatial correlation (now : 20 Hz estimation + 1Hz averaging, …)

➔ **Reduction of the noise level of the WFs before estimation**

Truncated Singular Value Decomposition for Noise Reduction

- SVD is a classical technique in signal processing (developped in 1940) sometimes used to « denoise » signals.
- Waveforms filtering by SVD was first investigated by **Annabelle Ollivier** in his PhD Thesis (2006) with very encouraging results
- The processing consists in :

Taking the S matrix representing the noisy signal (Wfs matrix)

$$S = \bar{S} + B$$

Computing the Singular Value Decomposition

$$S = U\Sigma V^*$$

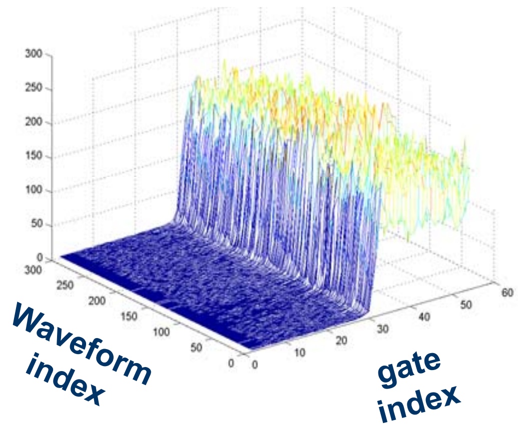
S (m,n) : WF matrix (m= 104; n=300)
U (m,m), V(n,n): unit matrices*
Σ (m,n) : diagonal matrix

$$S = \underbrace{\lambda_1 VS_1 + \ldots + \lambda_k VS_k}_{\text{signal + noise}} + \underbrace{\ldots \lambda_r VS_r}_{\text{noise}}$$

Discarding small singular values of S (which mainly represent the additive noise)

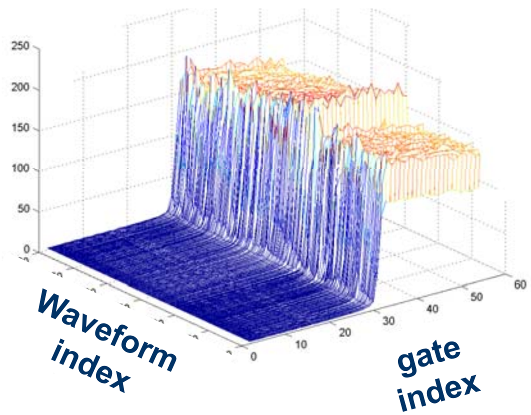
The rank-k matrix Ak represents a filtered signal

J1 Ku band raw waveforms



SVD

J1 Ku band filtered waveforms



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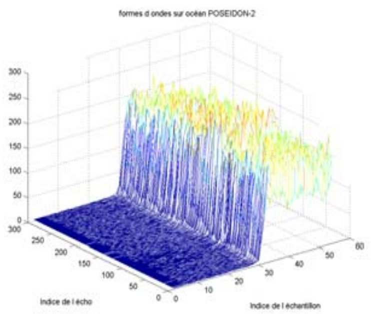
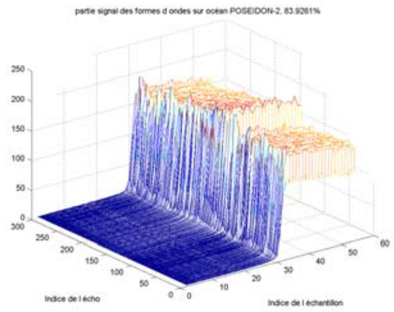
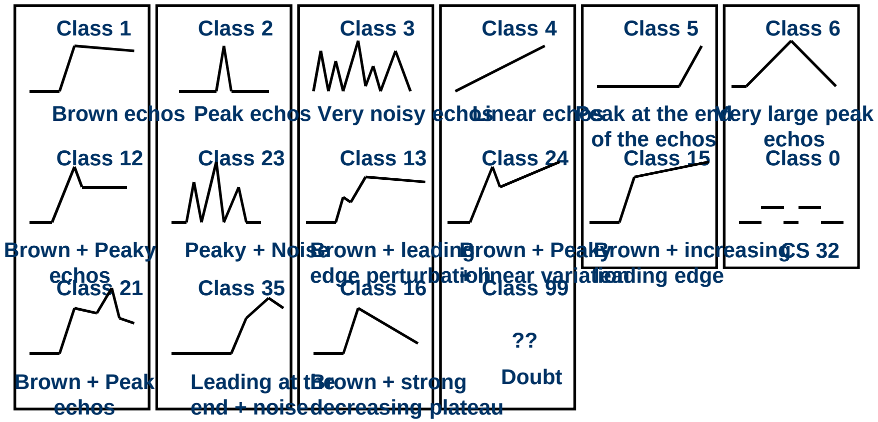
Truncated Singular Value Decomposition Noise Reduction

