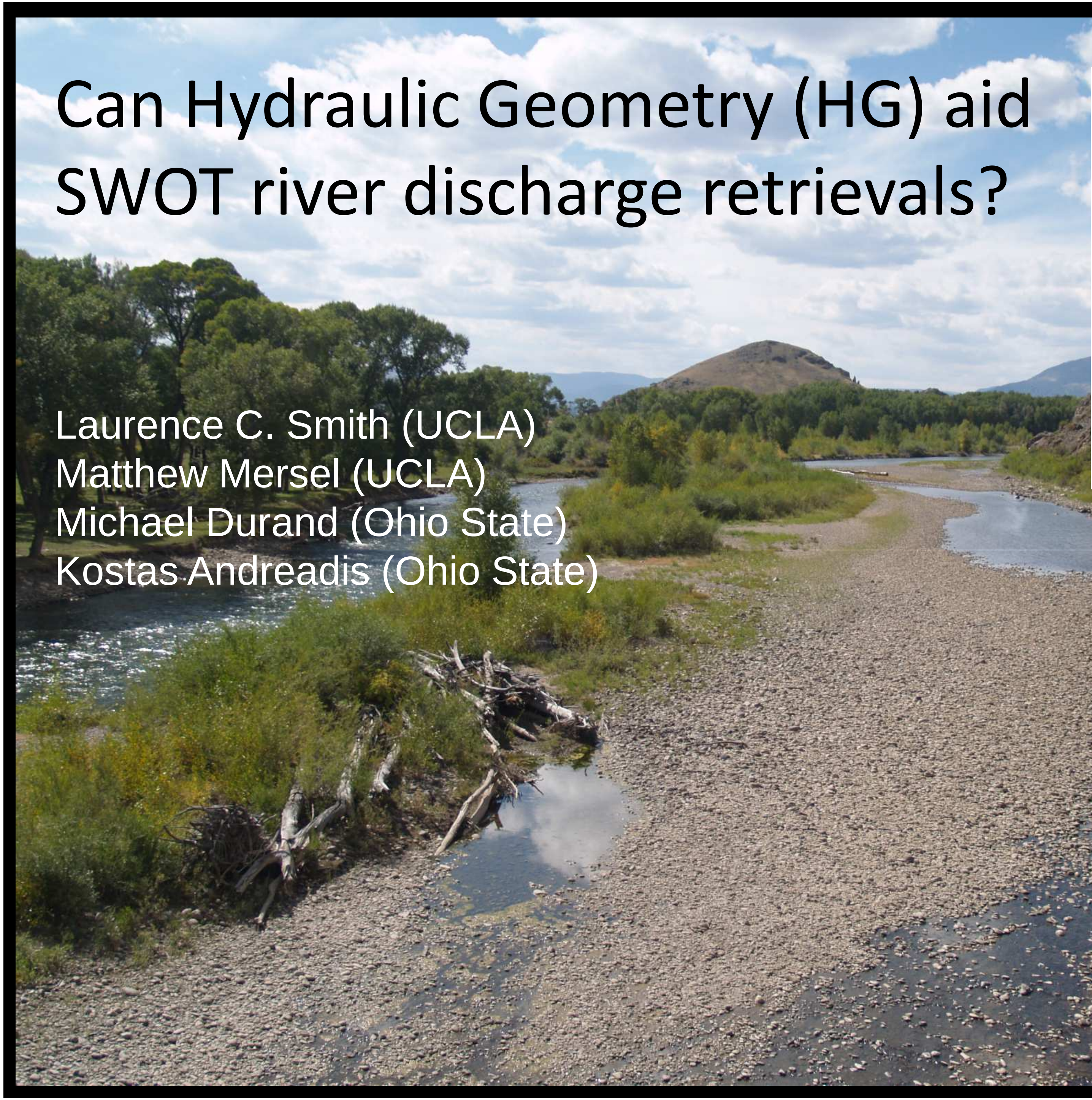

Can Hydraulic Geometry (HG) aid SWOT river discharge retrievals?

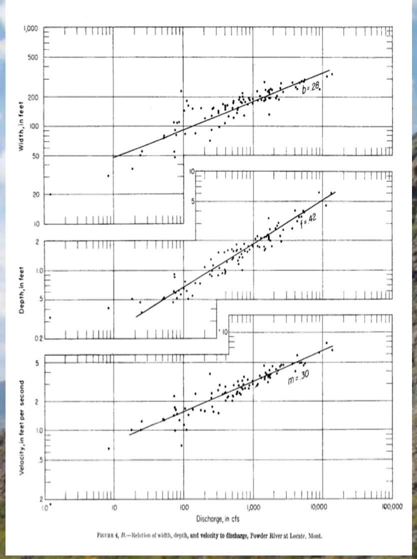

FIGURE 4, B.—Relation of width, depth, and velocity to discharge, Powder River at Locate, Mont.

Laurence C. Smith (UCLA)
Matthew Mersel (UCLA)
Michael Durand (Ohio State)
Kostas Andreadis (Ohio State)

SWOT will measure: A_t and dH/dt; dH/dx

Lakes: $(A_t)(dH/dt) = dS/dt$
(storage anomaly, m^3/dt)

Rivers: $(A_t/L)(dH/dt)(v_{est}) = dQ/dt$
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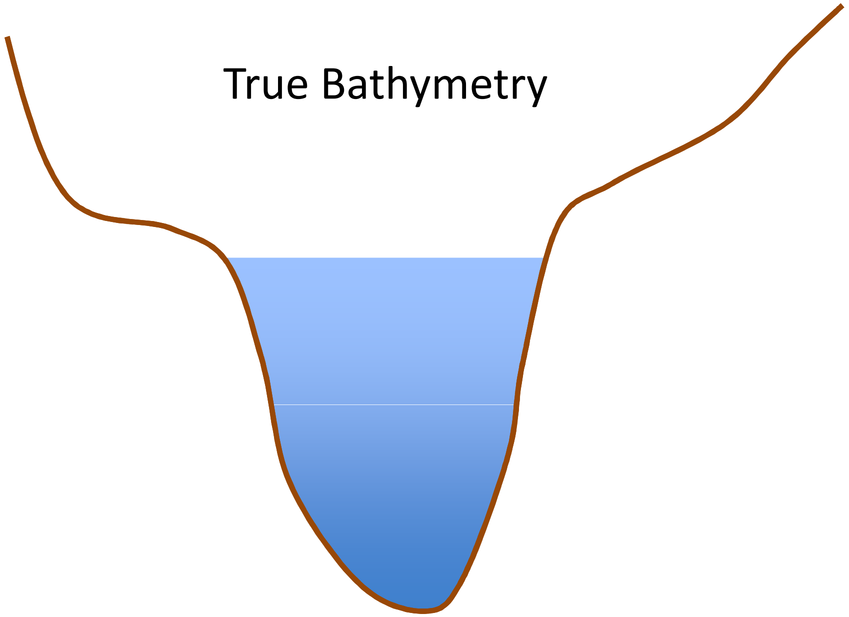
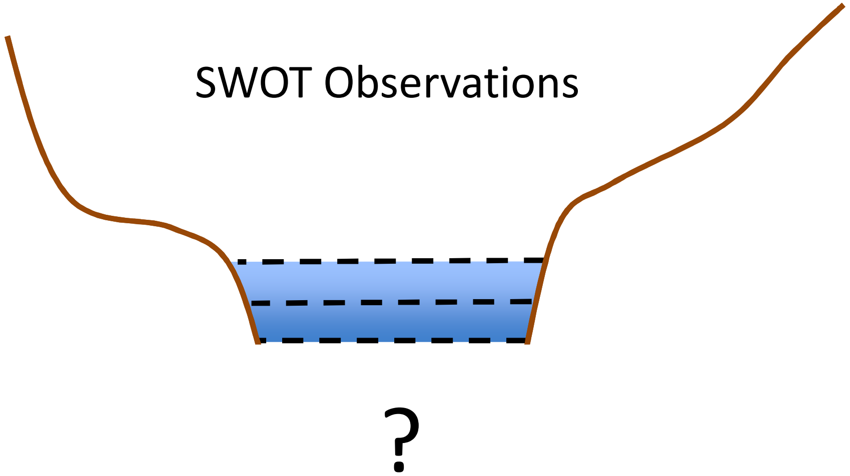
Rivers: $(A_t/L)(dH/dt)(v_{est}) = dQ/dt$
(discharge anomaly, $(m^3/s)/dt$)

(estimate from assimilation of SWOT observables into hydrodynamic model; and/or Manning’s equation)

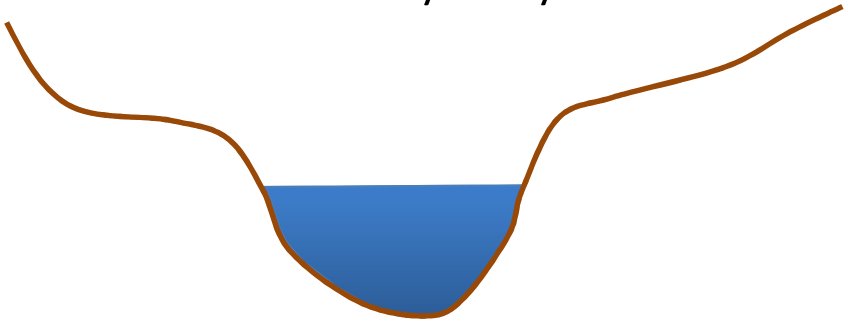
… but *true* discharge also requires flow depth



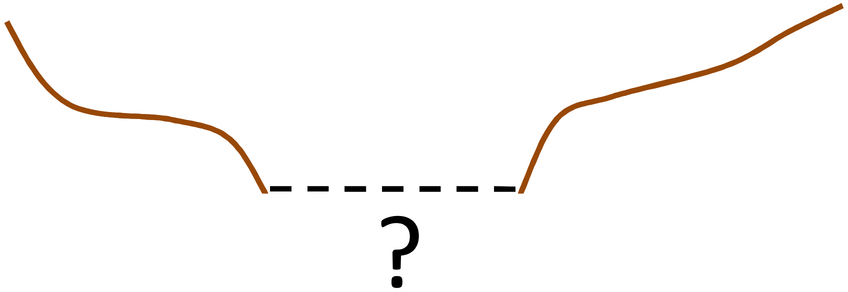


- SWOT will measure bathymetry down to the lowest water level encountered over the mission lifetime, but will not capture the entire channel bathymetry.
- This unknown baseflow depth limits the accuracy of SWOT discharge estimates.

