A theory for the vertical scale of sub-inertial currents

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Outline

- Difficulties with previous theory (*Charney*, 1971).
- Extension of theory.
- Testing principles of new theory with QG model.
- Analysis of 800 deep-water moorings.
- Conclusion.

Charney's theory on vertical scale of QG motions (*Charney*, 1971)

- Notes QG PV equation is invariant under rotation about the vertical axis and translations along the vertical axis.
- Concludes that the vertical scale D_u is anticipated to be related to the horizontal scale L_u by

$$D_u = \sigma L_u |f_0| / N, \tag{1}$$

with σ a nondimensional parameter that Charney assumes is of order unity.

Difficult applying theory of *Charney* (1971) to the ocean

- What's the value of Charney's σ parameter?
- QG PV equation is invariant about rotations about the vertical, but the flow is definitely not isotropic (*Scott et al.*, 2008). Related to the imprint of bottom topography on the surface flow, implying an anisotropy *throughout* the water column (NOCS seminar 2008).
- The buoyancy frequency is *not* almost constant in the ocean. On the contrary, it is strongly surface intensified.
- Most importantly, by focusing on the symmetry of the Laplacian operator defining quasigeostrophic potential vorticity, a more general principle may have been overlooked.

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How do we define the length scales?

The horizontal length scale of the geostrophic flow is simply,

$$L_u \equiv \left| \frac{\psi}{\zeta} \right|^{\frac{1}{2}}, \tag{2}$$

where the relative vorticity,

$$\zeta = \nabla^2 \psi.$$

Similarly the vertical scale follows (implicitly) from,

$$\left| \frac{\psi}{S} \right| = \frac{N^2}{f_0^2} \frac{D_u^2 D_N}{|D_N - D_u|}. \tag{3}$$

where D_N is the (known) vertical scale of N^2, and vortex stretching

$$S = \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \psi}{\partial z}\right).$$

When/How are D_u and L_u related?

When S and ζ have similar magnitude then the ratio of D_u to L_u is fixed, implicitly given by,

$$|S| \sim |\zeta| \to L_u = D_u \frac{N}{f_0} \sqrt{\frac{D_N}{|D_N - D_u|}}. \tag{4}$$

When, $D_N \gg D_u$, clearly

$$L_u = D_u \frac{N}{f_0}$$

When does ζ balance S? A statistical theory.

The *full* inviscid QG PV governing equation:

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0,$$

with

$$q = \zeta + S + \beta y,$$

Let us argue that, because the flow is highly turbulent, we expect ψ to visit a wide range of states. In a Lagrangian frame (following the horizontal movement of fluid parcels), q is constrained by conservation of PV, so we expect

$$\langle q^2 \rangle \quad \ll \langle \zeta^2 \rangle \tag{5}$$

$$\langle q^2 \rangle \quad \ll \langle S^2 \rangle \tag{6}$$

If meridional excursions on the mesoscale times (20 to 100 days) are also limited, then main balance between $\zeta \sim S$

$$q = \underbrace{\zeta + S} + \beta y,$$

$$\boxed{\langle (q - \beta y)^2 \rangle \ll \langle \zeta^2 \rangle \sim \langle S^2 \rangle,} \tag{7}$$

Implications for vertical length scale

In which case, Eq. 8 applies,

$$|S| \sim |\zeta| \rightarrow L_u = D_u \frac{N}{f_0} \sqrt{\frac{D_N}{|D_N - D_u|}}. \tag{8}$$

