Sea Surface State modelling for sea state bias evolution up to Ka-Band

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The two-scale integration scheme

Even simple methods like the Kirchoff approximation suffer from intractable numerical burden when it comes to integrate large ocean surfaces (some km^2) at the radar wavelength resolution (some cm^2)

We have adopted an exact EM model to explicitly integrate the longer scales and have addressed the shorter scales in a statistical manner, relying on some statistical methods.

Our model retains a crucial property of the ocean surface which is the non-homogeneous distribution of small roughness riding over long waves,(hydrodynamic modulation). Hence, the statistics describing ocean short-scale are not spatialy stationary but depend on the local properties of long scale features.

Lagrangian coordinates, particles trajectories

$$\frac{\partial^2 \boldsymbol{R}}{\partial t^2} - \boldsymbol{F} = \frac{1}{\rho}\nabla_{\boldsymbol{R}}\, p$$

- $\boldsymbol{R}$ position of a particle
- $\boldsymbol{F}$ external forces
- p pressure
- ρ volumetric mass

Solution : Gestner waves (e.g. 2D sinusoidal wave)

$$x = x_0 - a e^{kz_0}\sin(kx_0 - \omega t)$$
$$z = z_0 + a e^{kz_0}\cos(kx_0 - \omega t) \quad , \omega^2 = gk$$

Figure 1 shows the evolution of a Gestner wave during time.

Let us notice that for a set of equireparted initial coordinates $\{x_0\}$, the final set of $\{x\}$ presents close points on peaks and spread points on trough. → **longer scales**

The plain trochoids (the trajectories of particules during time) caracterize the orbital velocity of the particles → **shorter scales**

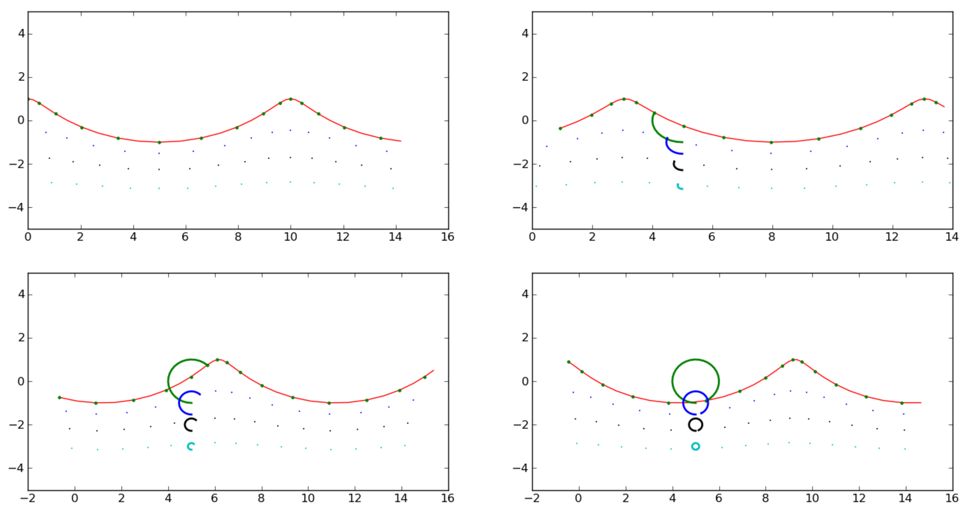

Figure 1 - Evolution of a Gestner wave during time at different depths

The ocean surface is coarsely subdivided (1m scale pixels) and the elevation at this scale is explicitly integrated.

Longer Scale

The Gestner waves introduce an horizontal displacement function of the vertical deplacement.

$$D(x,t) = -a\sin(kx_0 - \omega t) = Hilbert(z)$$

The knowledge of sea surface elevation spectrum allows us to write the sea surface elevation as a sum of cosinus.

Therefore the non Linearities can be described describes the Choppy Wave Model (CWM) [4] : generalisation of the horizontal displacement via the Hilbert transform

The new coordinates of the old linear grid are defined by

$$\boldsymbol{r} = \boldsymbol{r_0} + D(\boldsymbol{r_0}, t) \qquad \text{where } D(\boldsymbol{r_0}, t) = \int \mathrm{i}\frac{\boldsymbol{k}}{k} FT[\xi] e^{\mathrm{i}\boldsymbol{k}\boldsymbol{r_0}}\, d\boldsymbol{k}$$
$$\tilde{\xi}(\boldsymbol{r}, t) = \xi(\boldsymbol{r_0}, t)$$