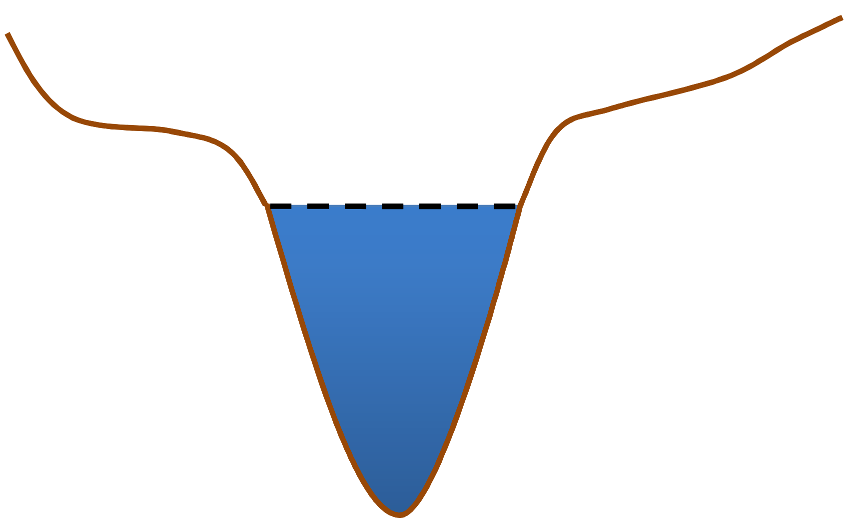
True Bathymetry

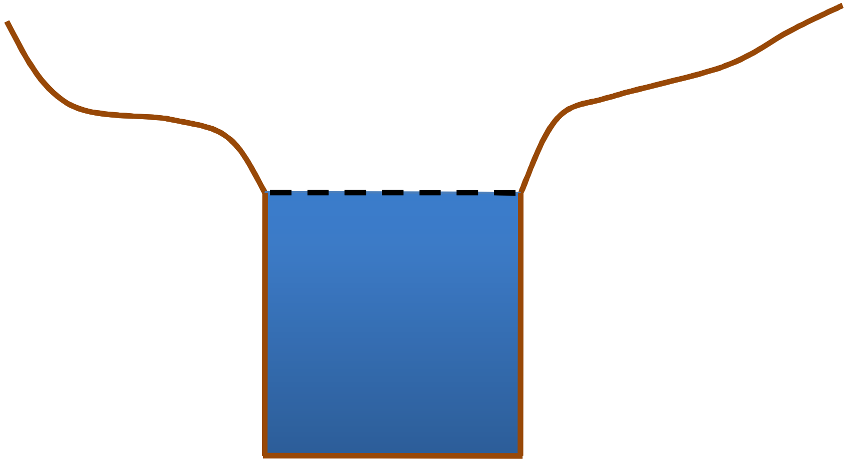


SWOT Observation

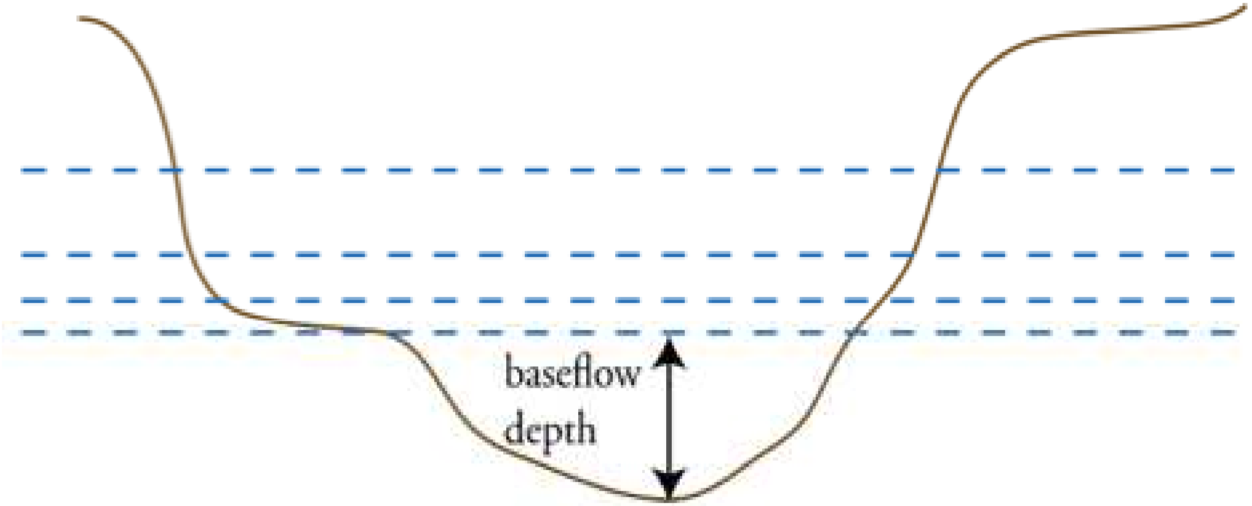


Approximations





SWOT Depth Estimation



SWOT will only measure channel bathymetry down to lowest exposed banks over mission lifetime

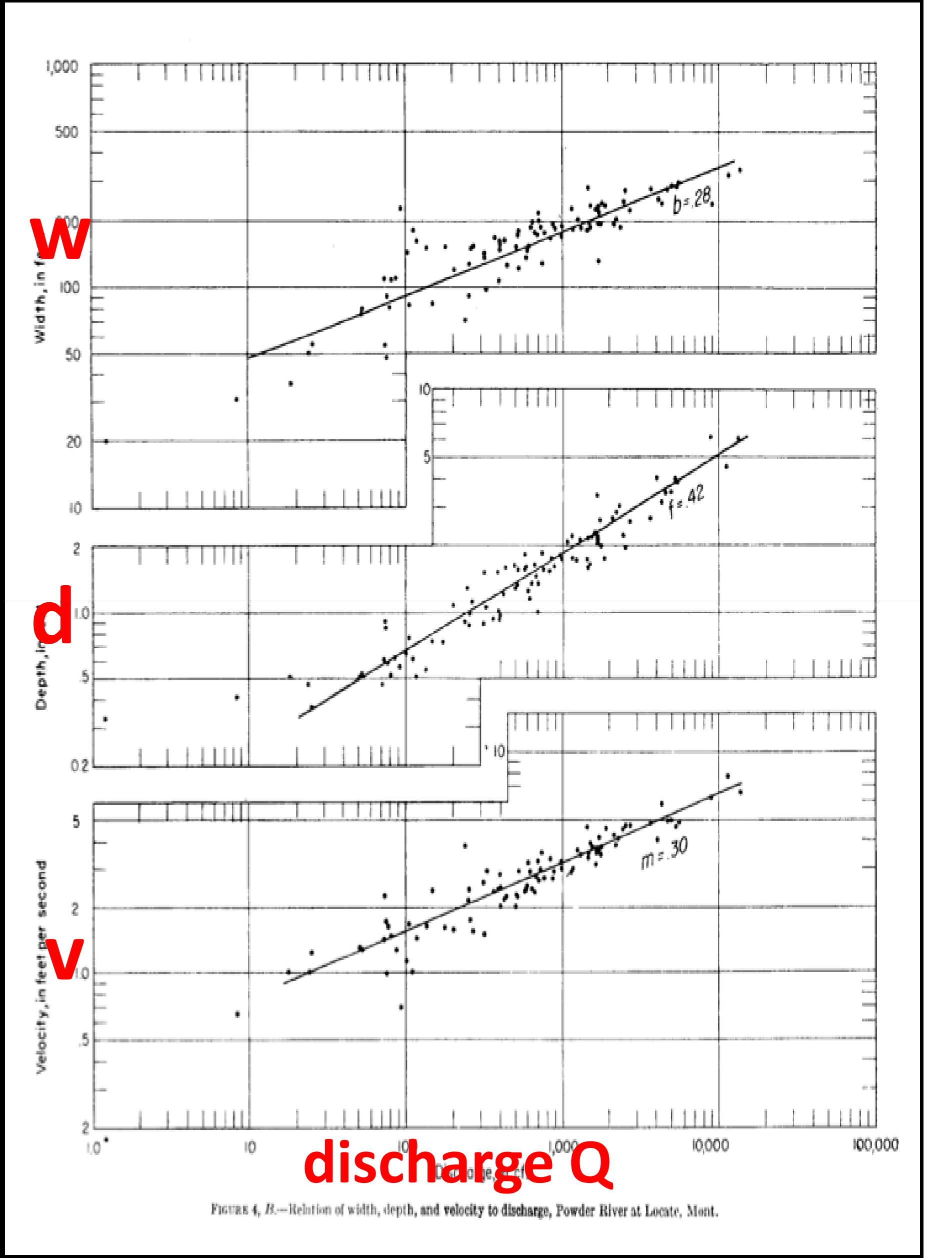
This unknown baseflow depth represents greatest risk to SWOT discharge estimates

HYDRAULIC GEOMETRY (HG)
(Leopold and Maddock, 1953)

$Q = wdv$

$w = aQ^b$; $d = cQ^f$; $v = kQ^m$

$b+f+m = 1$; $acv = 1$



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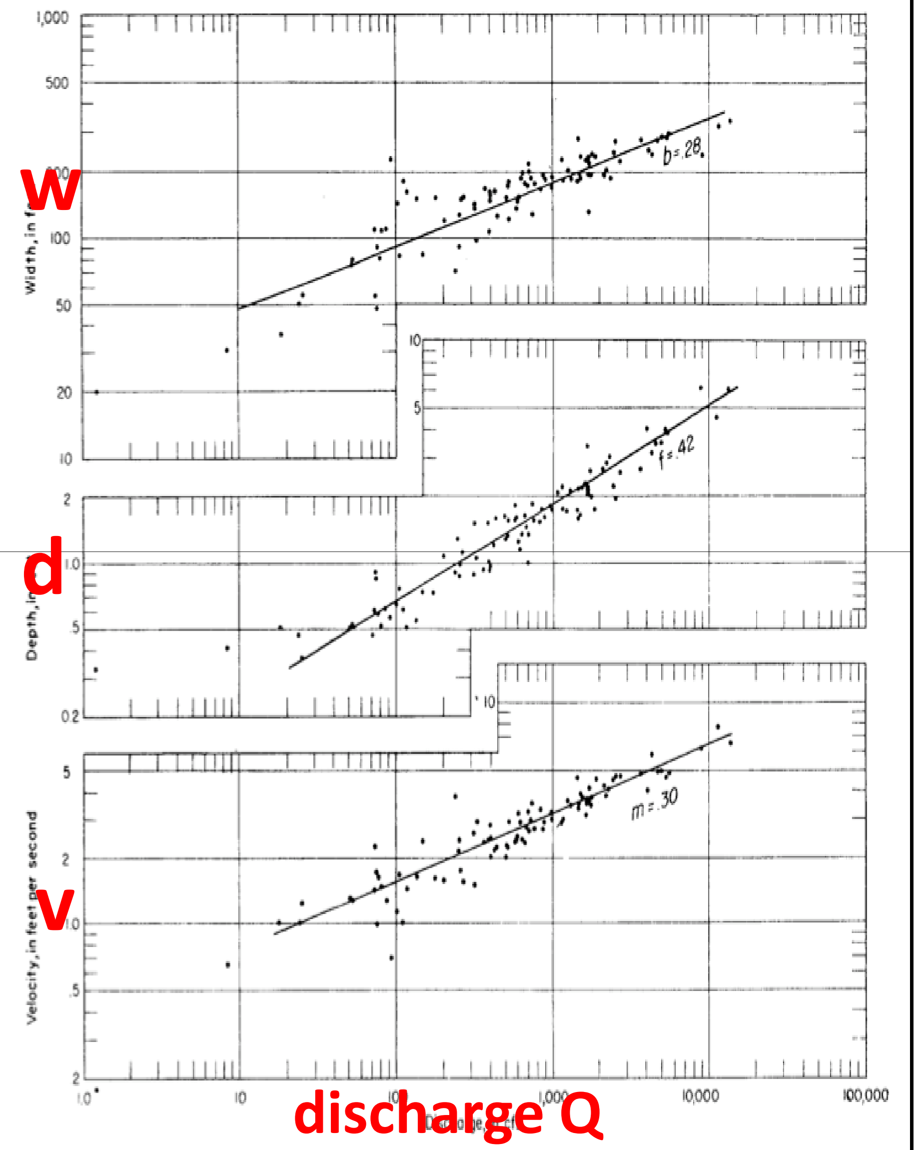