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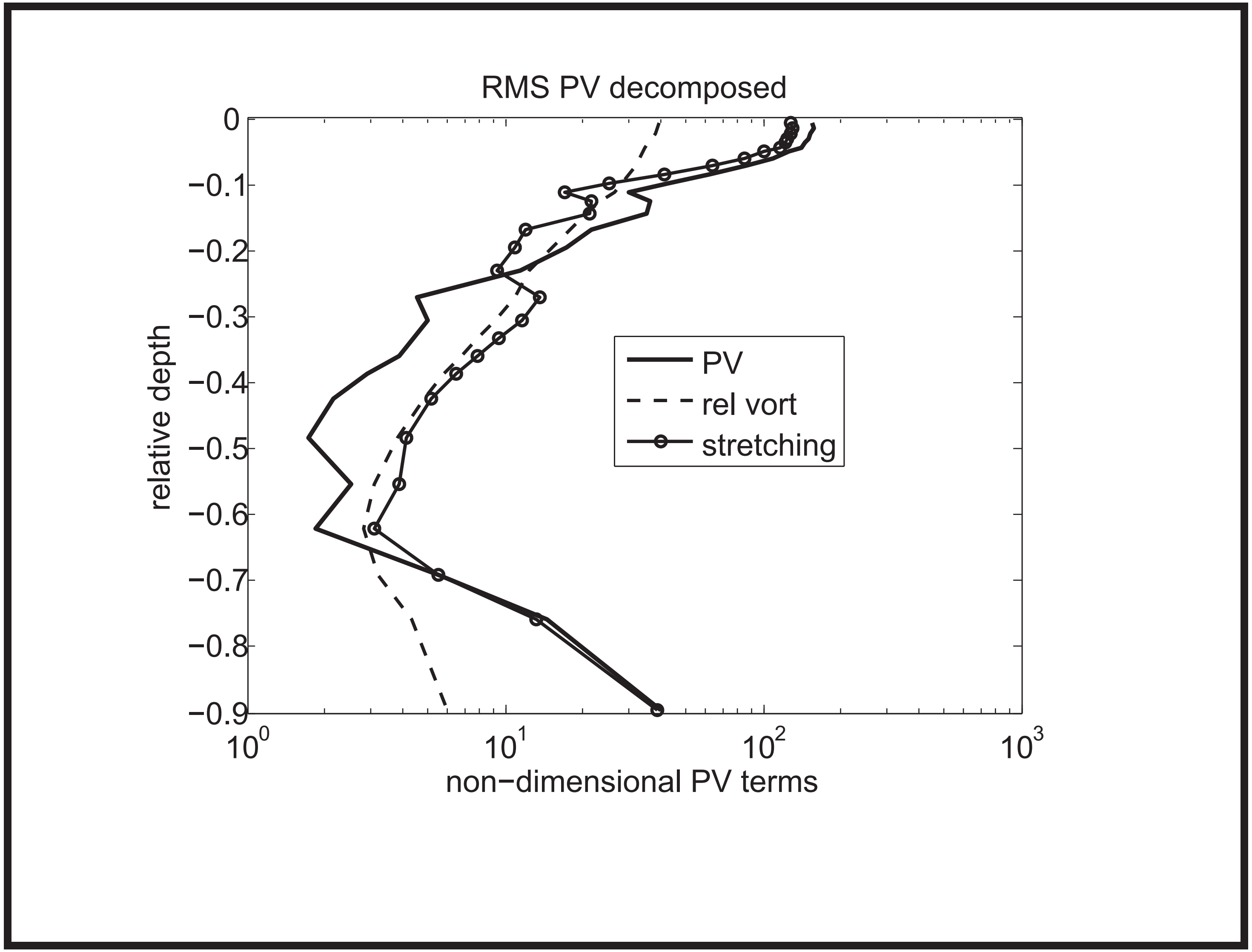
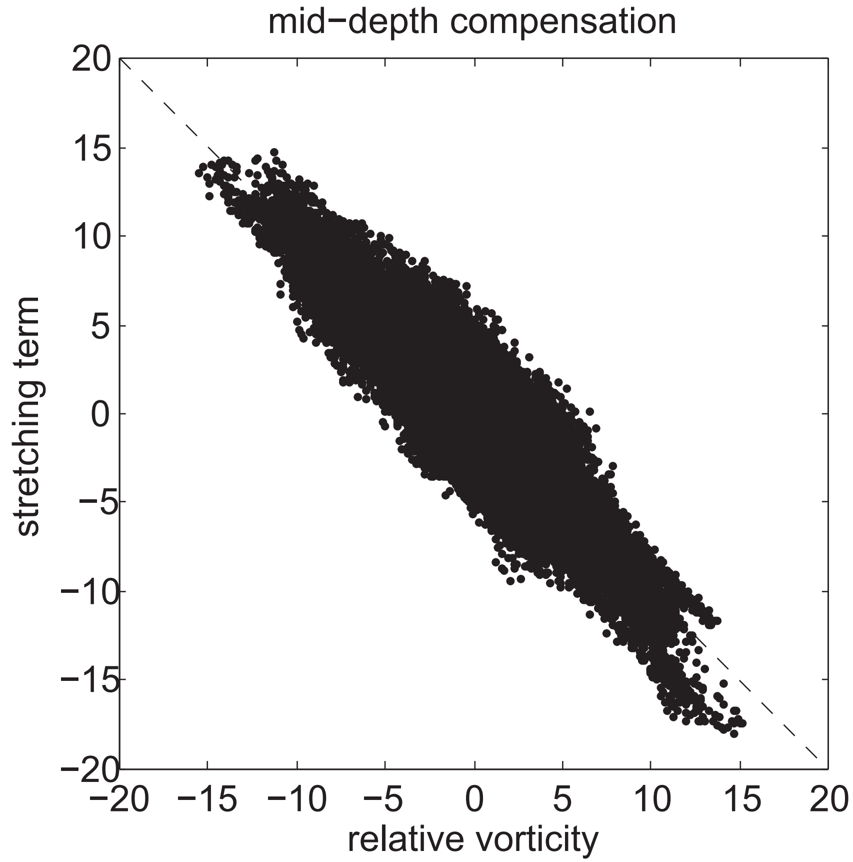
QG simulation, thanks to Shafer Smith

