Relation between mesoscale surface motions and baroclinic and SQG modes

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Upper oceanic layers at mesoscale

Classical paradigm

- QG turbulence driven by **interior potential vorticity**
  - Kinetic Energy in $\mathbf{k^{-3}}$ at mesoscales (Charney, 1971)
- The altimeter sees **1st baroclinic mode** (Stammer, 1997)
  - Transfer of surface (baroclinic) KE towards **small scales**

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In contradiction with recent results for ocean surface

- Kinetic energy spectra in $\mathbf{k^{-5/3}}$
  (Le Traon et al. 2008 using altimeter SSH,
  Klein et al. 2008, Capet et al. 2008 in PE simulations at HR)
- Transfer of surface Kinetic Energy towards **large scales**
  observed using SSH data (Scott and Wang 2005)

$\Rightarrow$ **What does the satellite really see in SSH?**

Potential vorticity inversion

QG PV inversion $\equiv$ invert an **elliptic equation** :

$$\underbrace{\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}}_{\text{relative vorticity}}+\underbrace{\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right)}_{\text{vortex stretching}}=PV$$

with **surface boundary condition**

$$f_0\left.\frac{\partial\psi}{\partial z}\right|_{z=0}=b|_{z=0} \qquad\qquad b=-\frac{g\rho}{\rho_0}$$

Important remark:

b **at surface plays the same role as interior PV!**

Separation in vertical modes

$$\underbrace{\psi(x,y,z)}_{\text{geostrophic streamfunction}} = \underbrace{\sum_j \phi_j(x,y)F_j(z)}_{\text{interior modes}} + \underbrace{E(x,y,z)}_{\text{surface mode}}$$

with interior (barotropic and baroclinic) modes satisfying

$$\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial F_j}{\partial z}\right) = -\lambda_j^{-2}F_j \quad \text{and} \quad \left.\frac{\partial F_j}{\partial z}\right|_{z=0} = 0$$

while surface (SQG-type) mode satisfying

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial E}{\partial z}\right) = 0 \quad \text{and} \quad f_0\left.\frac{\partial E}{\partial z}\right|_{z=0} = b|_{z=0}$$

Surface (SQG) mode $E(x,y,z)$ can be obtained using surface signal (e.g. SST or SSH) and typical stratification (Lapeyre et al. 2006)

Mode properties

Baroclinic modes $F_j(z) \approx \cos\left(\frac{N\,\lambda_j^{-1}\,z}{f_0}\right)$

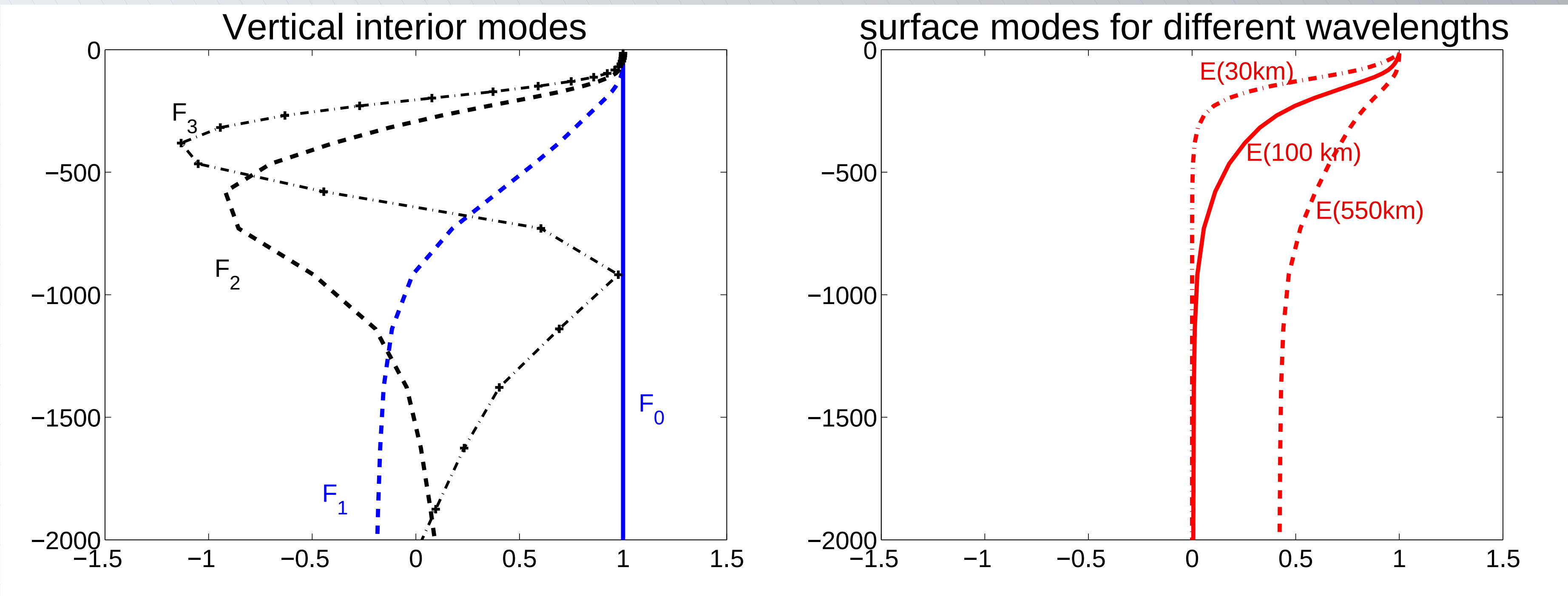
- associated with interior PV anomalies
- no surface density anomalies
- fixed deformation radius $\lambda_j = \frac{N\,H}{j\,\pi\,f_0}$ with $j = 0, 1, 2\ldots$

SQG mode $E(k, z) \approx \exp\left(\frac{N\,k\,z}{f_0}\right)$

- associated with surface density anomalies
- no interior PV anomalies
- depend continuously on horizontal wavelength k^{-1}