- Matricial method
- results are closely linked to size of the matrix (number of AT Wfs in the matrix)
- also to to the homogeneity of the waveform matrix
- results are closely linked to rank-k truncation



Waveforms classification → **SVD** → **Estimation Process (rtk) MLE4 here** →





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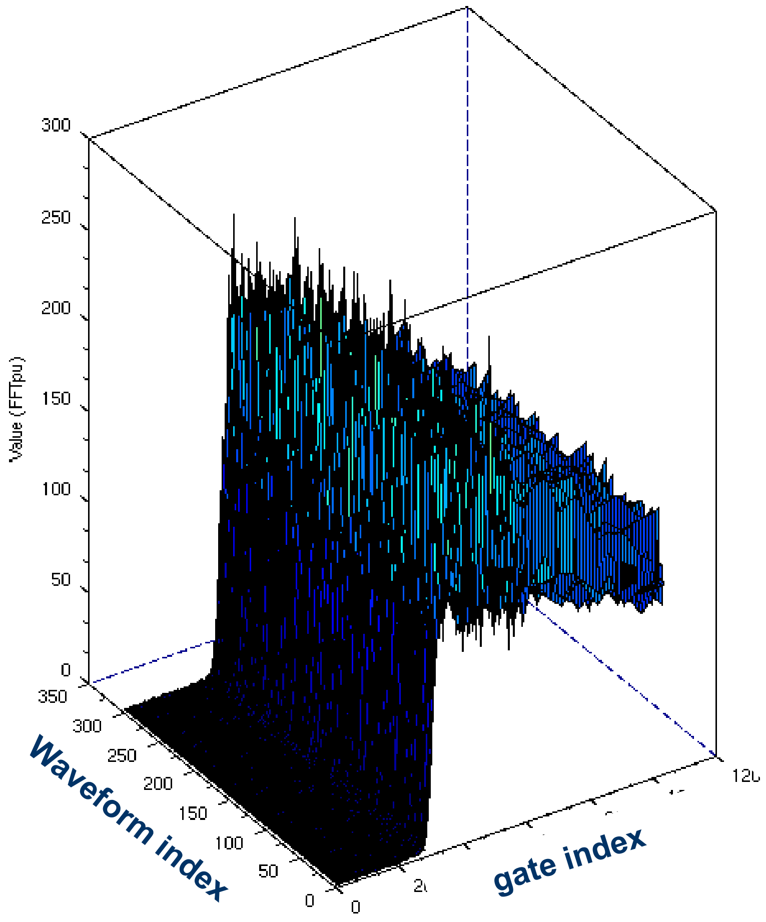
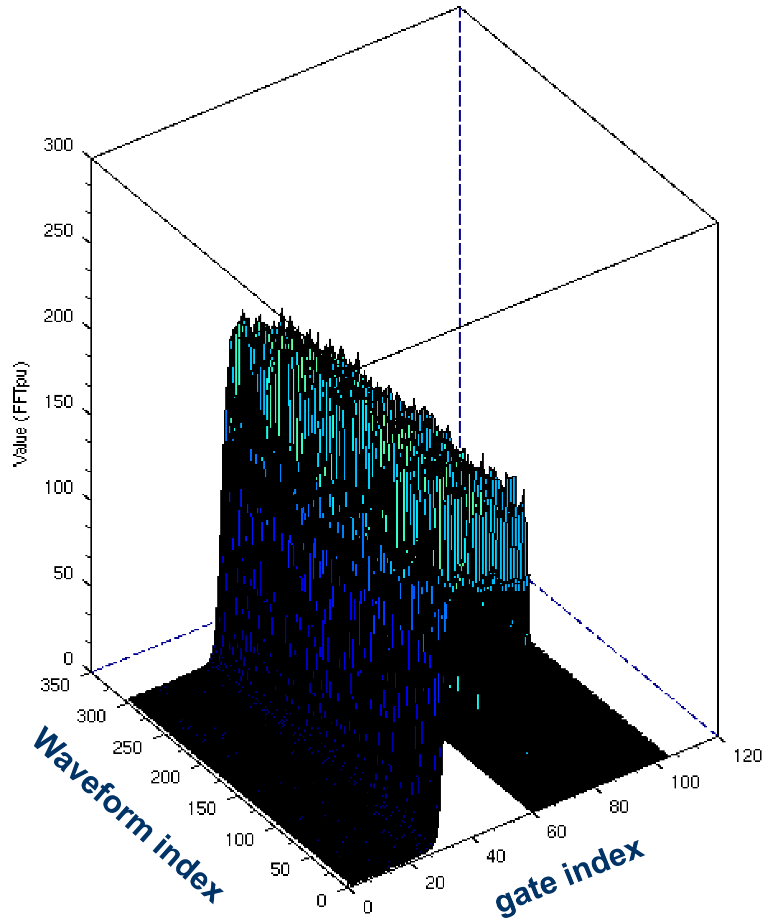
Truncated Singular Value Decomposition Noise Reduction