The model configuration similar to *Smith and Ferrari* (2009) but run here to represent Southern Ocean (142°E 51°S).

- QG model run on a doubly periodic, 1000 km square $\beta-$plane.
- Grid points: $256 \times 256 \times 29$.
- Prescribed mean flow kept the model baroclinically unstable state with energetic eddy activity.


RMS PV decomposed
relative depth
non−dimensional PV terms
PV
rel vort
stretching
0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
10^0
10^1
10^2
10^3


mid−depth compensation
stretching term
relative vorticity
20
15
10
5
0
−5
−10
−15
−20
−20 −15 −10 −5 0 5 10 15 20

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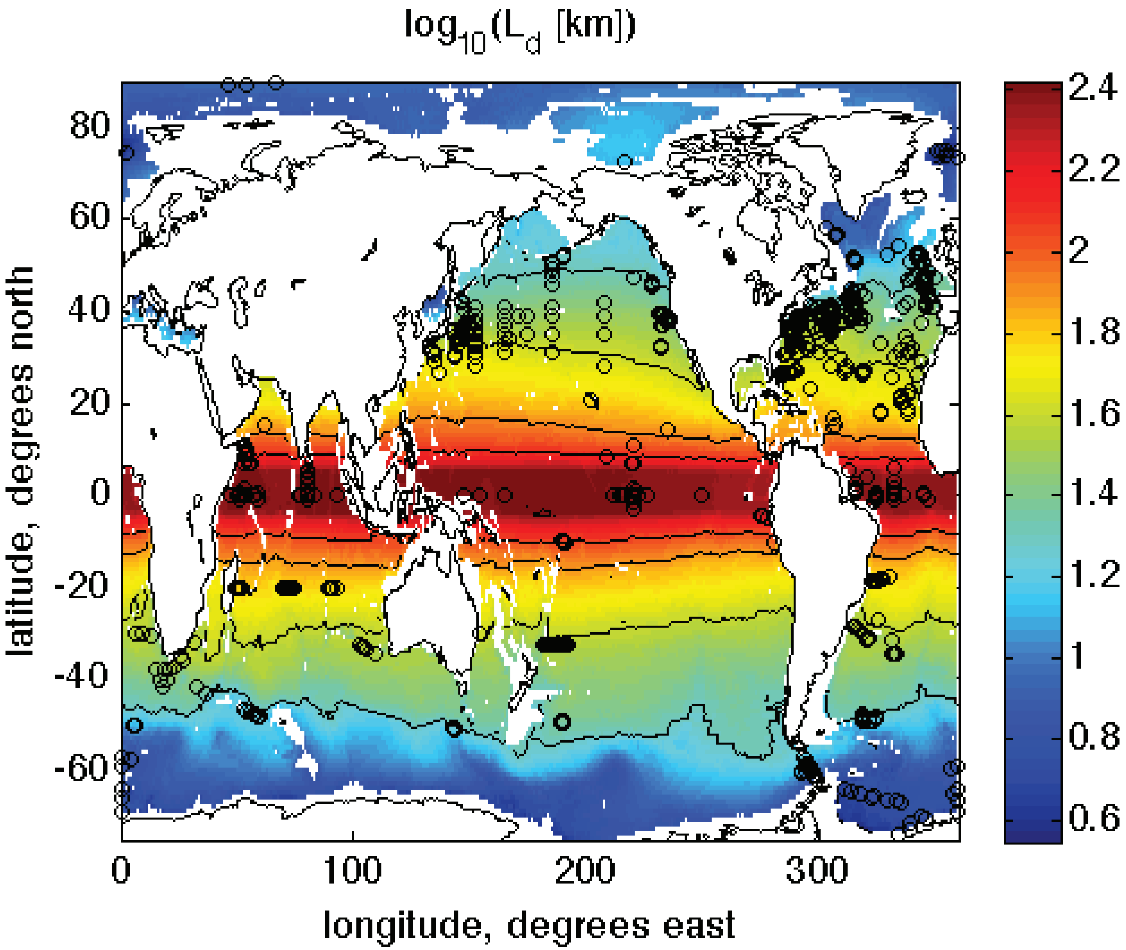