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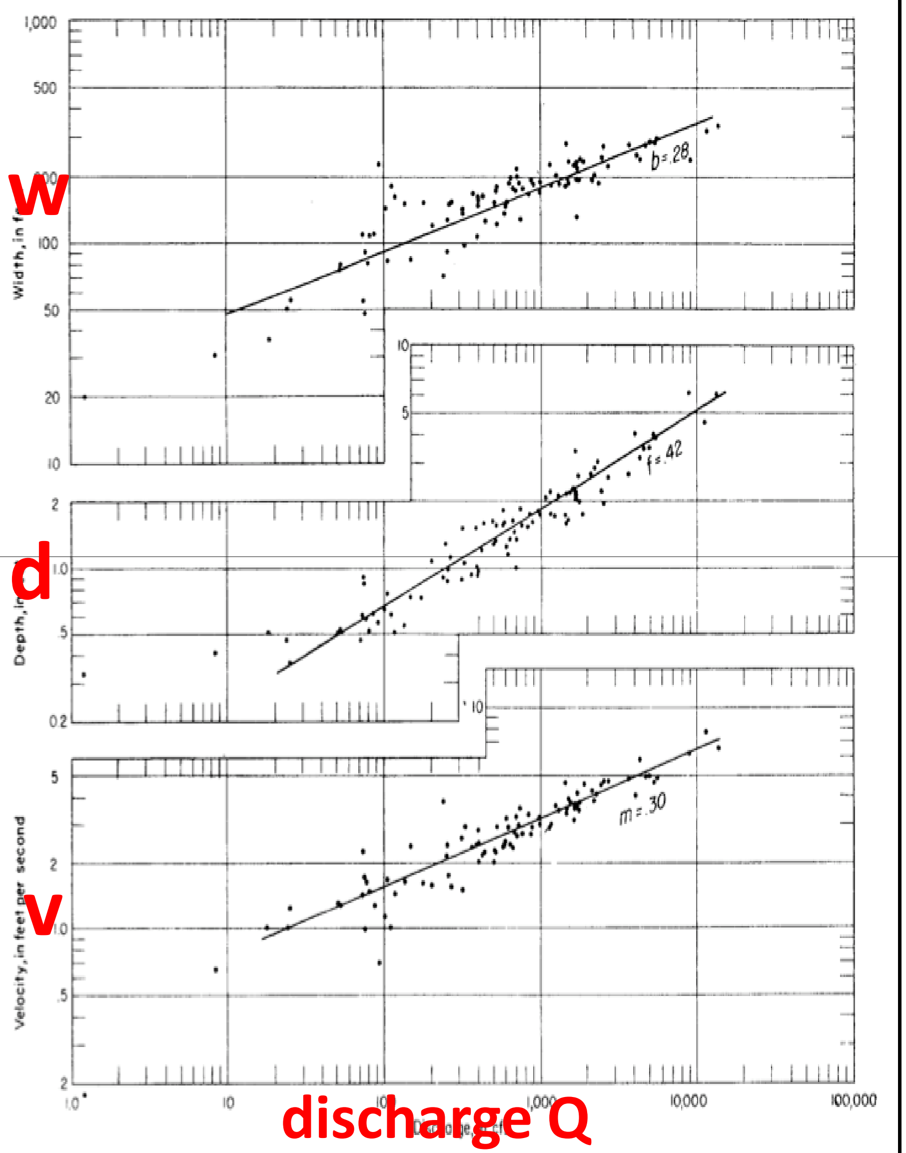
Type 1: At-a-station (AHG)

Type 2: Downstream(DHG)



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$Q = wdv$

$w = aQ^b$; $d = cQ^f$; $v = kQ^m$

$b+f+m = 1$; $acv = 1$

Type 1: At-a-station (AHG)

(site-specific, multi-temporal)

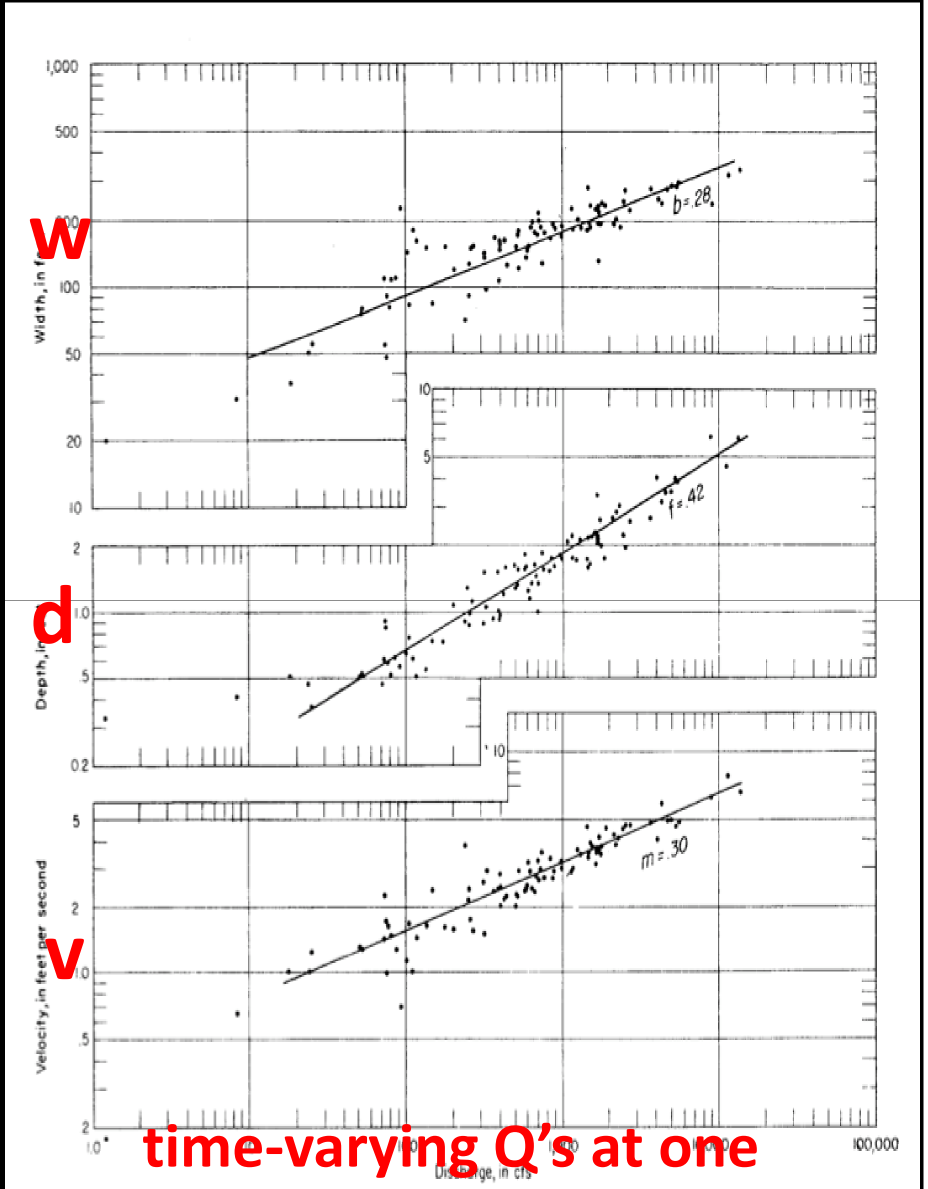
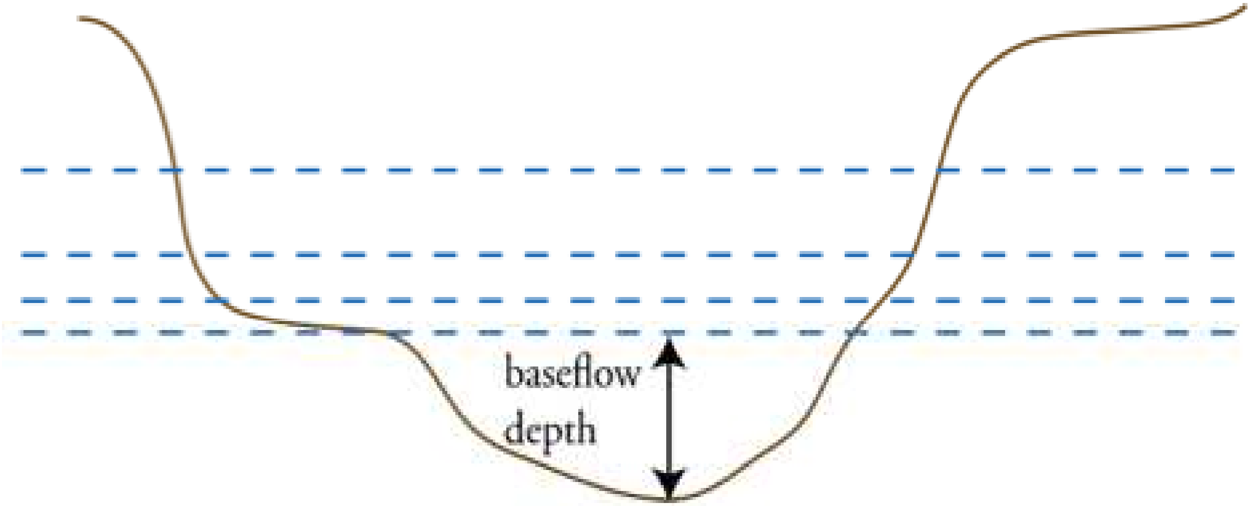


FIGURE 4 B.—Relation of width, depth, and velocity to discharge, Powder River at Locate, Mont.

time-varying Q's at one location in space

SWOT Depth Estimation

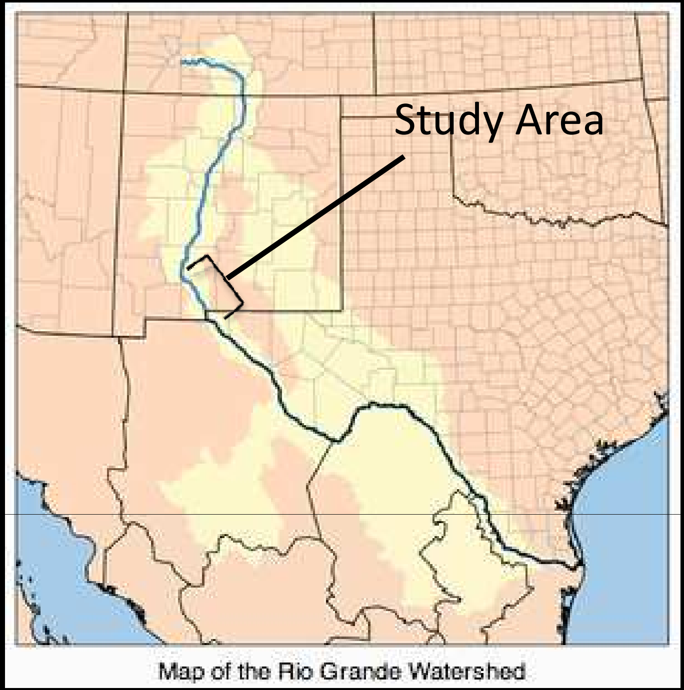


SWOT will only measure channel bathymetry down to lowest exposed banks over mission lifetime