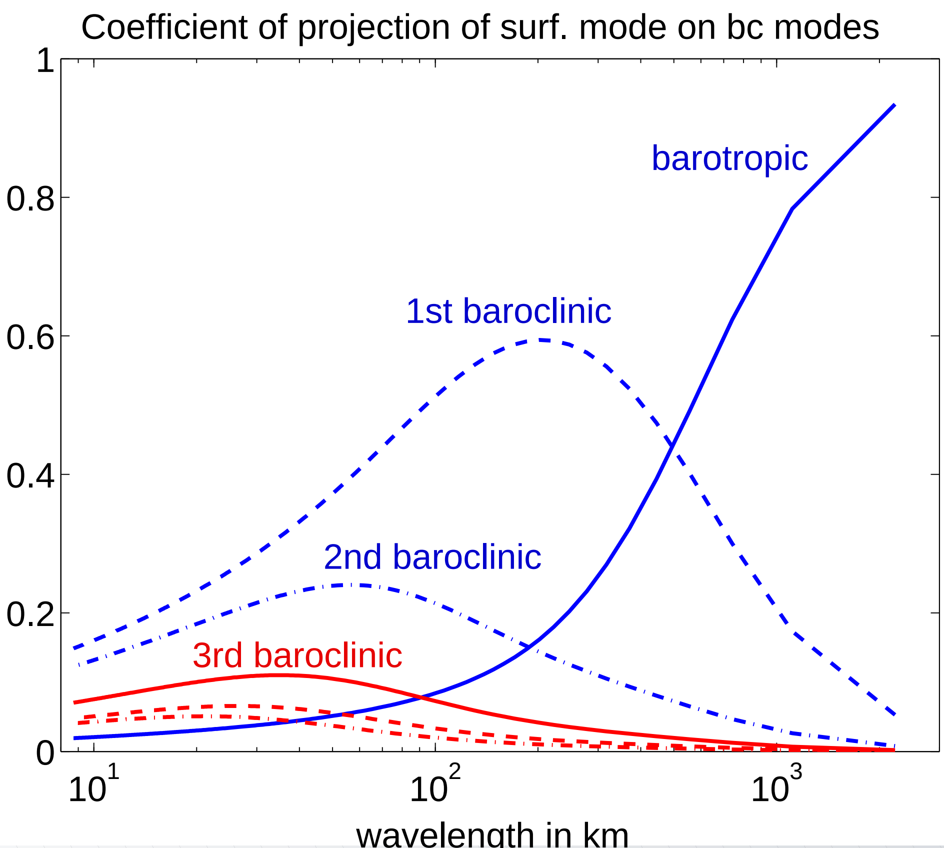
Vertical profile of modes

modes computed using stratification in the Gulf Stream



- Surface trapping of SQG mode and 1st baroclinic mode
- SQG mode decays more rapidly for smaller horizontal wavelength
- 1st baroclinic mode decays with depth more slowly

Non orthogonality of SQG and baroclinic modes



$$C_j(k) = \frac{\int_{-H}^{0} E(k,z) F_j(z) dz}{\int_{-H}^{0} F_j(z) F_j(z) dz}$$

using typical stratification in the Gulf Stream

- At mesoscales (60-400 km), SQG mode projects mainly onto first baroclinic mode
- Above 500 km, SQG mode projects onto barotropic mode
- Below 30 km, SQG mode projects evenly between baroclinic modes

Importance of the surface SQG mode?

Surface mode:

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\right) = 0 \qquad \text{with} \quad \left.\frac{\partial\psi}{\partial z}\right|_{z=0} = -\frac{g}{f_0}\frac{\rho|_{z=0}}{\rho_0}$$

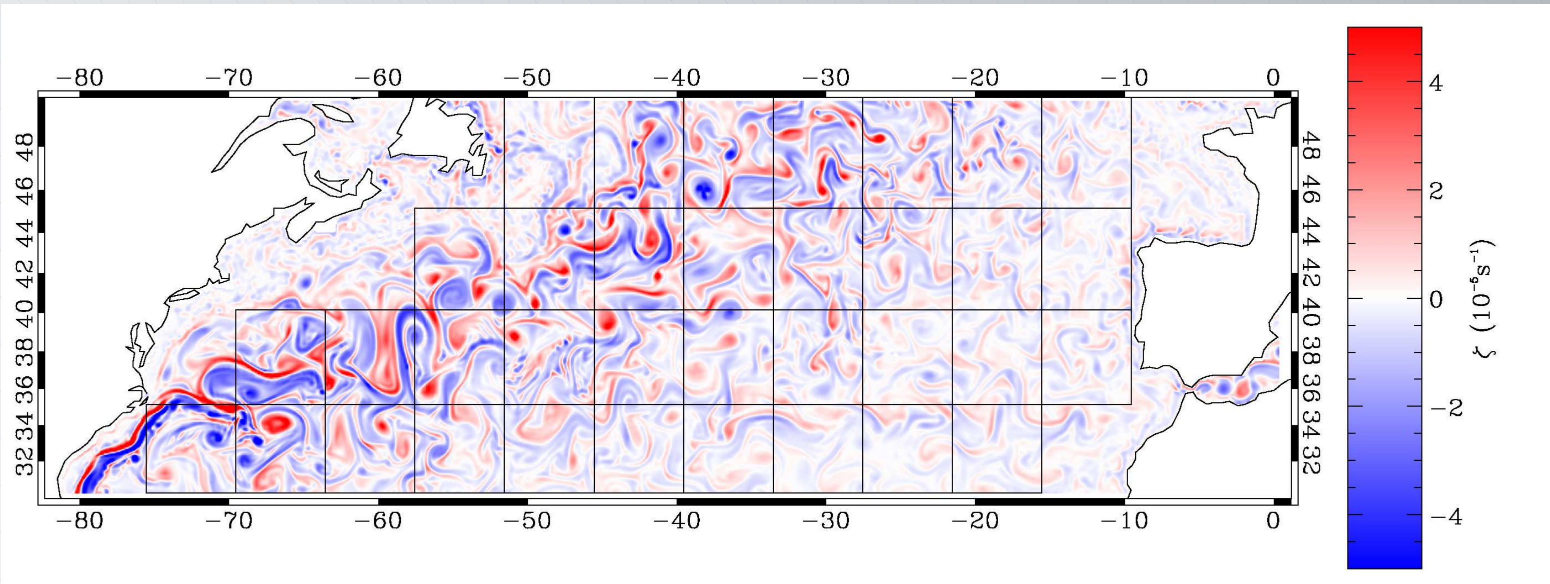
Solution with constant N^2

$$\widehat{\psi}(\vec{k}, z) \;=\; -\frac{g}{\rho_0 N}\;\frac{\widehat{\rho}_{\mathbf{s}}(\tilde{\mathbf{k}})}{|\mathbf{k}|}\;\exp\left(\frac{N}{f_0}|k|\,z\right)$$