Figure 1: Locations of 824 moorings in water at least 3000 m deep. Black lines indicate contours at 20 km, 40 km, 80 km, and 120 km.

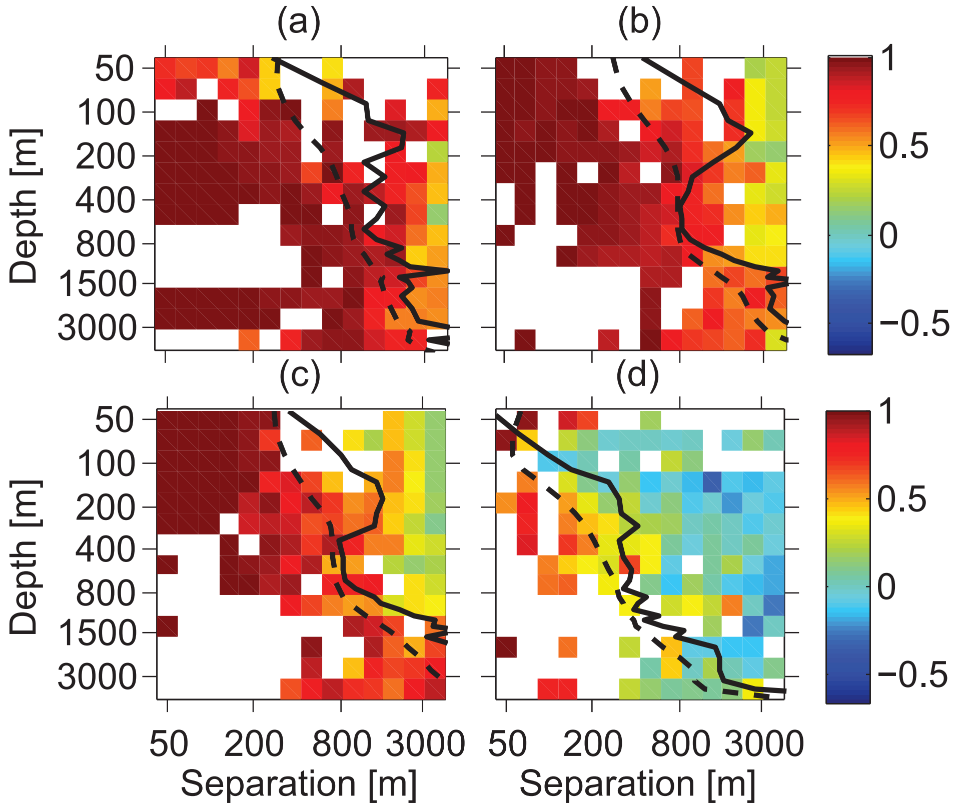