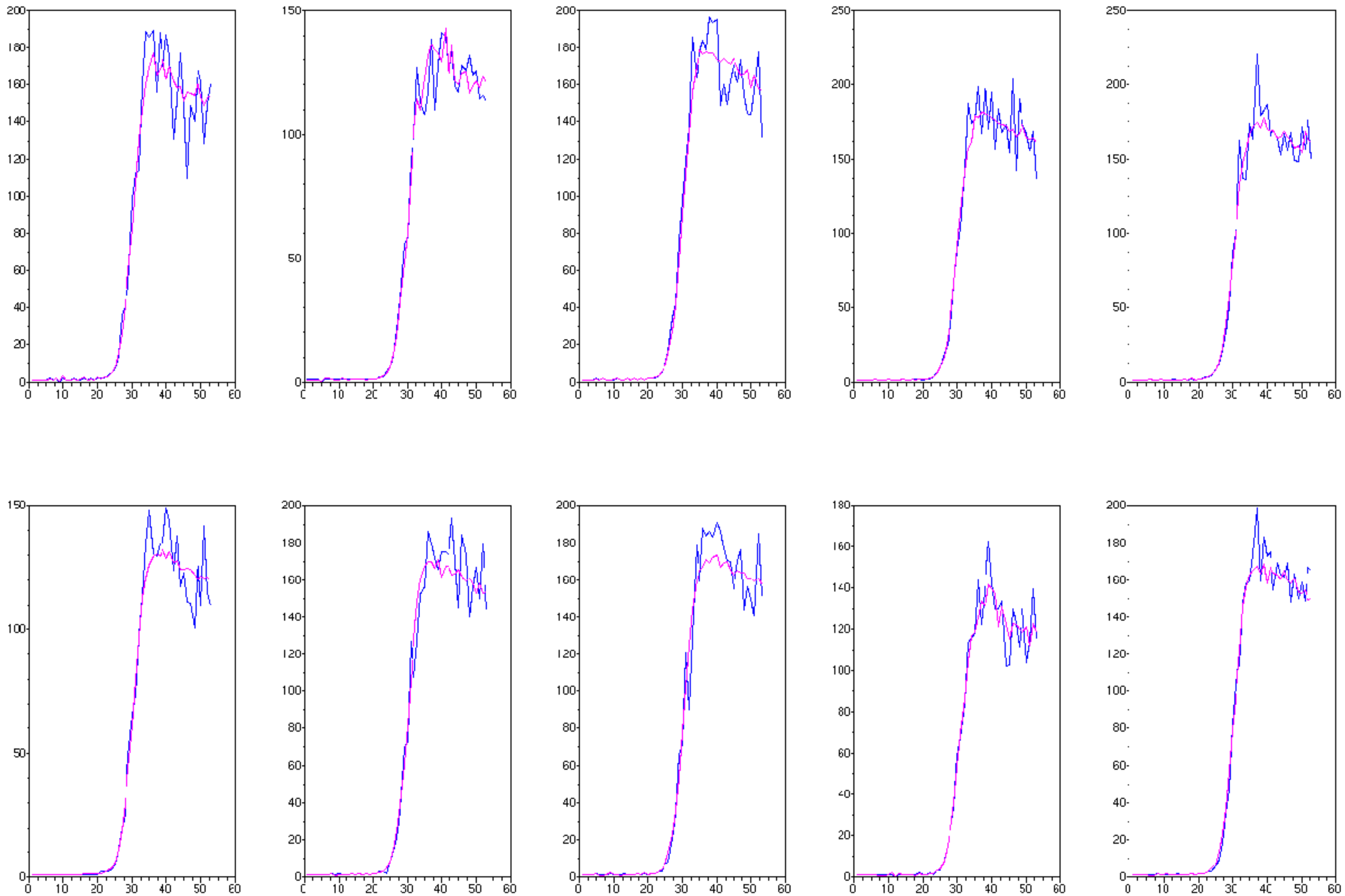
J1 Ku band raw waveforms



J1 Ku band filtered waveforms

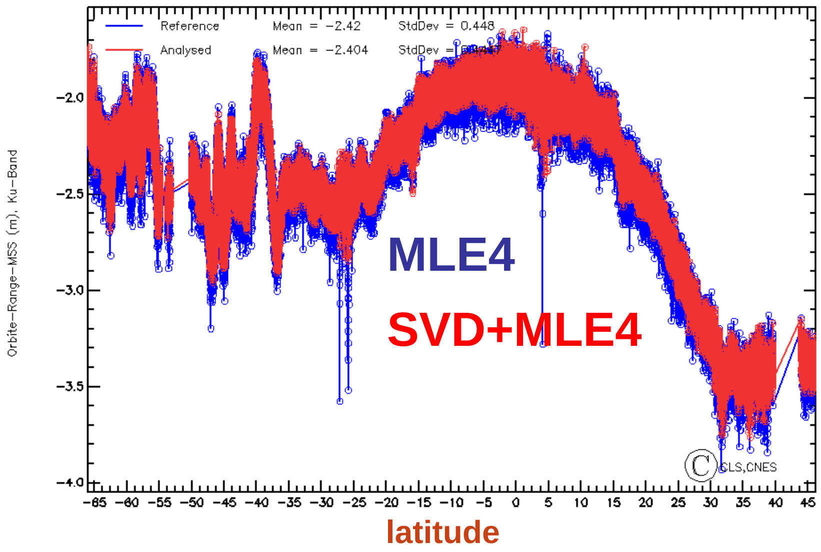


Truncated Singular Value Decomposition Noise Reduction