- SQG balance implies $\mathbf{SSH} = \mathbf{k^{-1}}\ \mathbf{SST}$ in Fourier space
- Information like SWOT allows to know 3D current above 500m

  (Lapeyre and Klein, 2006, Isern-Fontanet et al. 2006, Klein et al. 2009)
- SQG model studied by Held et al. (1995) among others
- Lesieur and Sadourny (1981) first proposed its usefulness for the ocean to explain chlorophyl spectra

Decomposition in a numerical simulation of the North Atlantic



$$\psi(x,y,z) = \underbrace{\sum_j \phi_j(x,y)F_j(z)}_{\text{interior modes}} + \underbrace{E(x,y,z)}_{\text{surface mode}}$$

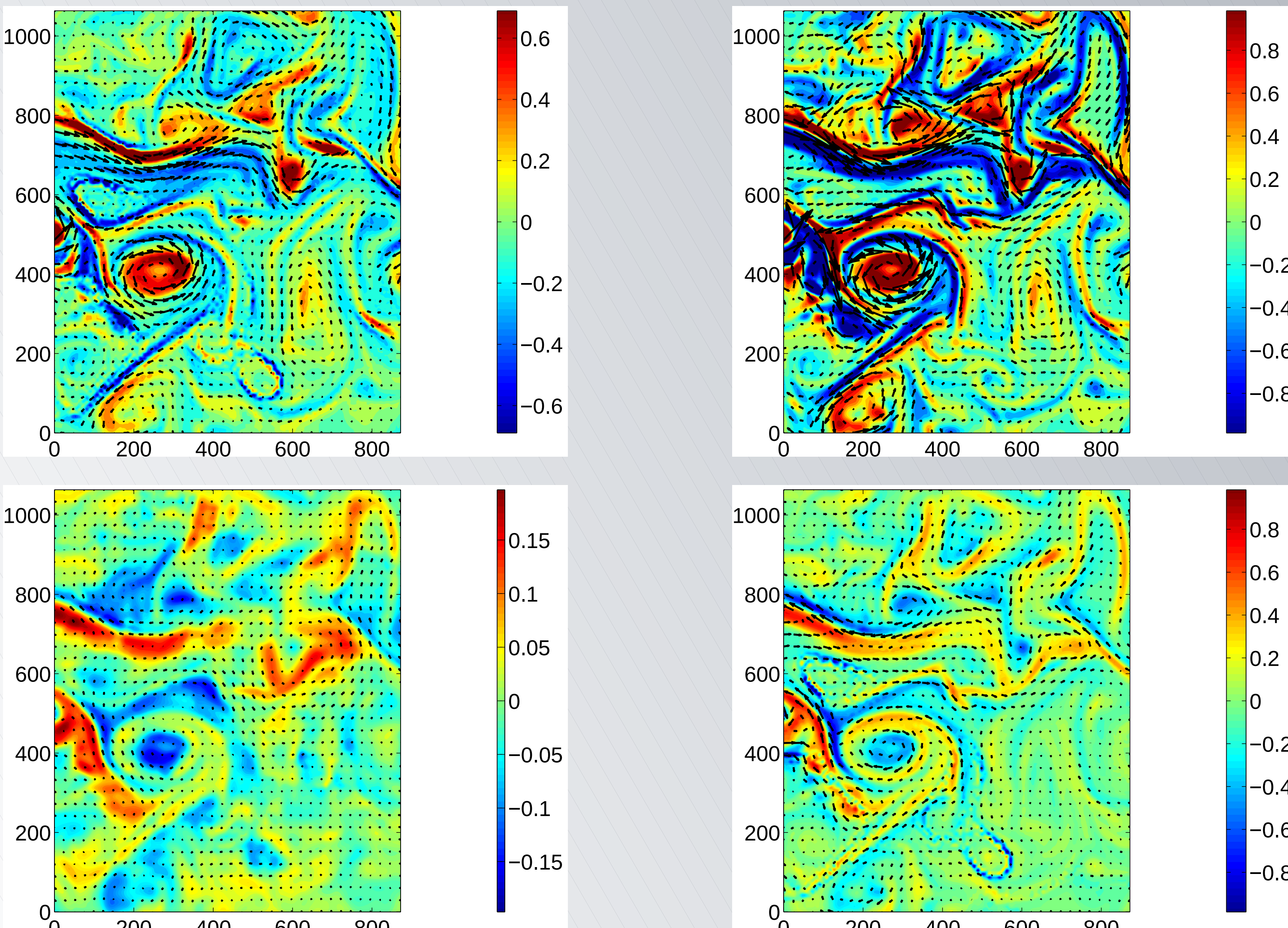
- POP model at $1/10^\circ$ and 40 vertical levels
- forced with realistic winds and heat fluxes