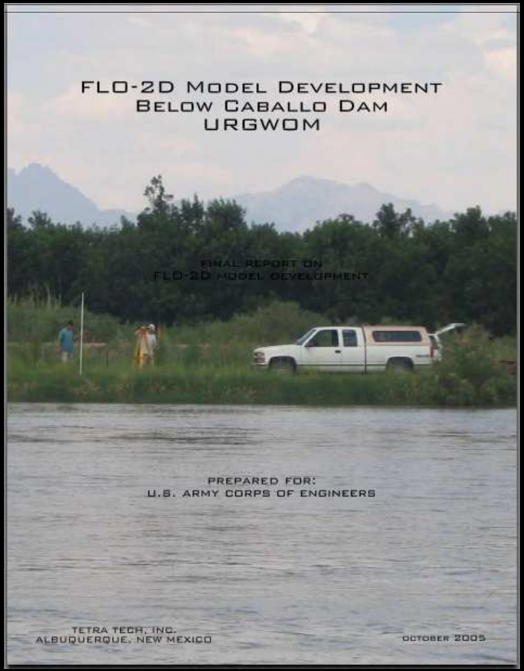
This unknown baseflow depth represents greatest risk to SWOT discharge estimates

Case study: Rio Grande River

- 4th longest river in the U.S. (>3000 km, ~472,000 km^2 watershed)
- High quality cross-section data (145 surveyed; 1,235 interpolated)
- 100-year flood flow rates



Map of the Rio Grande Watershed


FLO-2D MODEL DEVELOPMENT
BELOW CABALLO DAM
URGWOM

FINAL REPORT ON
FLO-2D MODEL DEVELOPMENT

PREPARED FOR:
U.S. ARMY CORPS OF ENGINEERS

TETRA TECH, INC.
ALBUQUERQUE, NEW MEXICO

OCTOBER 2005

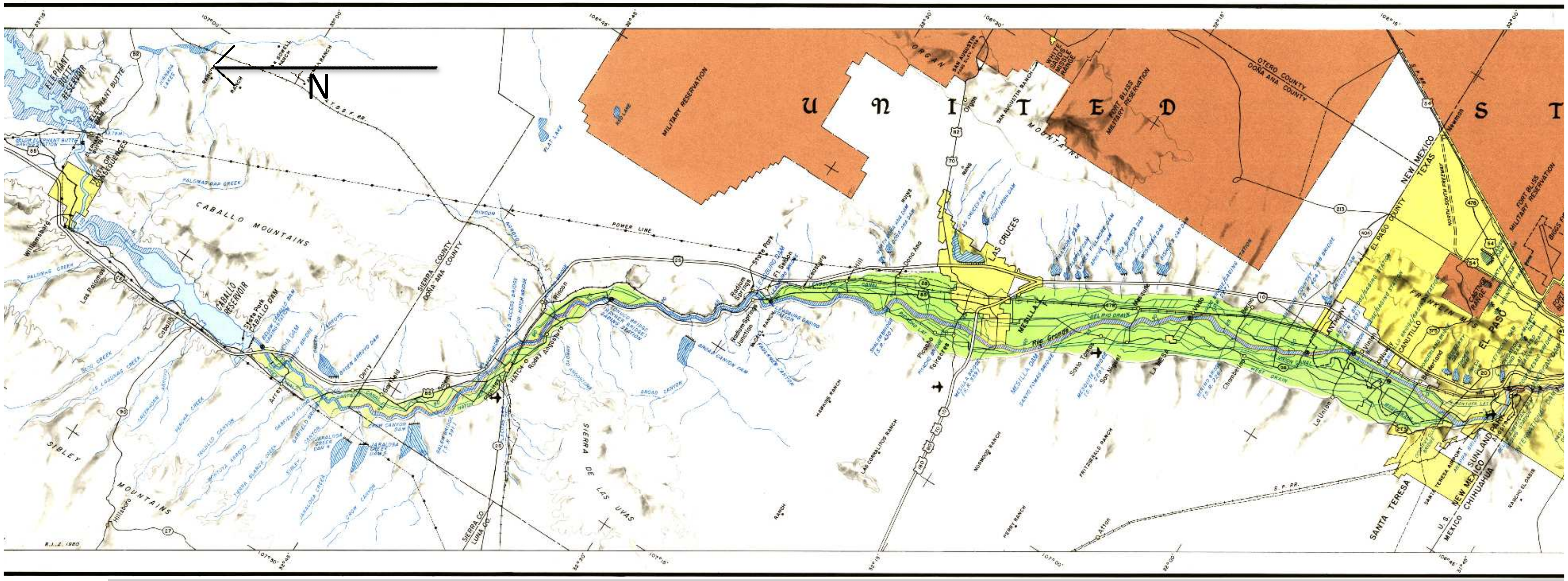
Used HEC-RAS to simulate AHG's (widths, depths, velocities versus range of simulated discharges) at 145 surveyed cross-sections

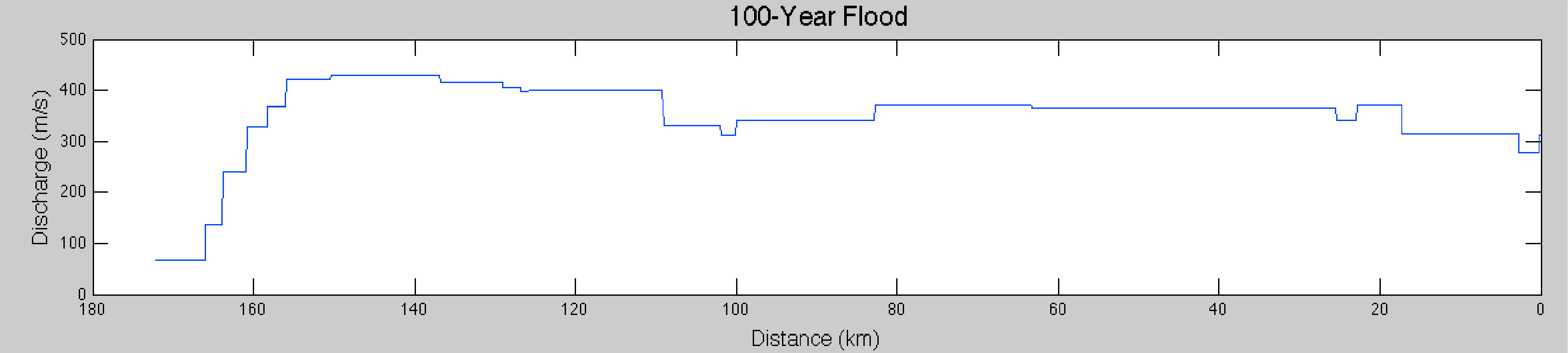
-Steady state discharges

-Fixed bed

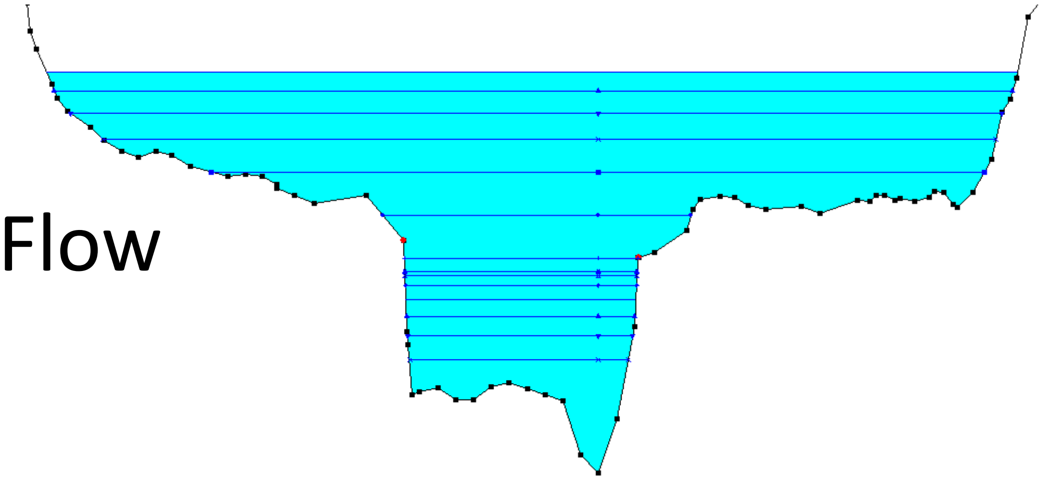
Caballo Dam to American Dam

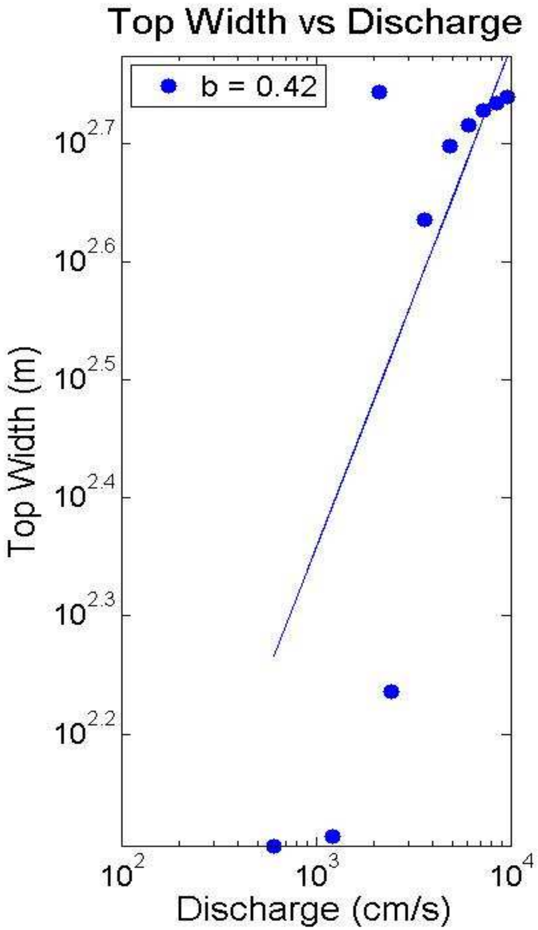
- Contributing Watershed ≈ 2315 sq km
- ~89% in upper half of reach