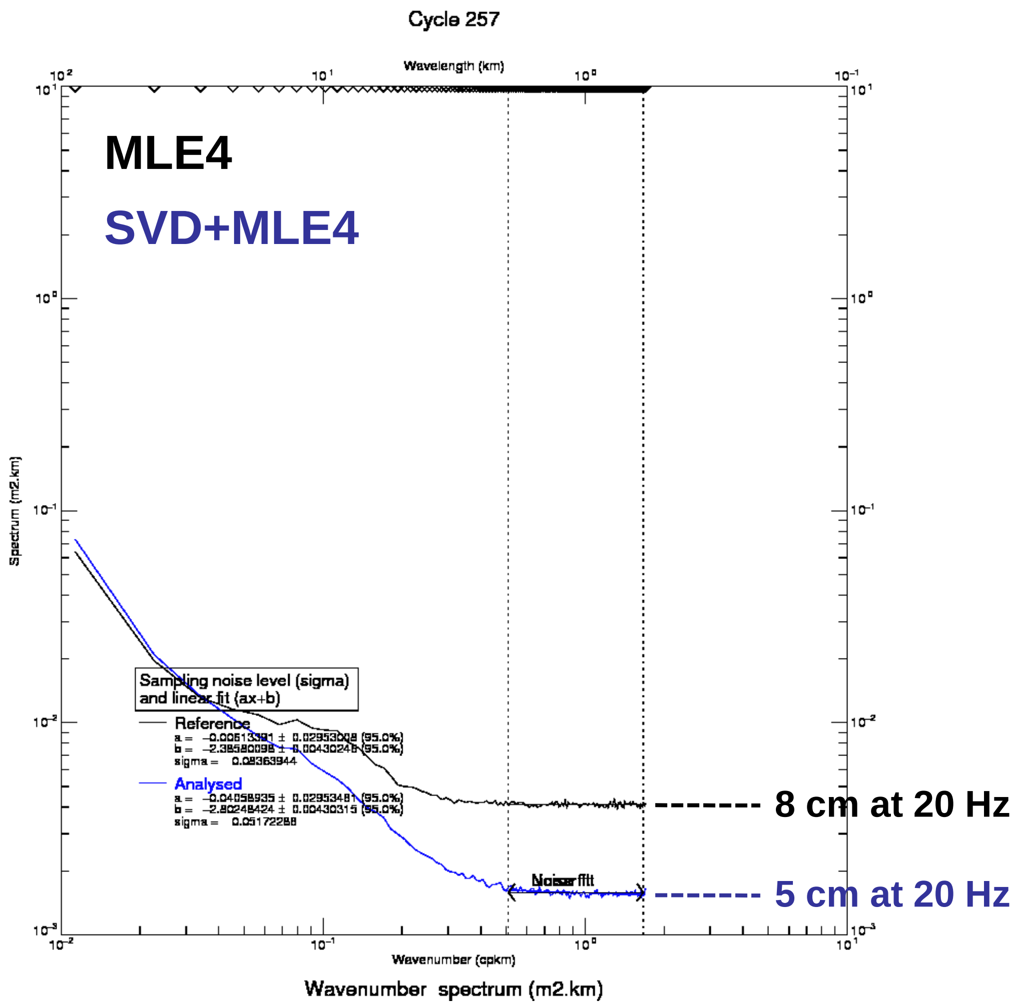


Impact on range

SLA variation

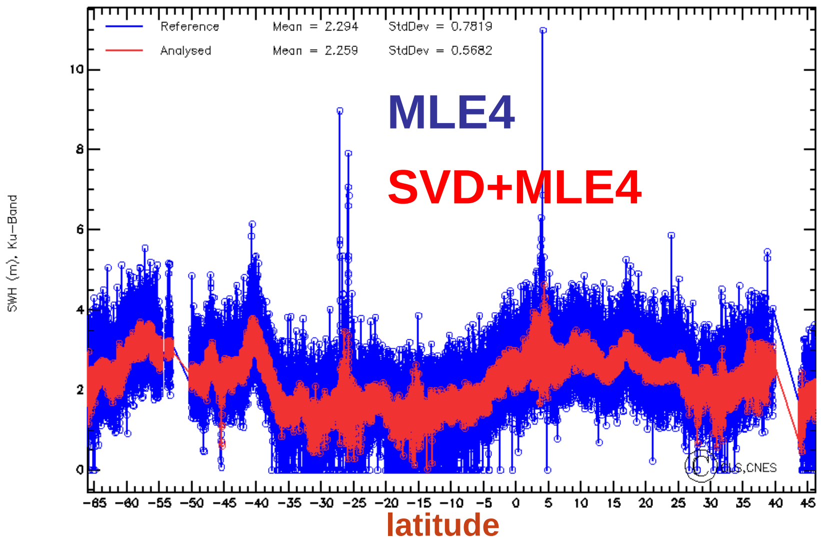


SLA spectrum – Ku band

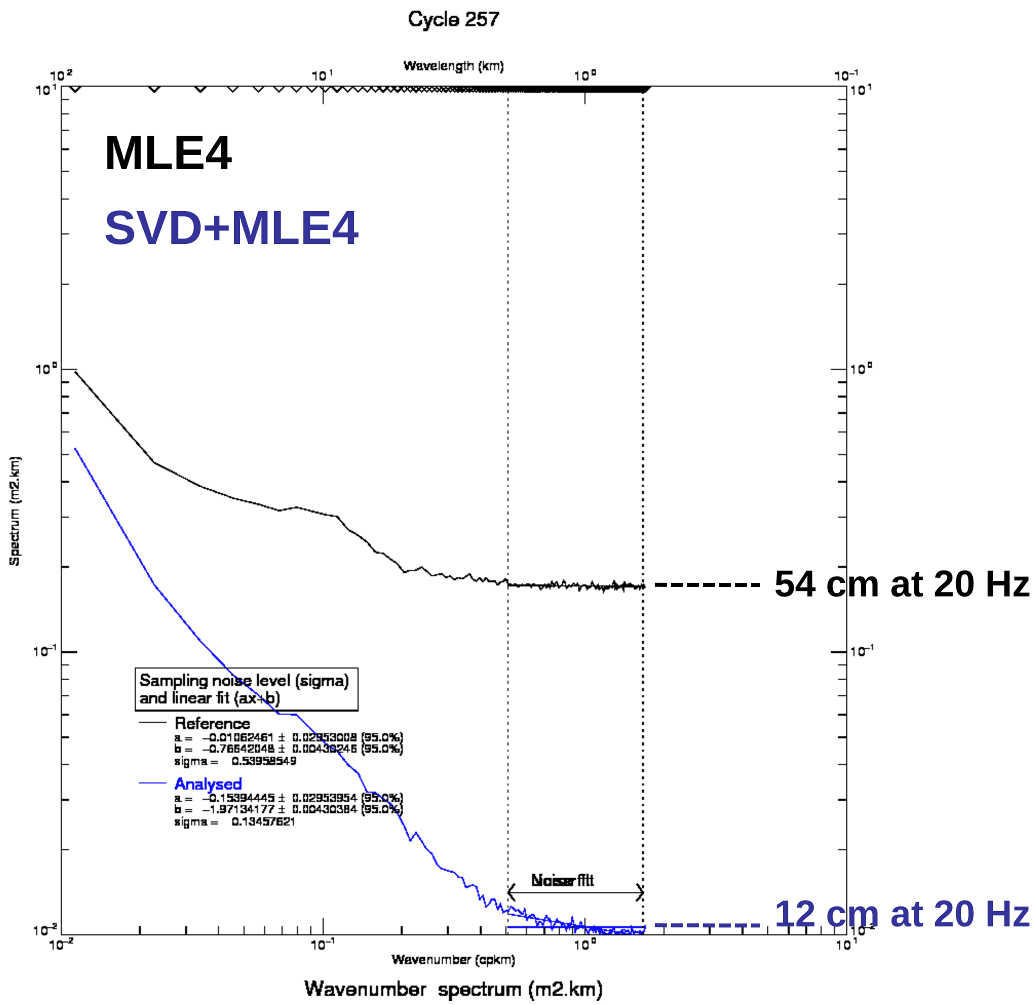


Impact on SWH

SWH variation