Decomposition into 8 barotropic$+$baroclinic modes and surface mode

$\Rightarrow$ satisfies surface boundary condition in density

Surface mode dominates the surface signal in Gulf Stream

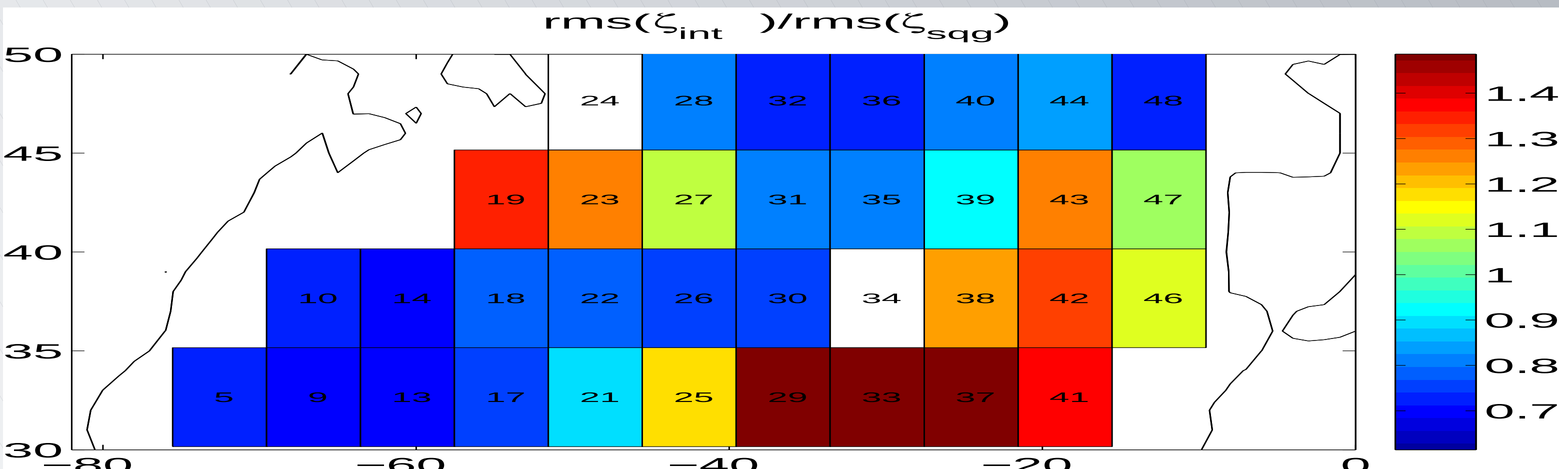
surface vorticity **surface (SQG) mode**



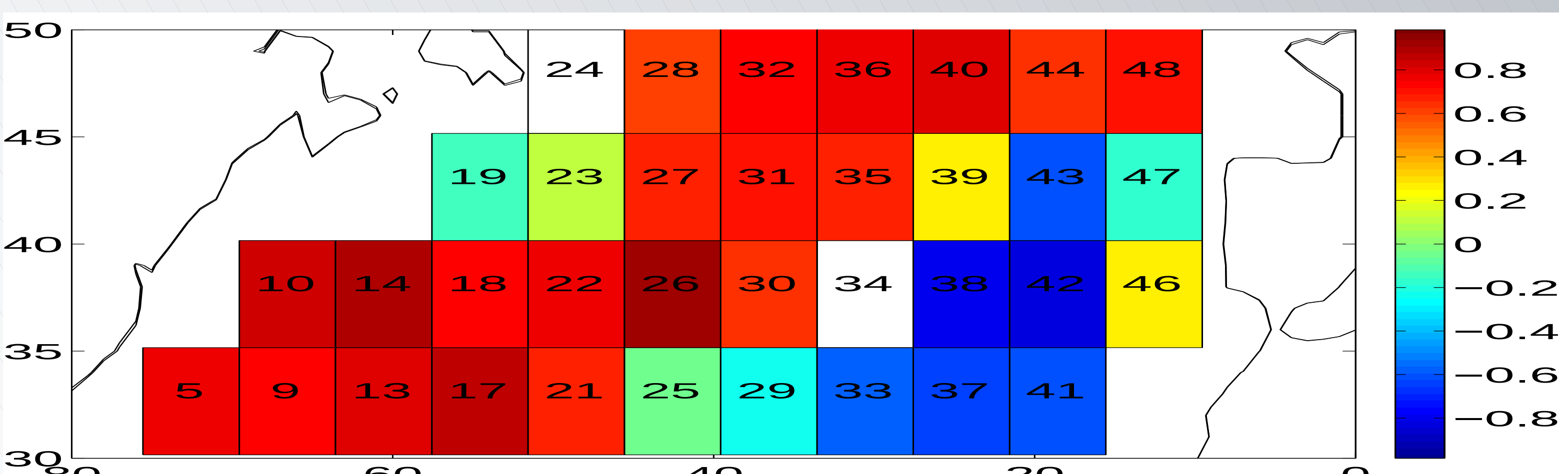
barotropic mode **1st baroclinic mode**

Surface mode dominates the surface signal in general

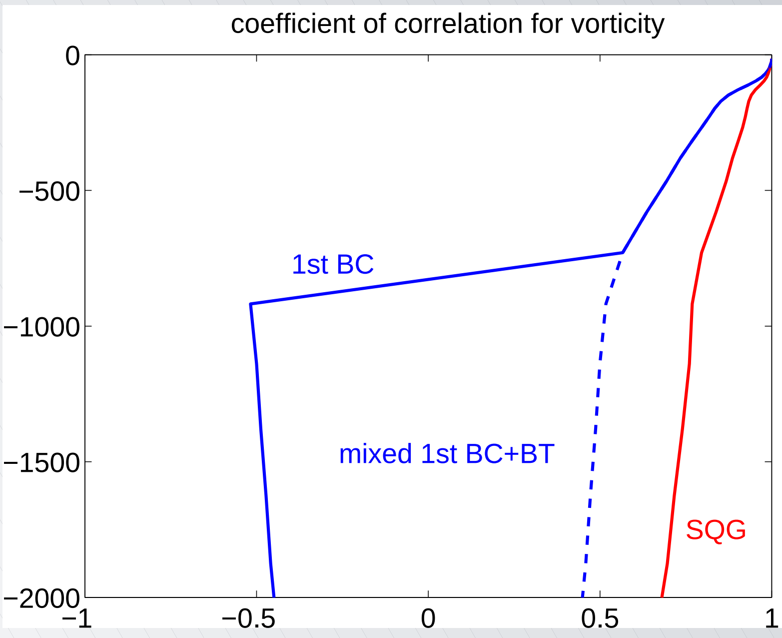
ratio of rms vorticity of interior modes vs surface (SQG) mode