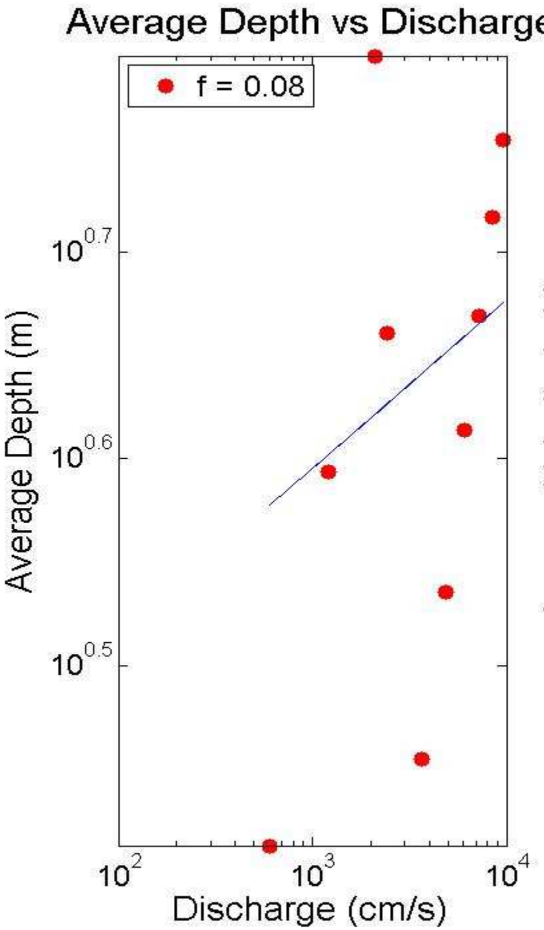


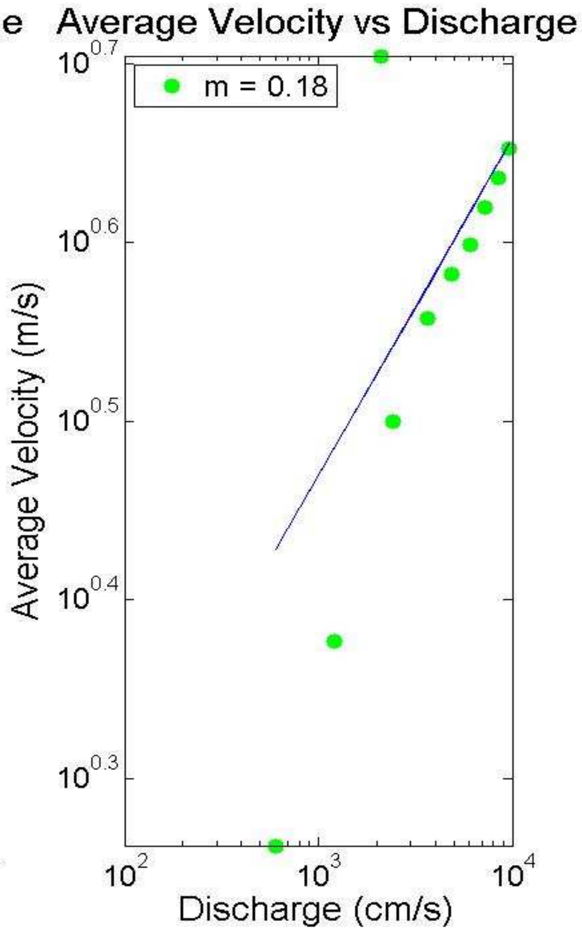


Overbank Flow

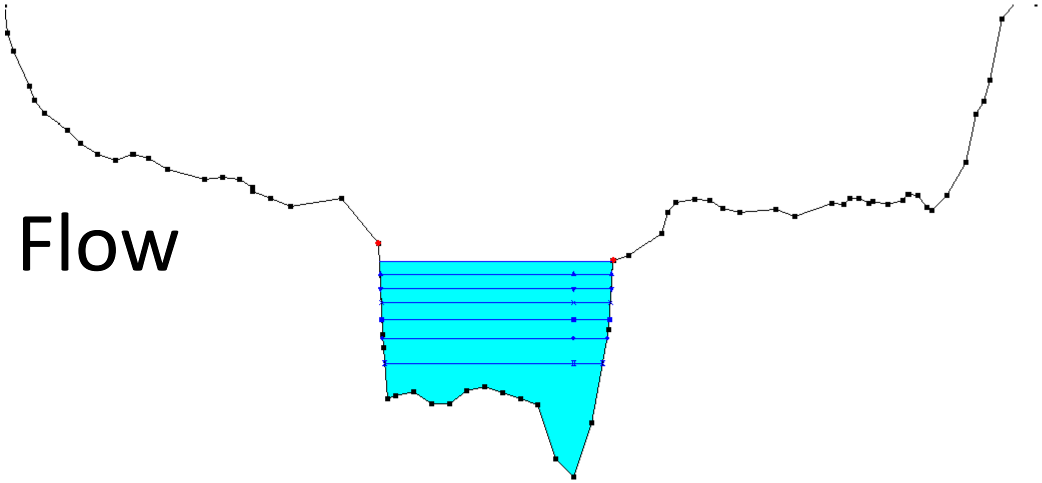


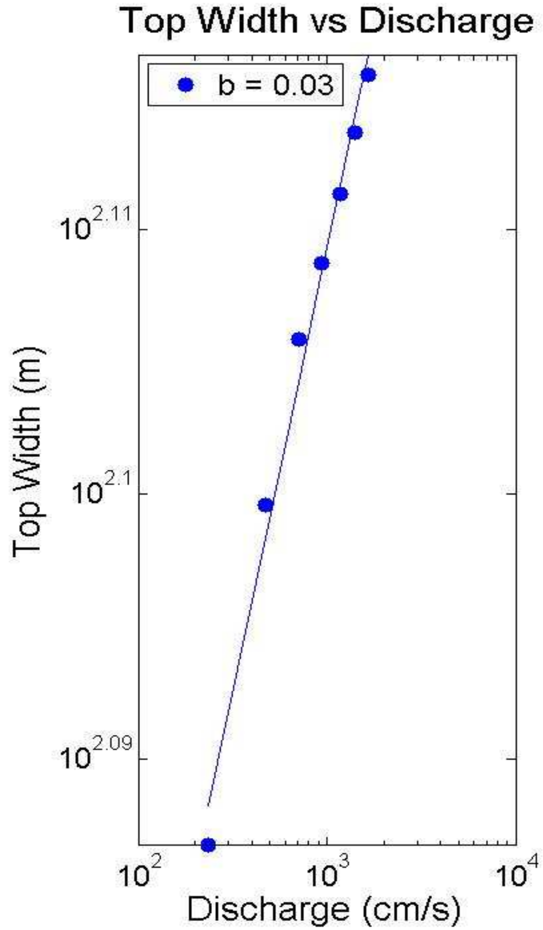


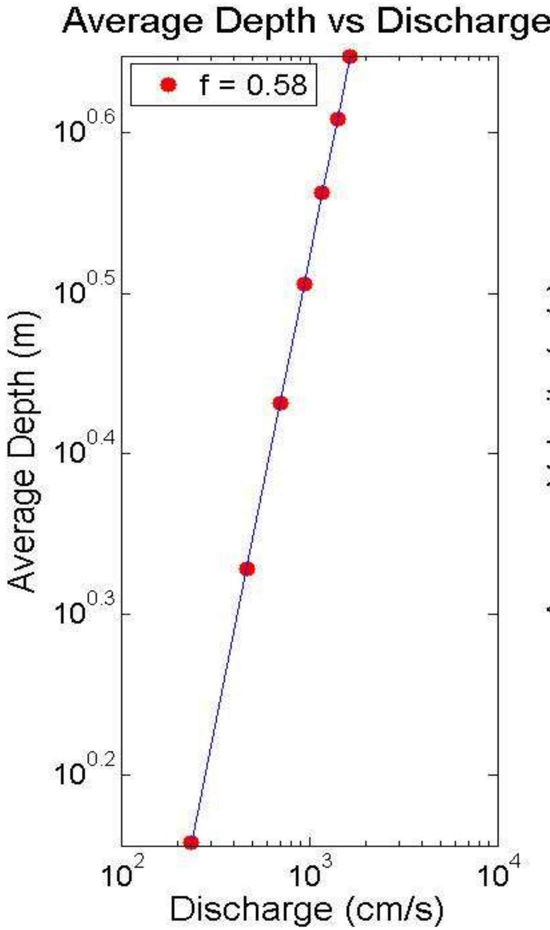


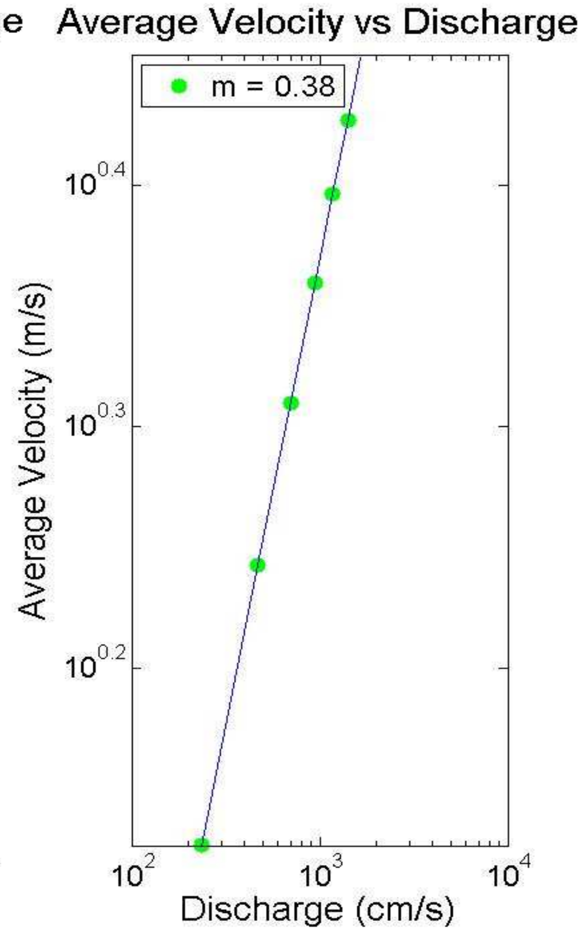


In-Channel Flow

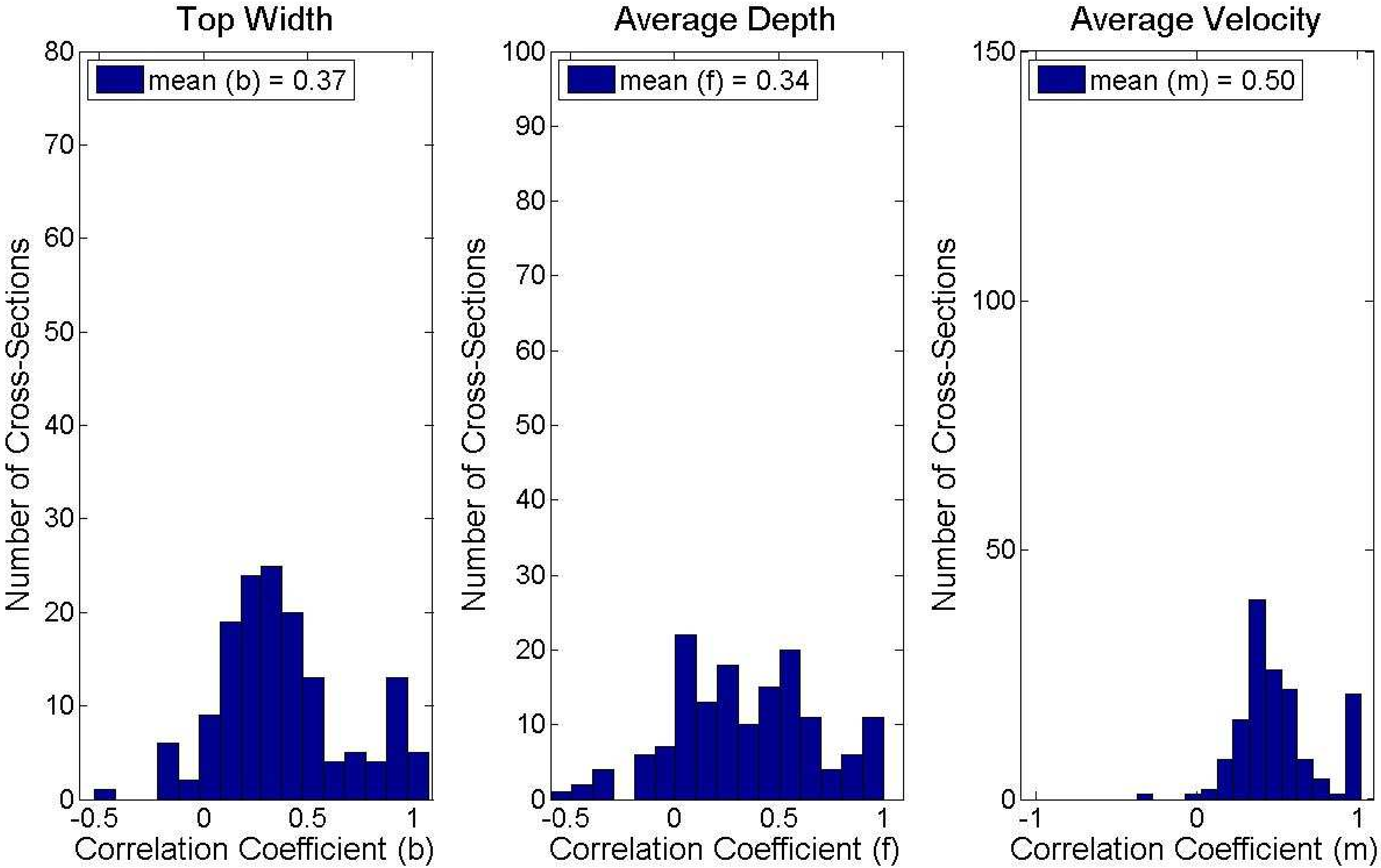