SLA spectrum – Ku band



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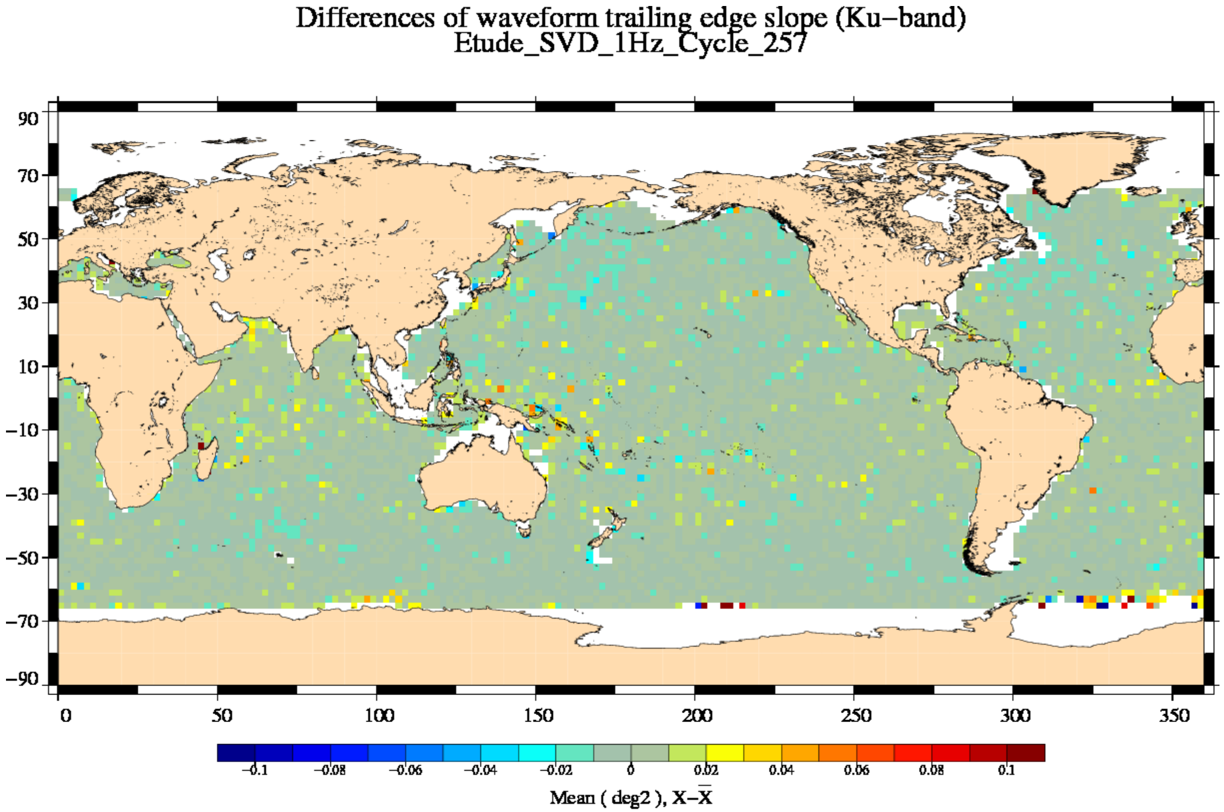
No impact of the SVD on mispointing angle



Nb of data	:	8194	St. Dev	:	0.009588	Skewness	:	3.576945	Minimum	:	–0.149812
Mean	:	0.018105	Rms	:	0.020487	Kurtosis	:	119.677332	Maximum	:	0.256380

Conclusions

- SVD allows a strong noise reduction on SWH and range
- SVD allows a gain in SLA rms measurements by a factor between 1.2 (weak waves) to 2 (strong waves).
- SVD allows to pass from a 7 km resolution (present 1 Hz products) to a 1.2 resolution (6 Hz) with an equivalent noise (precision of the SLA).

Thank you !