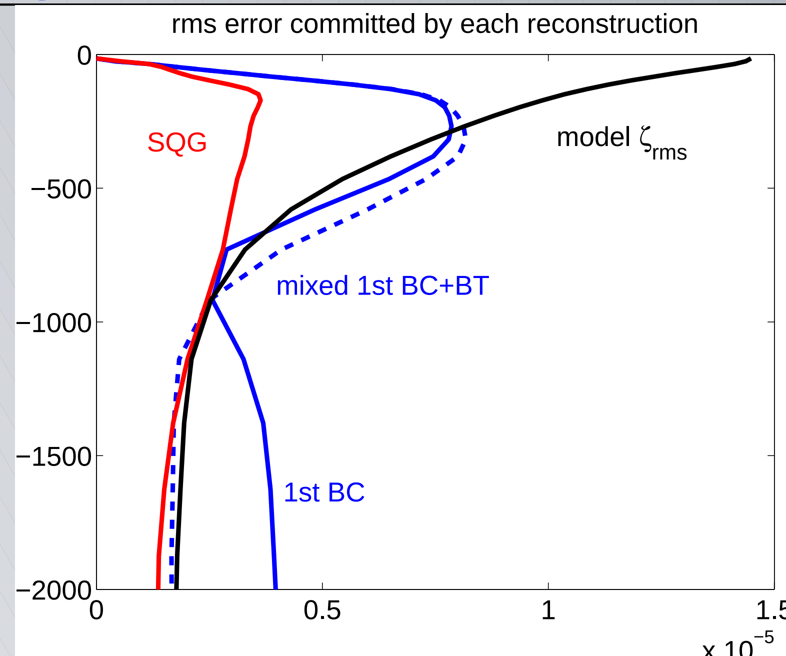


correlation surface (SQG) mode and observed vorticity at surface



Comparison of reconstruction using 1st baroclinic and SQG modes





– SQG reconstruction: $\zeta^{SQG}(z) = \zeta(z=0)\exp\left(\frac{N_0 kz}{f_0}\right)$

– reconstruction using $F_1(z)$: $\zeta^{BC}(z) = \zeta(z=0)\,F_1(z)/F_1(0)$

- - using $G(z) = aF_0(z) + bF_1(z)$ such that $G(z=-H) = 0$

$\zeta^{mixed}(z) = \zeta(z=0)\,G(z)/G(0)$ (proposed by R. Scott)

all using surface relative vorticity $\zeta(z=0)$ in Gulf Stream region

Conclusions

- Two types of modes:
  - baroclinic (interior) modes related to interior PV anomalies
  - SQG-type (surface) mode
    related to surface buoyancy anomalies
- Non-orthogonality of SQG surface mode and baroclinic modes
- In most part of the North Atlantic,
  - **SQG mode contribution dominates the surface signal**
  - 1st baroclinic mode is the second contribution
    but dominates only in the recirculating part of the gyre
- Link between SSH and SST for SQG mode
- **The altimeter sees the SQG signal and not the 1st baroclinic mode**

(Lapeyre 2009, JPO)

Non orthogonality of surface and baroclinic modes

Defining dot product corresponding to total energy norm:

$$\iiint \left(\nabla\psi \cdot \nabla\phi + \frac{f_0^2}{N^2}\frac{\partial\psi}{\partial z}\frac{\partial\phi}{\partial z} \right) \, dxdydz =$$

$$\int \oiint \psi \nabla\phi \cdot d\vec{n} \, dz + \iint \left[\frac{f_0^2}{N^2}\psi\partial_z\phi \right]_{-H}^{0} \, dxdy$$

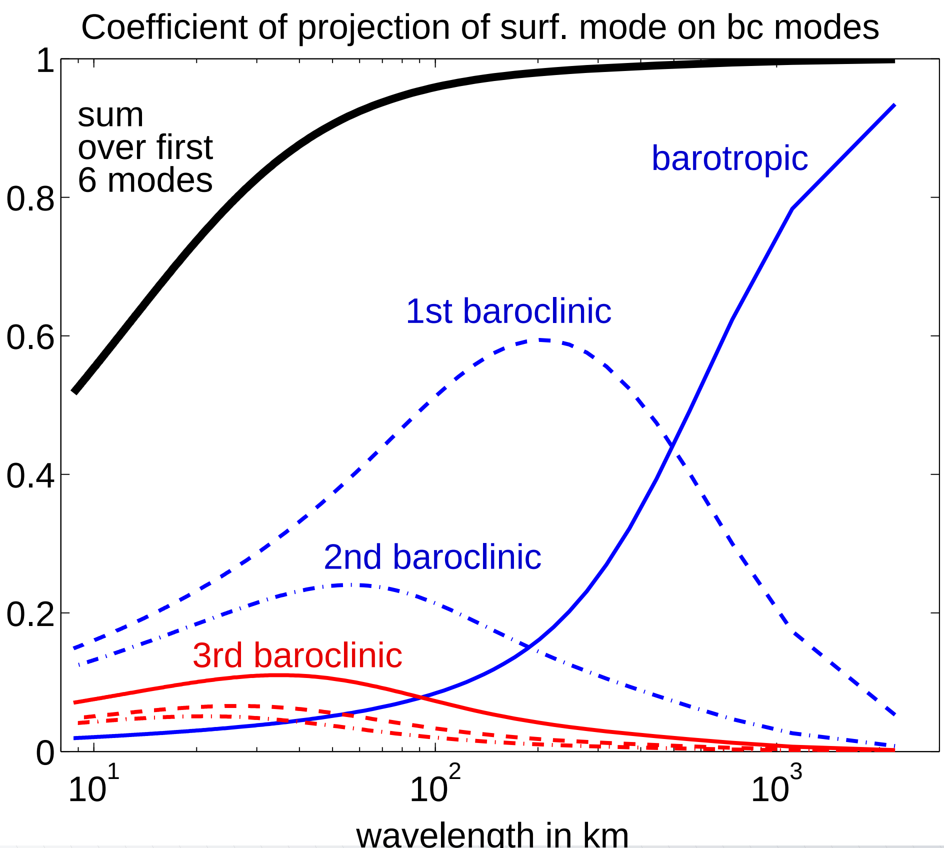
$$- \iiint \psi \left(\nabla^2\phi + \left(\frac{f_0^2}{N^2}\partial_z\phi \right) \right) \, dxdydz$$