Top Width vs Discharge

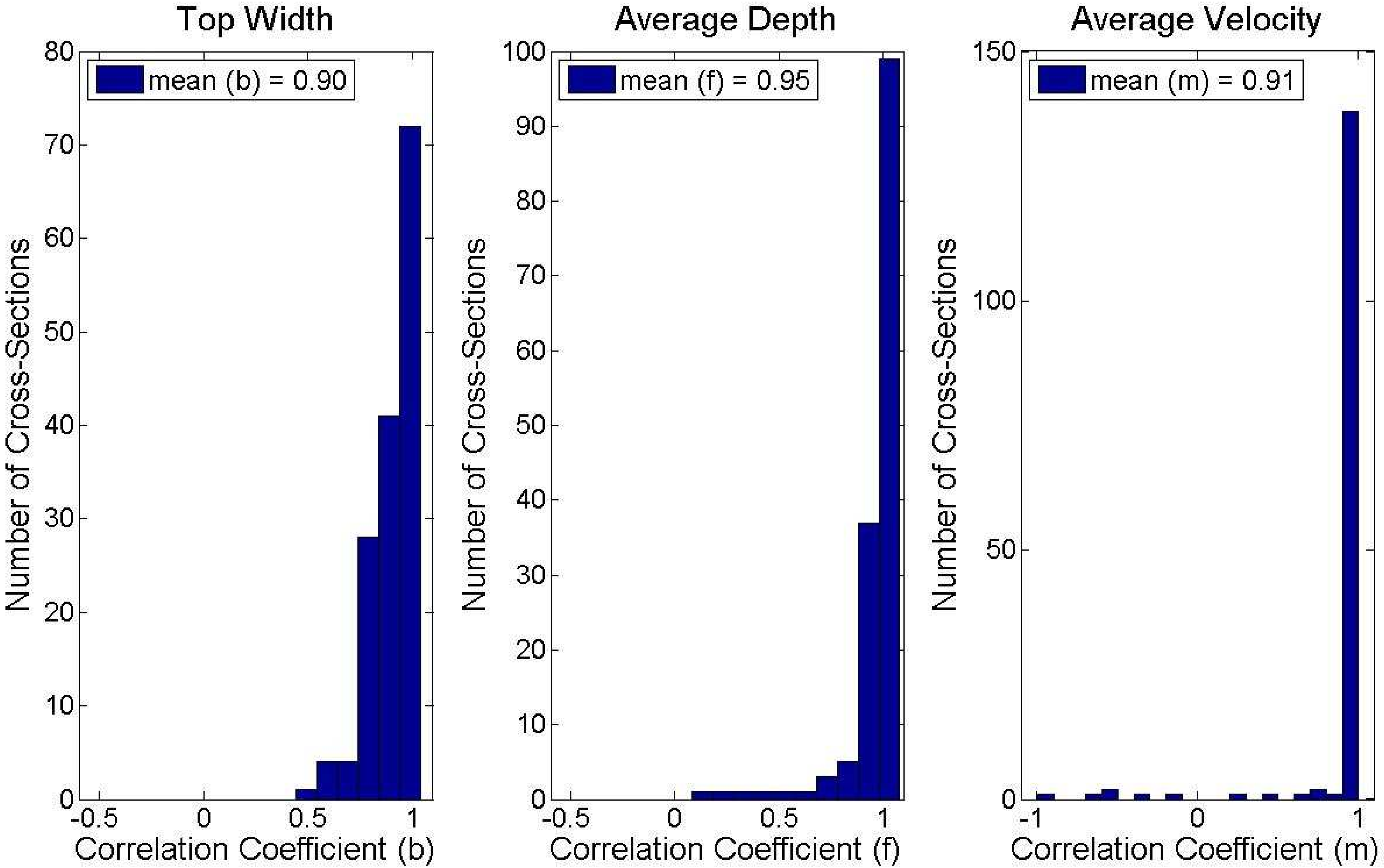

Average Depth vs Discharge


Average Velocity vs Discharge

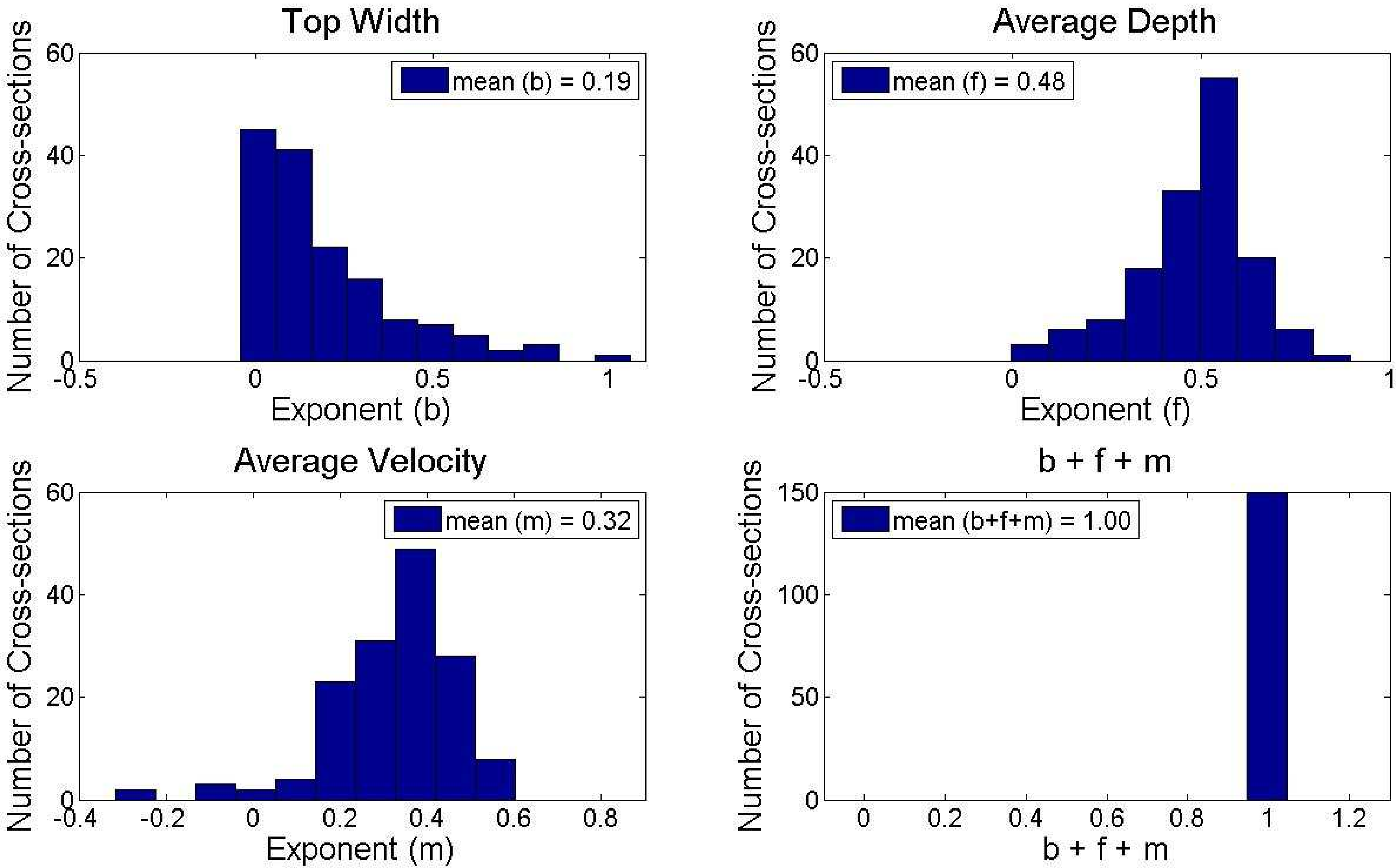
AHG goodness-of-fit (n=145)
(overbank flows)

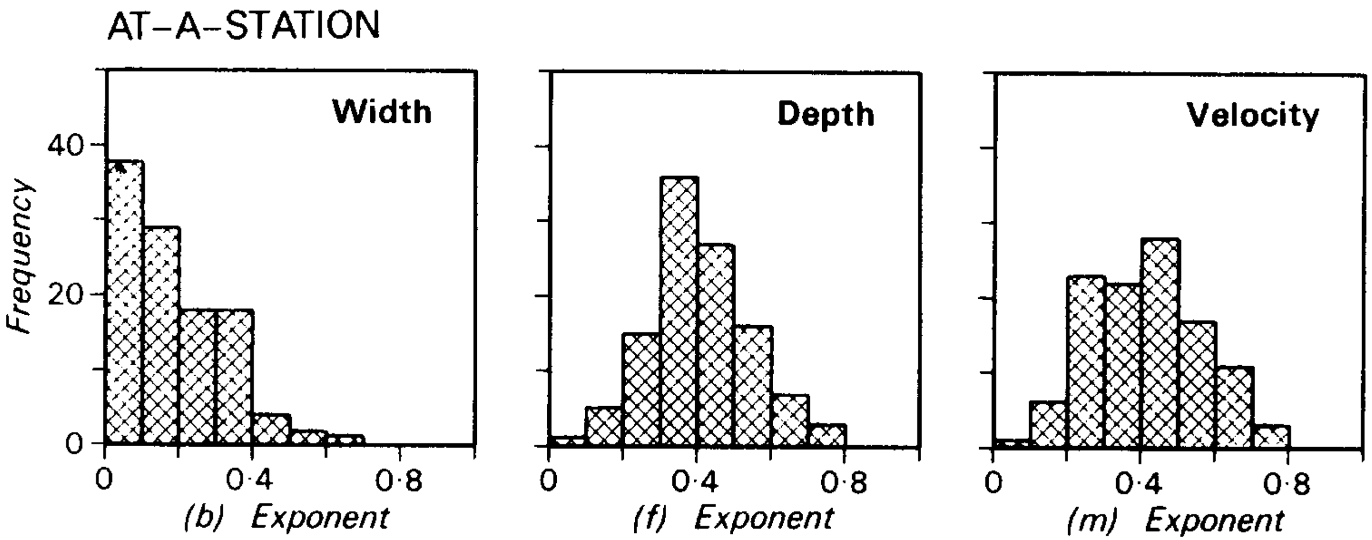


AHG goodness-of-fit (n=145)
(within-bank flows)