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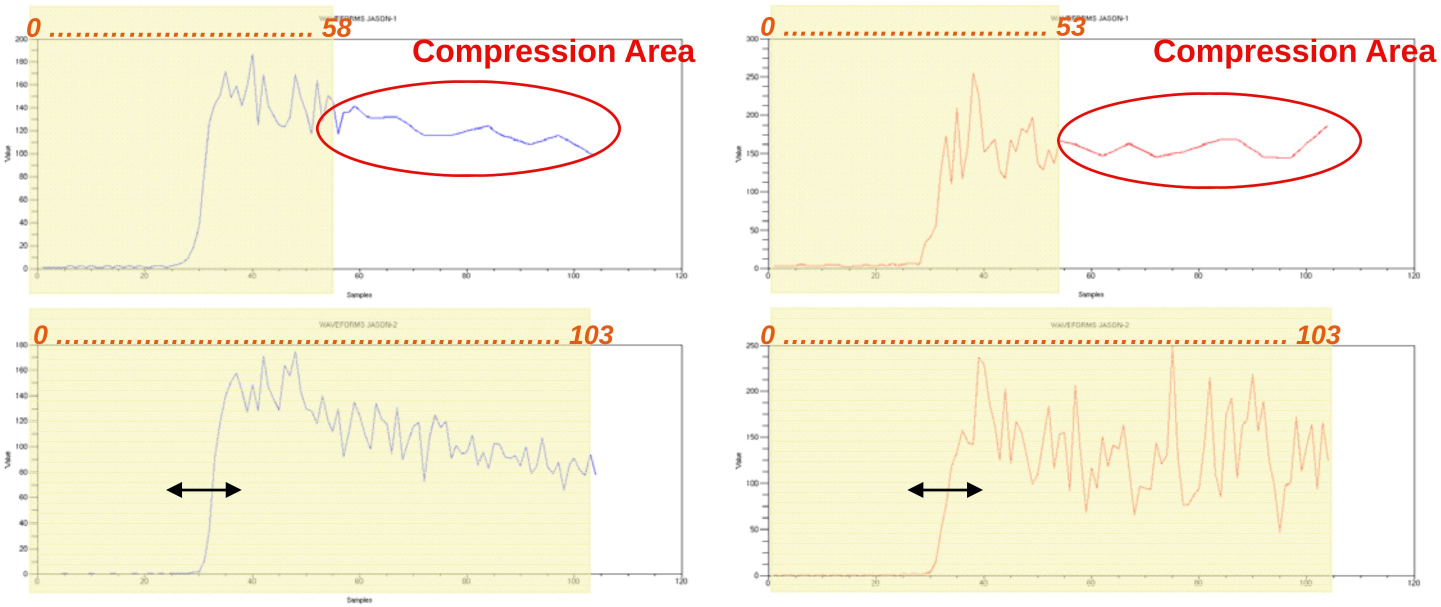
Differences between Jason-1 and Jason-2 waveforms

➔ Due to telemetry rate, Jason-1 waveforms are compressed

Ku band waveforms | C band waveforms

Jason-1

Jason-2

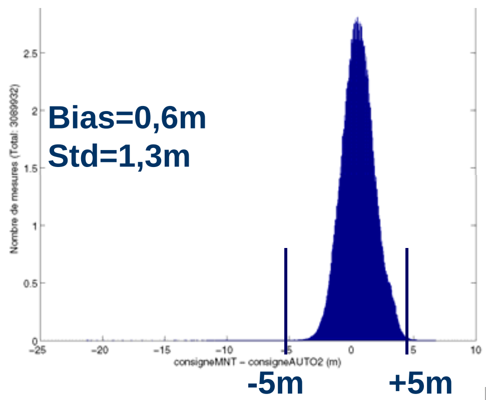


➔ <u>SGT tracker on Jason-1</u> = echos are centred on gate 44

➔ <u>Median and DIODE/DEM on Jason-2</u> = Lateral motion of the echos in the window

- ➢ for median : depends on the waveheight (as for Envisat)
- ➢ for DIODE : depends on the local value of the DEM
- ➢ AGC tracking loop : small differences in C band loop coefs

Differences between median and DDEM trackers



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