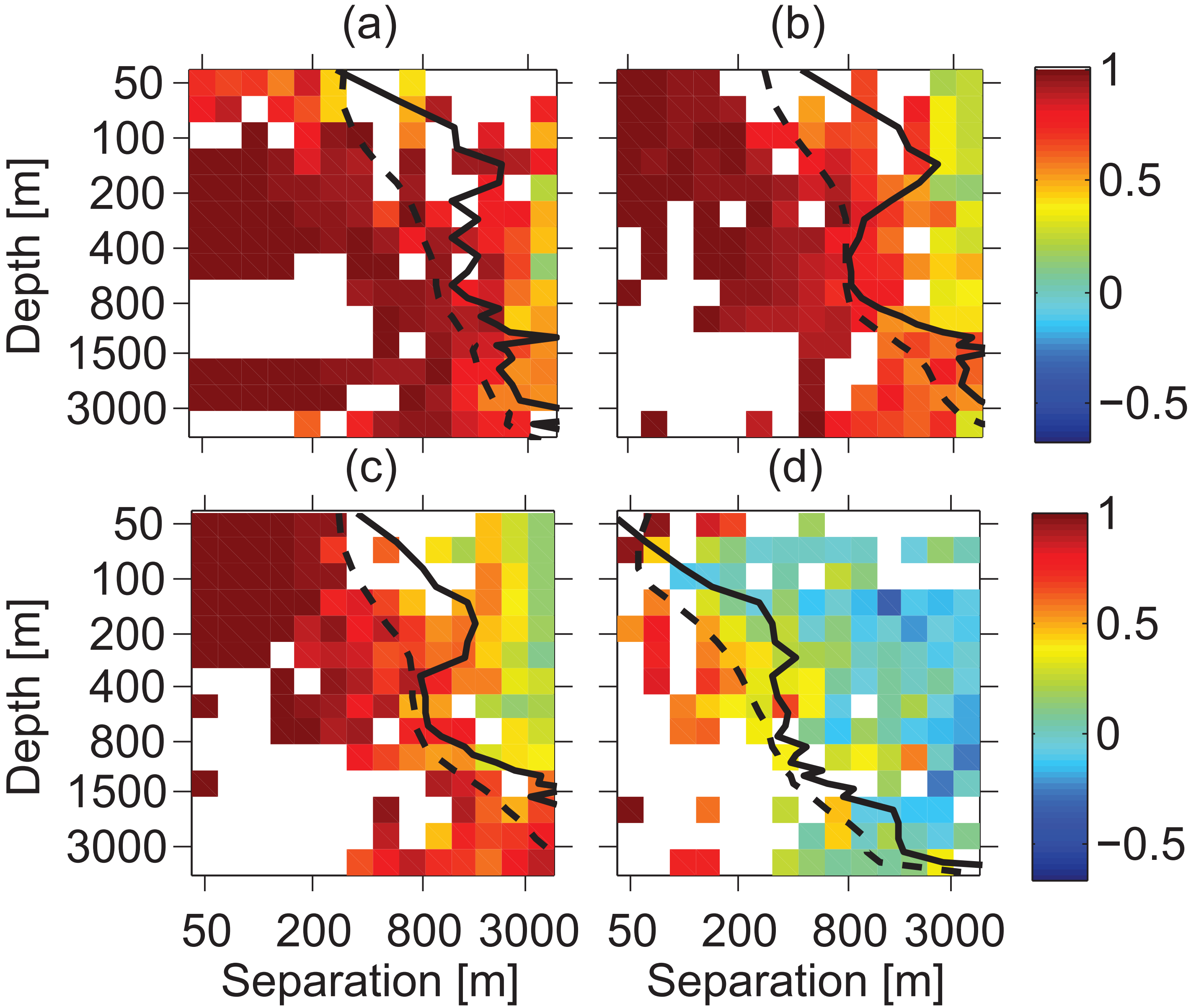
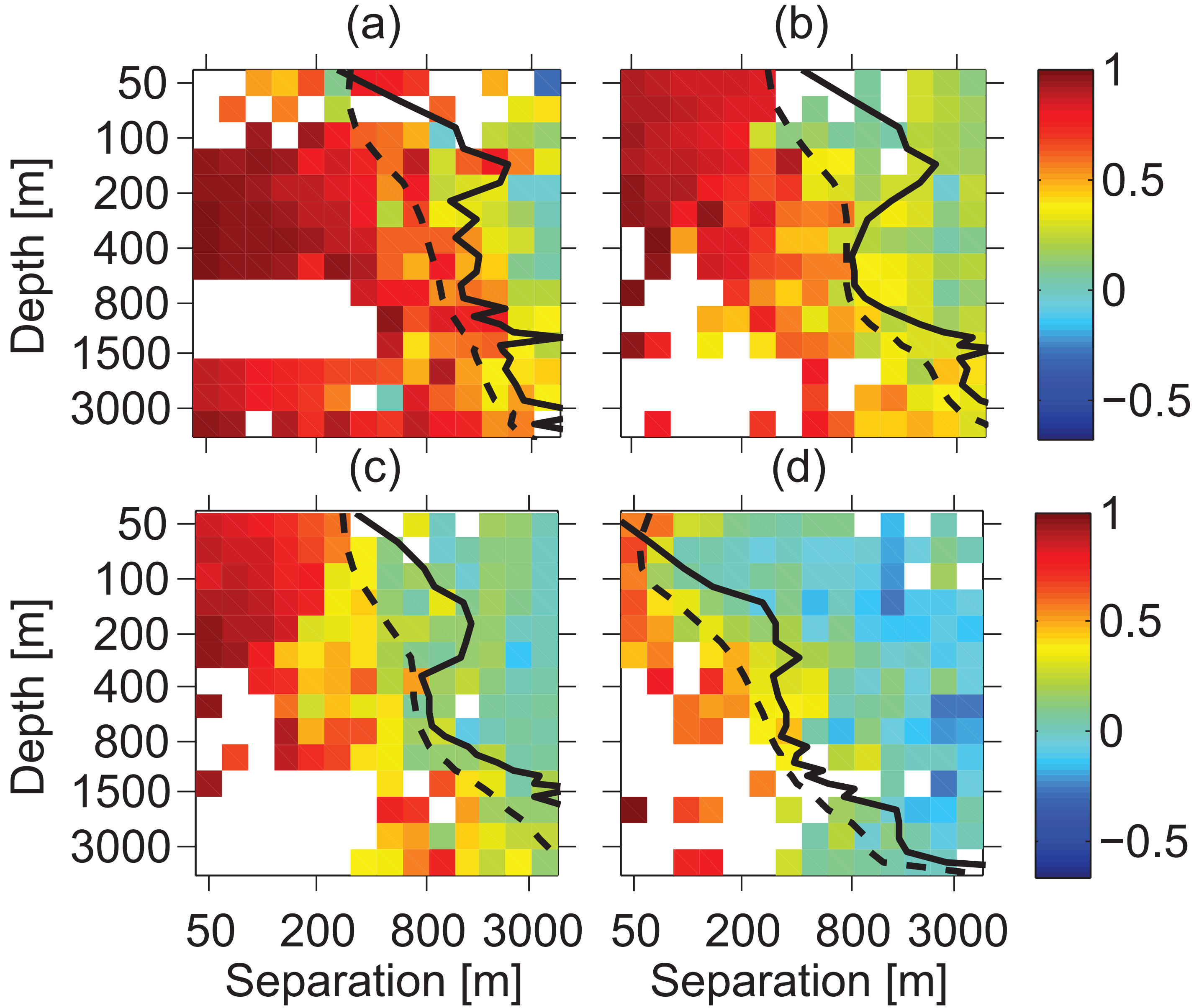