If ψ projects onto 1 baroclinic mode,
and ϕ projects onto 1 surface mode, then

$$\iiint \nabla\psi \cdot \nabla\phi + \frac{f_0^2}{N^2}\partial_z\psi\partial_z\phi \, dxdydz = \iint \left[\frac{f_0^2}{N^2}\psi\partial_z\phi \right]_{-H}^{0} \, dxdy \neq \mathbf{0}$$

with $\partial_z\phi = -\frac{g}{f_0\rho_0}\rho$

Non orthogonality of surface and baroclinic modes



$$C_j(k) = \frac{\int_{-H}^{0} E(k,z) F_j(z) dz}{\int_{-H}^{0} F_j(z) F_j(z) dz}$$

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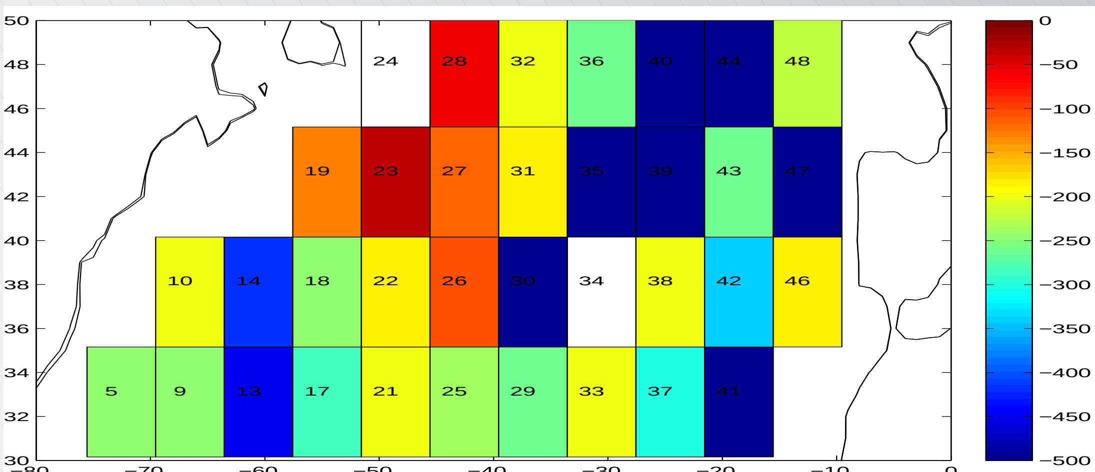
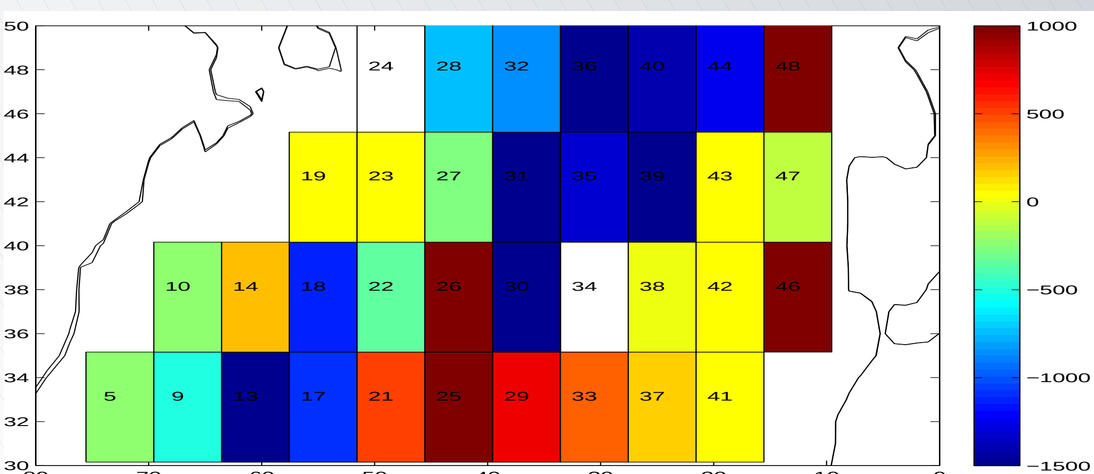
Charney scales