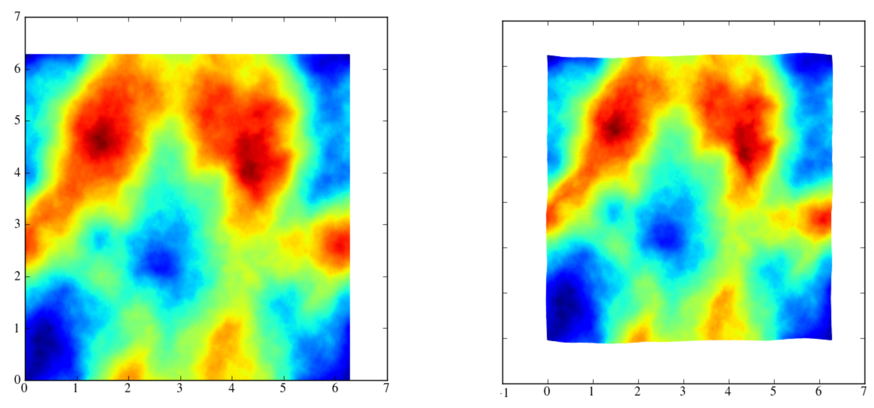

Figure 2 -Linear elevation map of a 6m x 6m patch of the sea surface and the corresponding non linear elevation map (CWM)

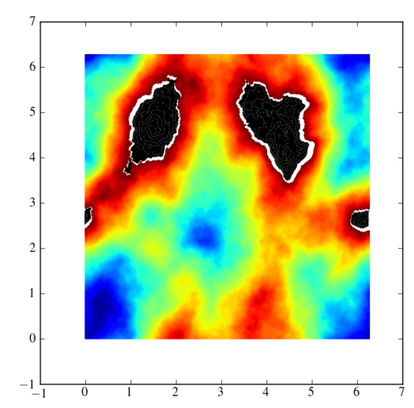

Figure 3 -The linear elevation map (in colors) has been cut above a certain threshold and the non linear elevation map (in black) under this threshold

The Figure 3 shows that the pics are sharper after the CWM process.

We note that the spectrum of the nonlinear process is different from the spectrum of the linear process. We do have to transform the input spectrum so that the output spectrum will fit the observations. [4] have develloped an iterative process to undress a measured spectrum, so that the CWM transform will produce an observed like spectrum.

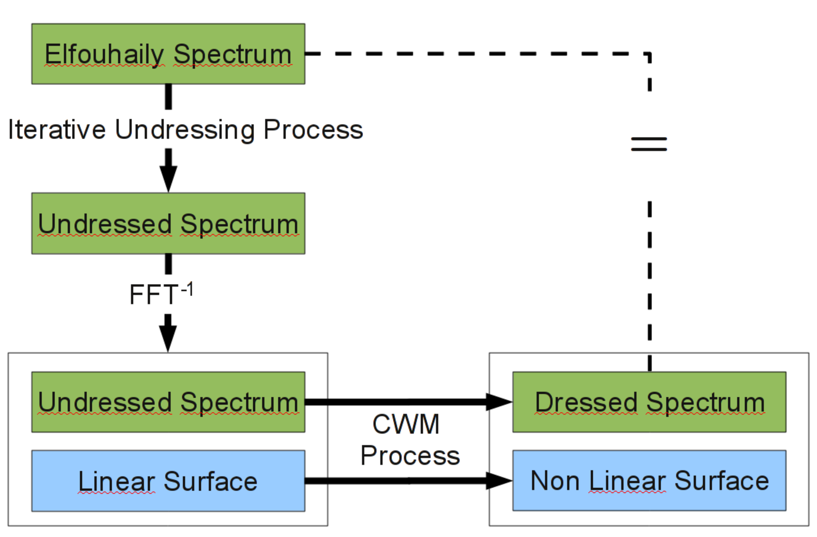

Figure 4- Block diagram of the spectrum transformations

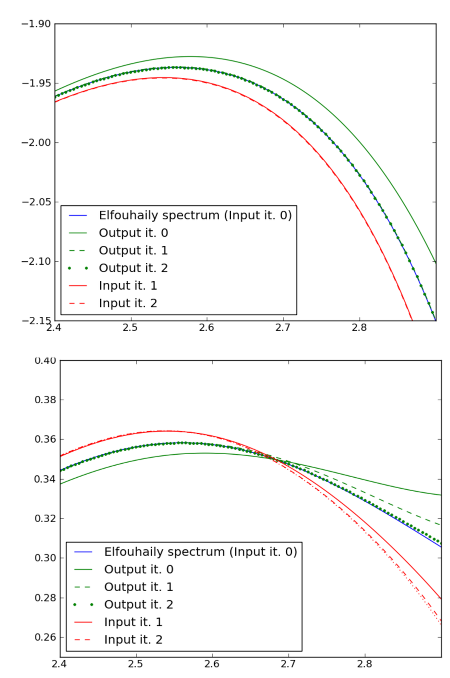

Figure 5- View of the iterative process of Input (undressed) spectrum calculations.

The contribution coming from shorter scales within the pixels of resolution is accounted for in a statistical manner, relying on an average cross-section

Shorter Scale

Under the Geometric Optics (GO) assumption

$$\sigma_0 = \frac{R^2}{2} T_x\, pdf_{sw}(\xi_x, \xi_y)$$

(ξ_x, ξ_y) are the slopes of the pixel carrying the rugosity, T_x is the effect of the local geometry also known as Tilt Modulation.