AHG b,f,m exponents, n=145 (within-bank flows)





Summary of distribution characteristics of hydraulic geometry exponent data

	Width	Depth	Velocity
At-a-station (*n* = 139):			
Range	0.00—0.59	0.06—0.73	0.07—0.71
Modal class	0.0 —0.1	0.3 —0.4	0.4 —0.5
Theoretical*[1]	0.23	0.42	0.35

(Park, J. Hydrol., 1977)

w << d, v

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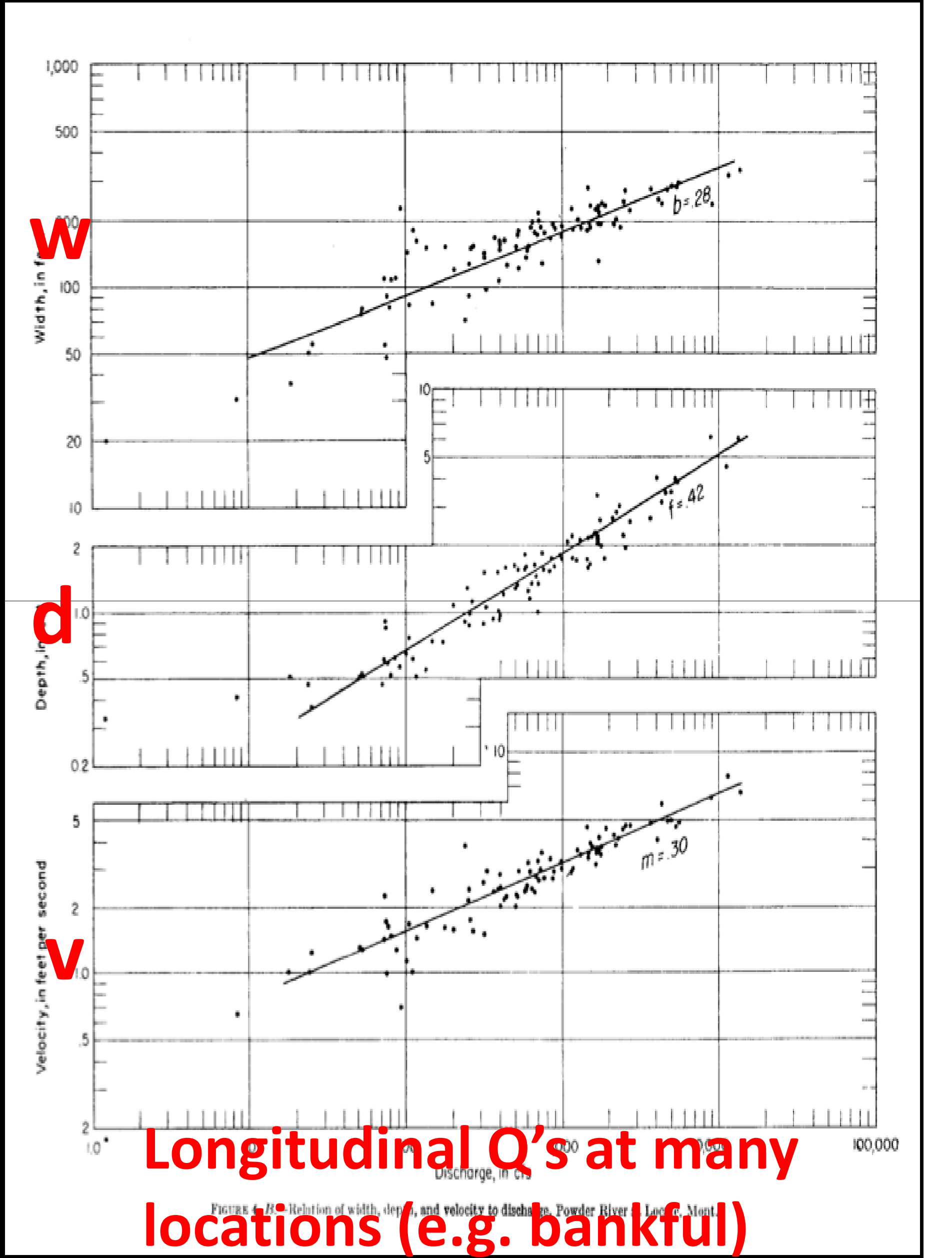
Type 1: At-a-station (AHG)

Type 2: Downstream (DHG)

(landscape scale, steady-state)

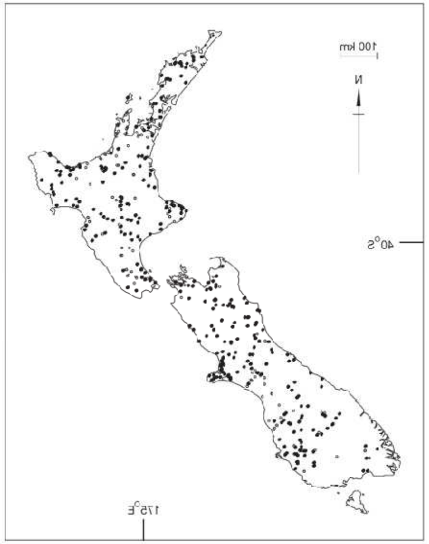


w

d

v

Longitudinal Q's at many locations (e.g. bankful)



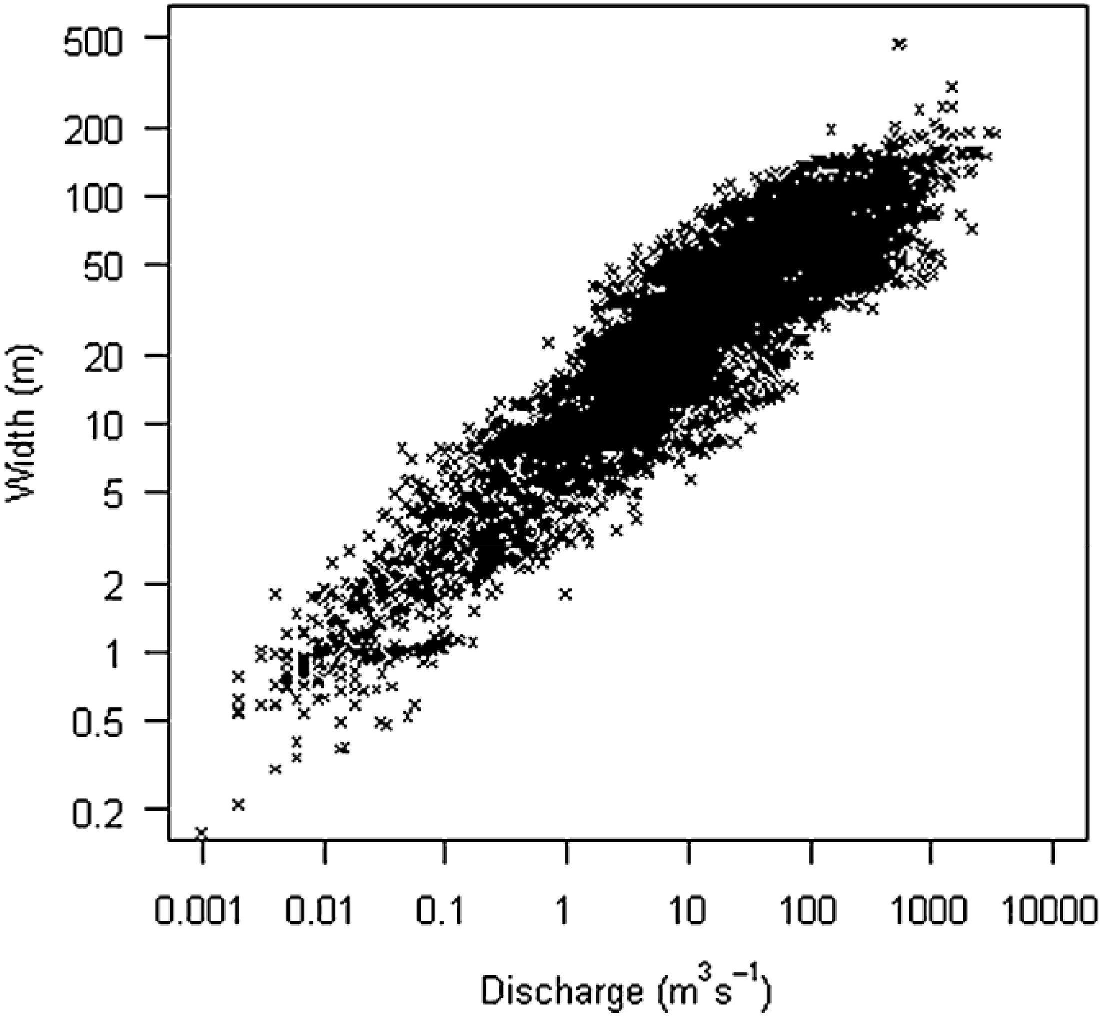
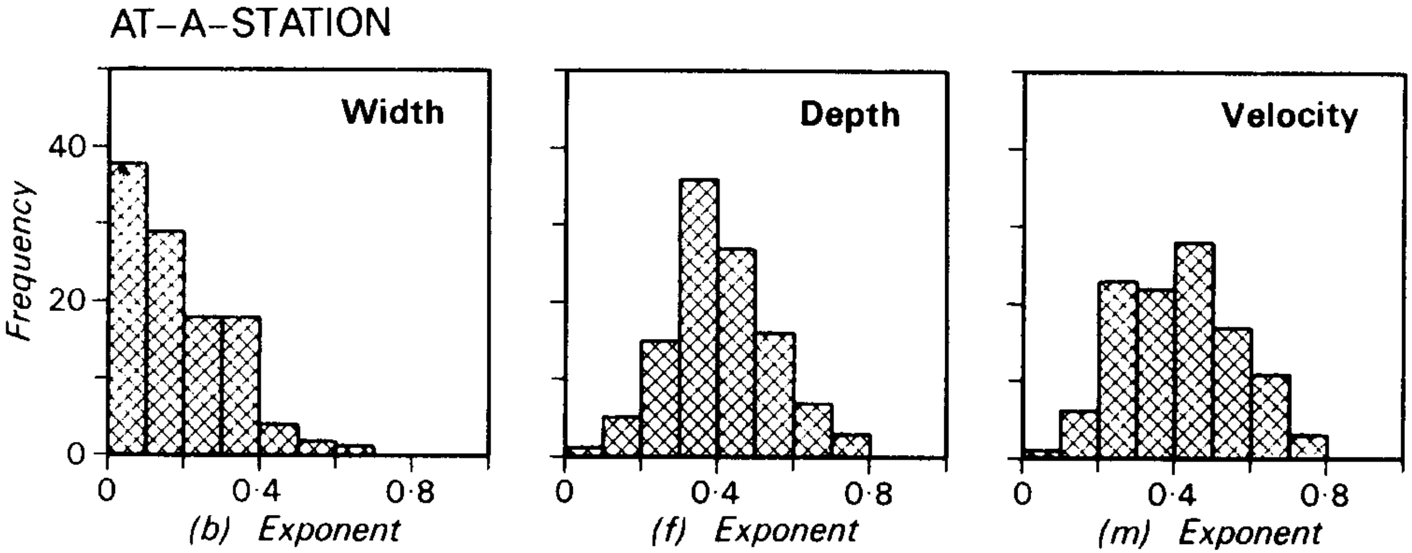
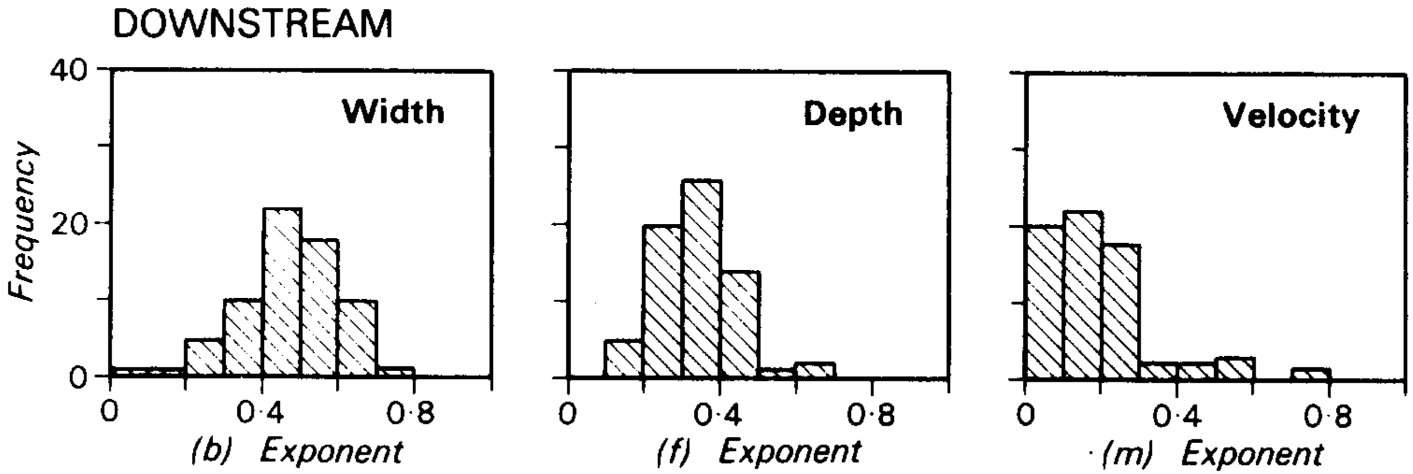