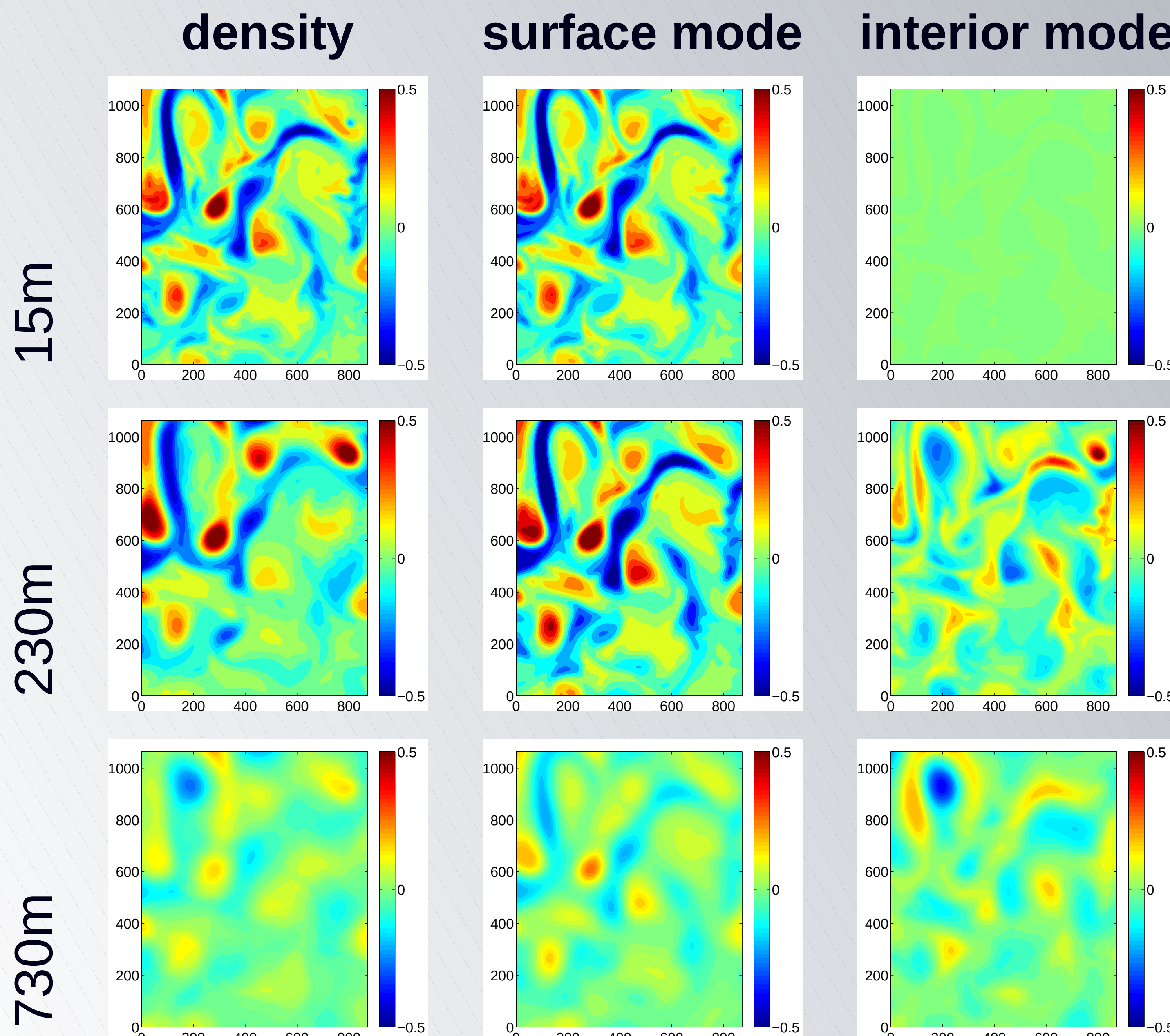




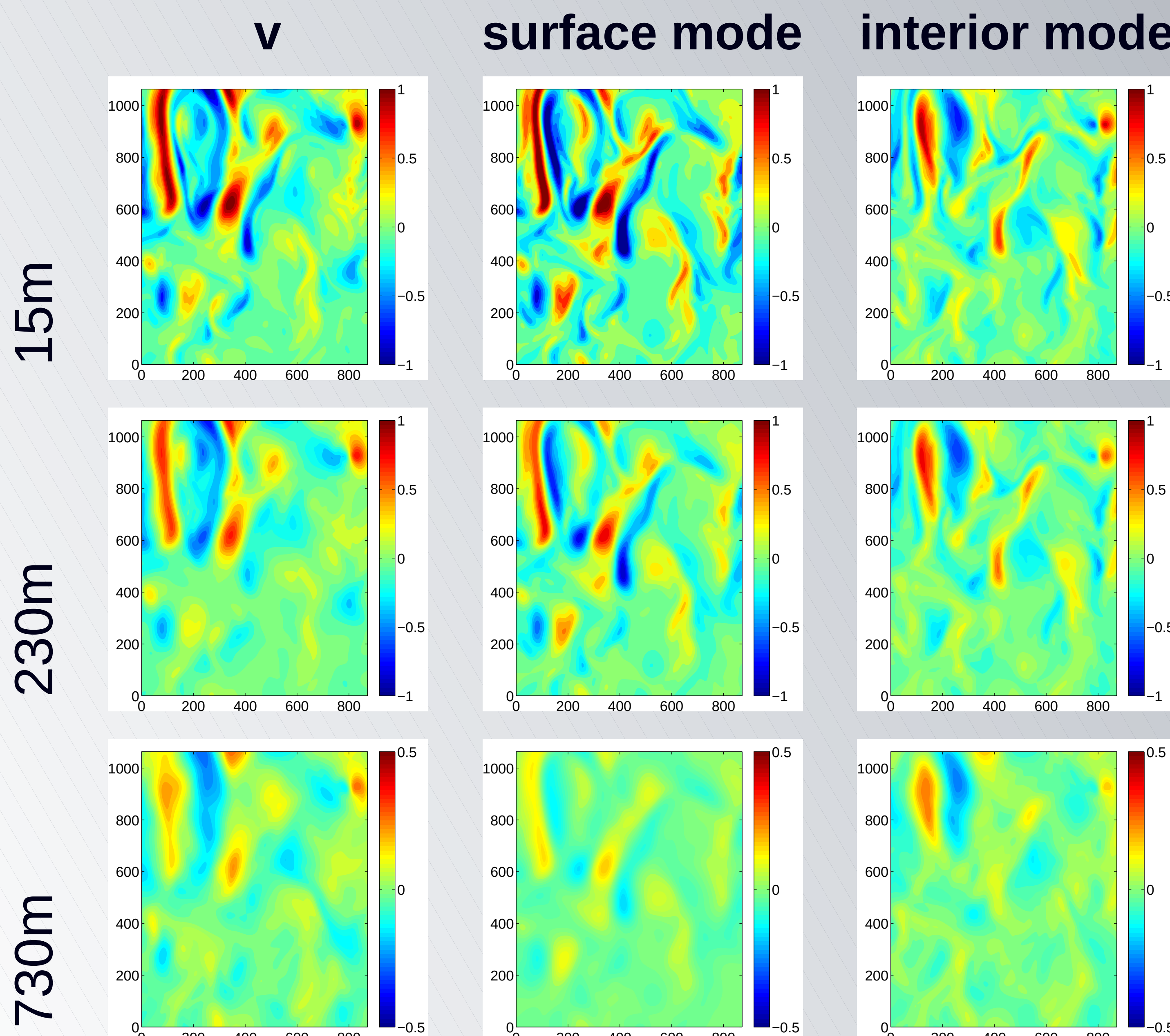
Depth of change of sign of $\partial_y PV$