Figure 2: Weighted average of Pearson correlation coefficient (a) $4 \leq L_d < 20$ km; (b) $20 \leq L_d < 40$ km; (c) $40 \leq L_d < 80$ km; (d) $120 \leq L_d < 260$ km. Solid and dashed lines show weighted average of theoretical D_u and $L_u = L_d$.


(a)
(b)
(c)
(d)
Depth [m]
50
100
200
400
800
1500
3000
Separation [m]
50
200
800
3000
1
0.5
0
−0.5



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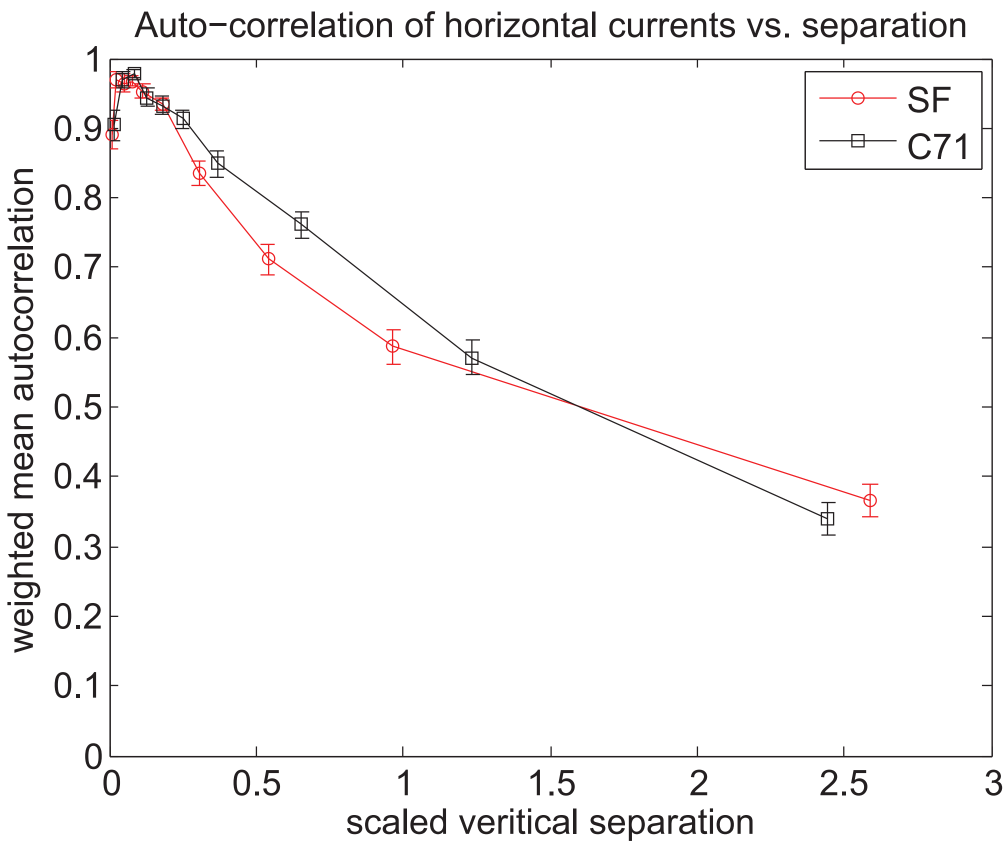

Figure 3: Scale actual vertical separation by theoretical length scale

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- Charney's theory sort of works, but justification (based upon symmetry) is not applicable to ocean, needs to be extended.
- Simple extension based upon different fundamental principles – not symmetry of governing equations, but balance between relative vorticity and vortex stretching that arises, perhaps, because of statistical mechanics.
- Principles supported with idealized QG simulation, but only away from top and bottom boundary.
- Theory confirmed by comparison with horizontal velocity auto-correlation as a function of upper current meter depth, and vertical separation between upper and lower current meter.
- New theory matches auto-correlation function better than C71.

References

Charney, J. (1971), Geostrophic turbulence, *J. Atmos. Sci.*, *28*, 1087–1095.

Scott, R. B., B. K. Arbic, C. L. Holland, A. Sen, and B. Qiu (2008), Zonal versus meridional velocity variance in satellite observations and realistic and idealized ocean circulation models, *Ocean Modelling*, *23*, 102–112.

Smith, K. S., and R. Ferrari (2009), The production and dissipation of compensated thermohaline variance by mesoscale stirring, *J. Phys. Oceanogr.*, *39*, 2477–2501.

on shoulders of Charney (1971)