With the assuption of a Gaussian pdf for short waves

$$\sigma_0 = \frac{R^2}{2}\frac{\left(1+\boldsymbol{x}_L^T\boldsymbol{x}_L\right)^2}{\sqrt{|\boldsymbol{V}_s|}} e^{\frac{-1}{2}\boldsymbol{x}_L^T \boldsymbol{V}_s \boldsymbol{x}_L} \quad \text{, with } \boldsymbol{x}_L = \begin{pmatrix}\xi_x\\ \xi_y\end{pmatrix} \quad \text{, and } \boldsymbol{V}_s = \begin{pmatrix} mss_x & 0 \\ 0 & mss_y\end{pmatrix}$$

mss_x and mss_y are the mean square slopes in the x and y direction

A relative variation can be readily expressed as a weighted sum of relative variations of each of the slope moments.

$$\sigma = \sigma_0\left(1+\frac{\delta\sigma_0}{\sigma_0}\right) \quad \text{, with } \frac{\delta\sigma_0}{\sigma_0} = f_x(\boldsymbol{x}_L, \boldsymbol{V}_s)\frac{\delta\, mss_x}{mss_y} + f_y(\boldsymbol{x}_L, \boldsymbol{V}_s)\frac{\delta\, mss_y}{mss_y}$$

The next challenge is to write the variation of short-wave statistics in terms of the hydrodynamic modulation. The solution is set by the Wave Action Balance which express the energy of the short wave spectrum under the action of energy sources and sinks.

$$N(\boldsymbol{k}_s, x) = \frac{\omega_s}{k_s}\Psi = N_0(\boldsymbol{k}_s) + N_1(\boldsymbol{k}_s, x) = N_0\left(1 + TF_{k_L}^{-1}\left[MTF(\boldsymbol{k}_s, \boldsymbol{k}_L)\, TF[\xi(x,t)]\right]\right)$$

The linear (first-order) ModulationTransfer Function can be written as

$$MTF(\boldsymbol{k}_s \boldsymbol{k}_L) = -\frac{\omega_L - i\mu}{\omega_L^2 + \mu^2}\frac{\omega_L}{k_L}(\boldsymbol{k}_L.\boldsymbol{k}_s)\frac{\boldsymbol{k}_s}{N_0}\frac{\partial N_0(\boldsymbol{k}_s)}{\partial \boldsymbol{k}_s}$$

where μ is a relaxation parameter which describe the growth rate of wave under the action of the sources and sinks used in the WAB (mostly the wind). ω_L characterizes the effect of orbital velocity of the longer waves on the MTF.

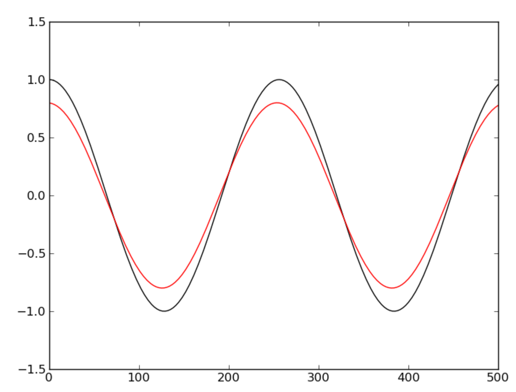

Figure 6- Trends (in red) of the mss_x modulation for a single sinusoidal modulated wave (in back) and a wind of 10m.s^{-1}

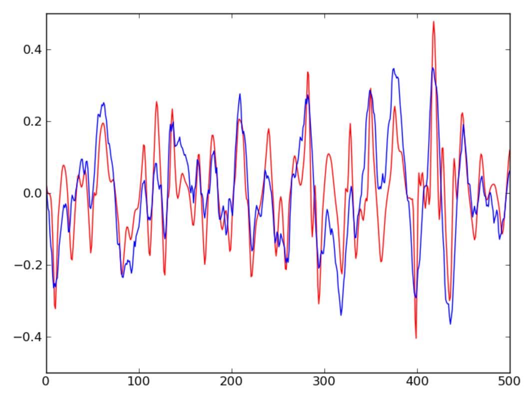

Figure 7- Trends (in red) of the mss_x modulation for a single sinusoidal modulated wave (in back) and a wind of 10m.s^{-1}

The comparison with linear hydrodynamic modulation [3] in figure 7 shows good agreement beetween the 2 approches. The linear hydrodynamic modulations can be written as

$$\frac{\delta\sigma_0}{\sigma_0} = -0.18\frac{Elevation}{\sigma_h}$$

We achieve to simulate a reliable sea surface geometry and EM model that take into account the main physics phenomena that carries the SSB. The two-scale integration scheme is then the key input component of a wave form simulator. The validation of the results of simulations under different radar wavelengths are under progress.

Some improvements are already under work, to take into account the second order of the MTF and the limitation of mms growth due to breaking phenomena.

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