$$h_c = \frac{g f_0^2}{\rho_0 N^2} \frac{\partial_y \rho_s}{\partial_y PV}$$

Density decomposition



Meridional velocity decomposition



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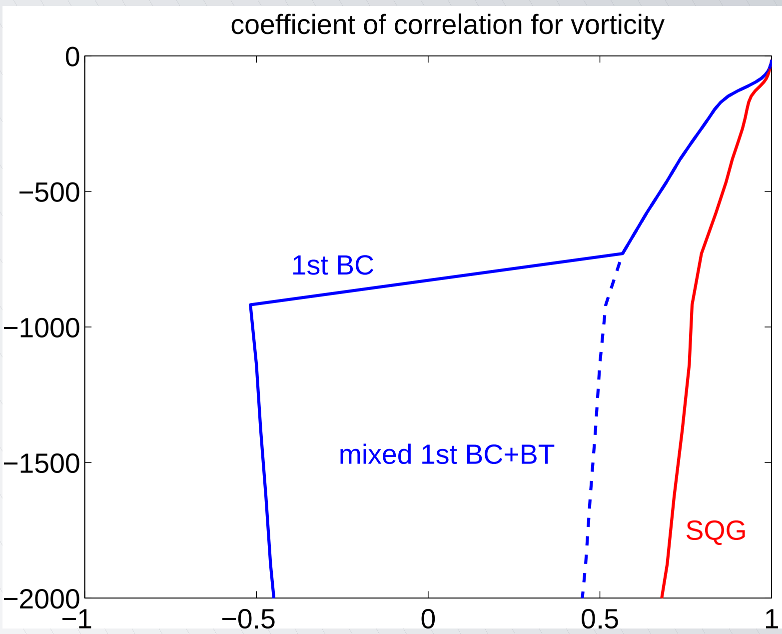

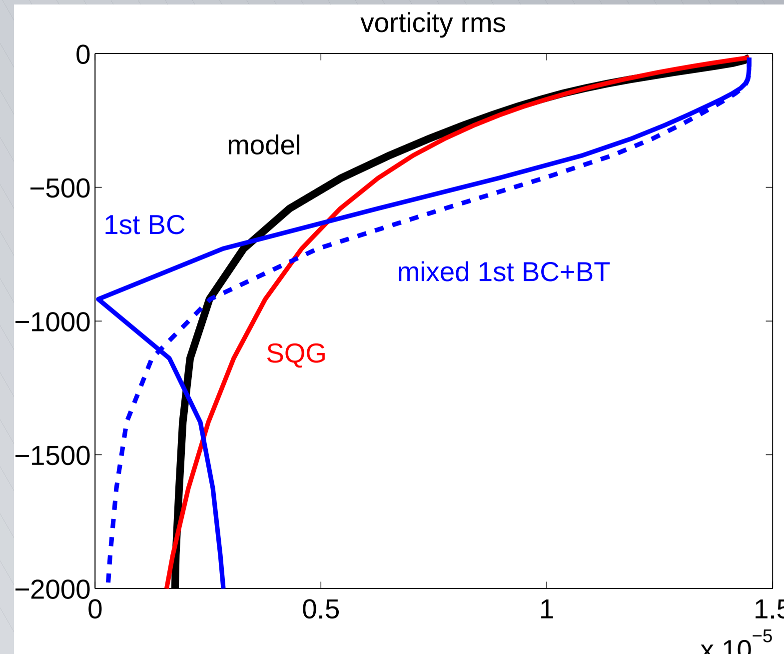

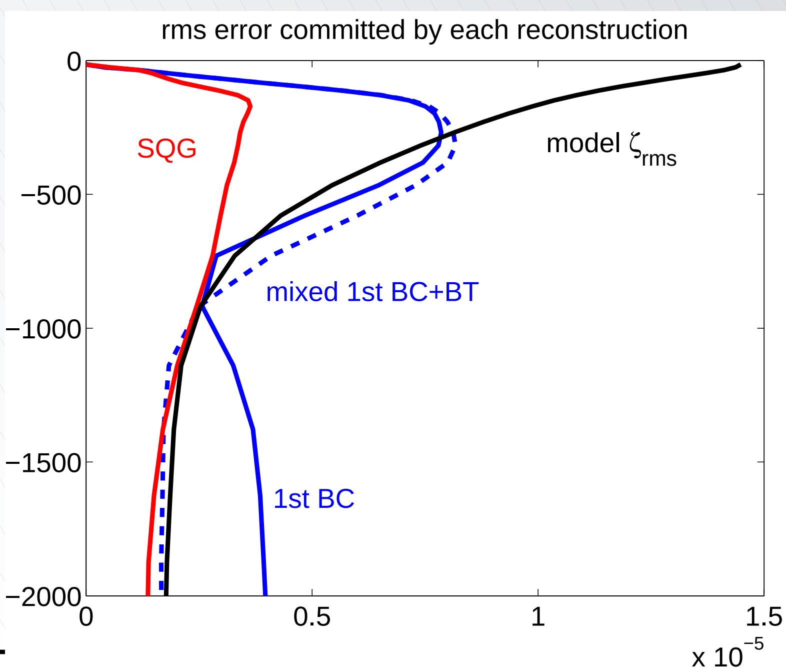

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