Figure 1. Width against flow at 328 gauging stations

(Booker, ESPL, 2010)

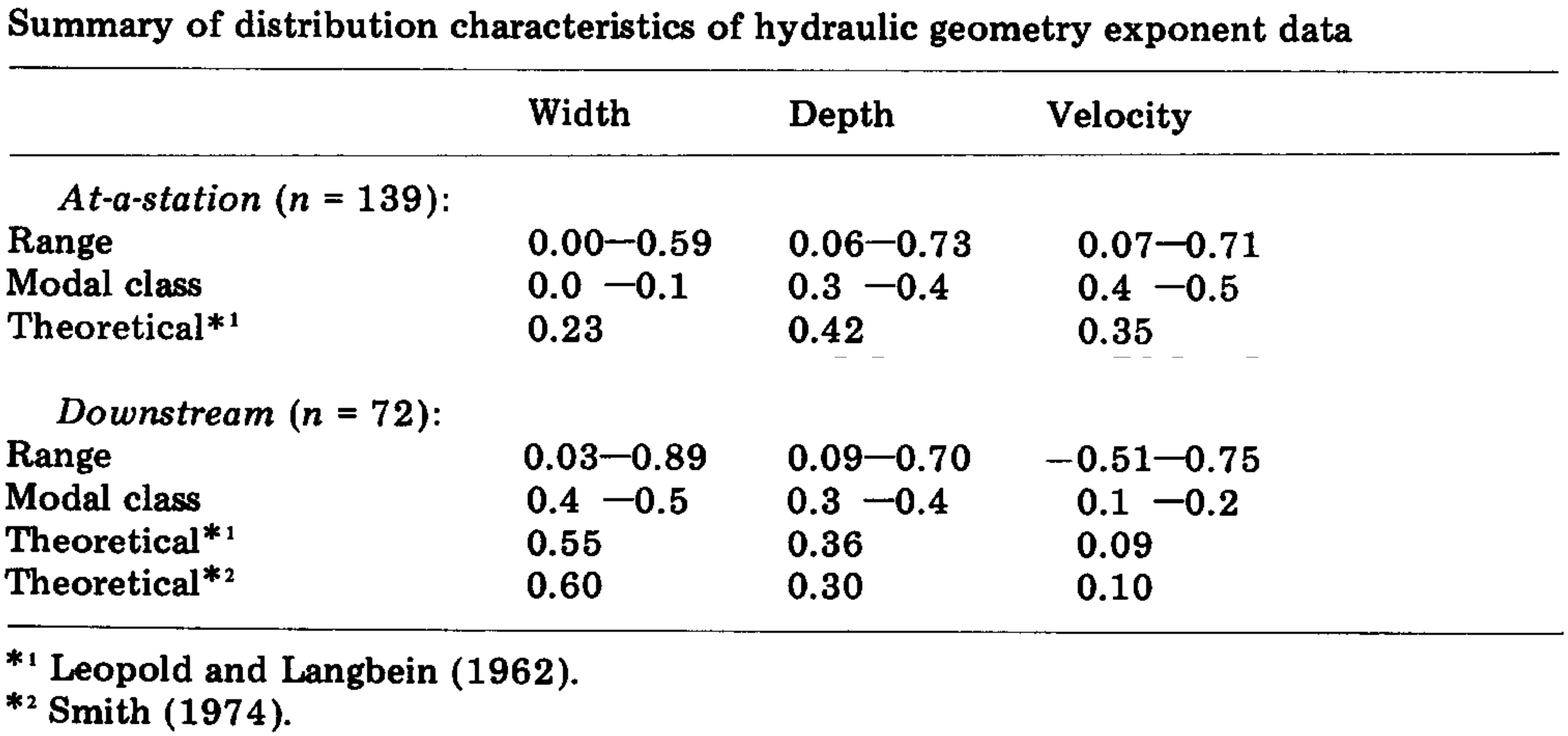


w << d, v



v<<w > d

(Park, J. Hydrol., 1977)



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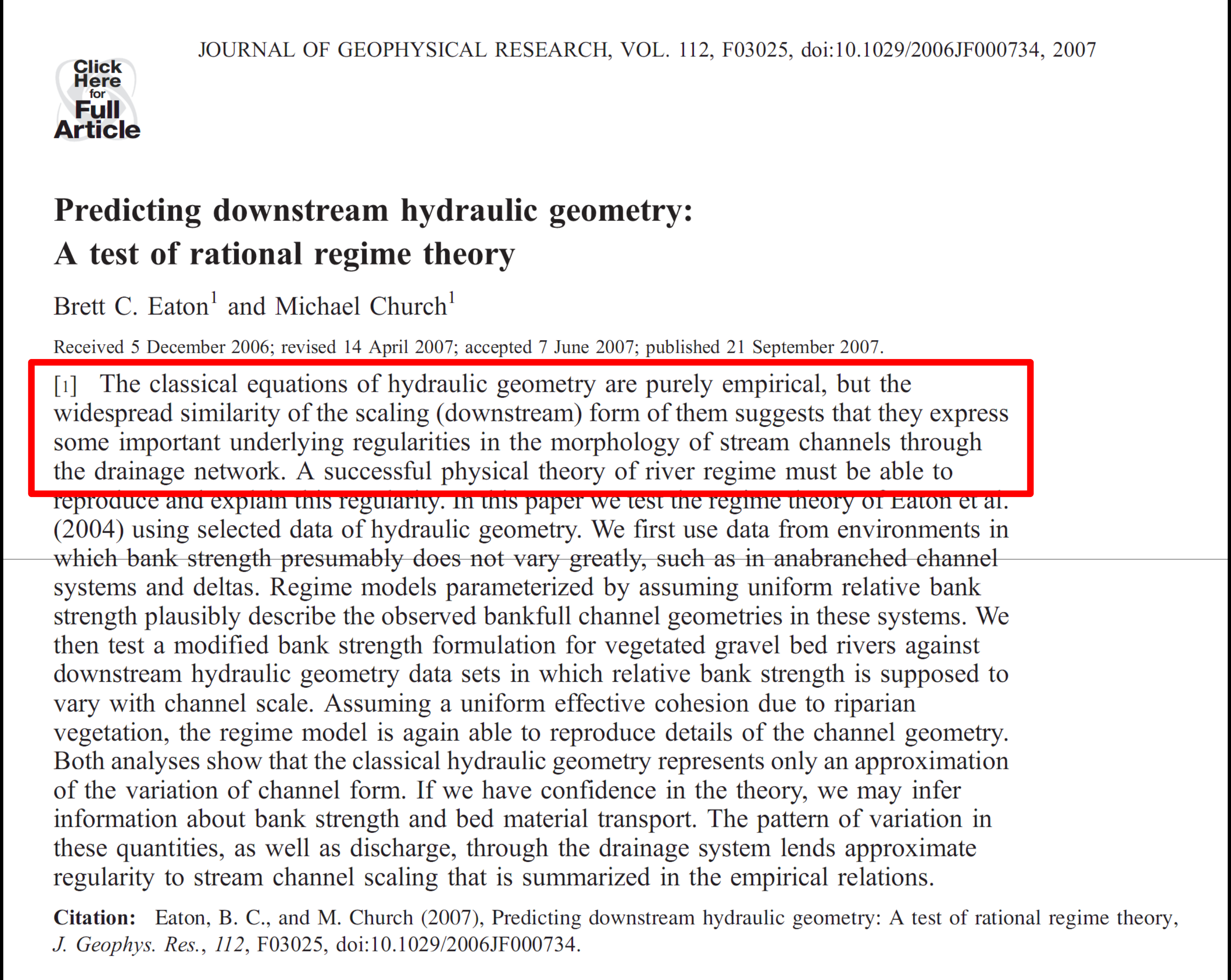
Theoretical*[1]	0.23	0.42	0.35
Downstream (*n* = 72):			
Range	0.03—0.89	0.09—0.70	−0.51—0.75
Modal class	0.4 —0.5	0.3 —0.4	0.1 —0.2
Theoretical*[1]	0.55	0.36	0.09
Theoretical*[2]	0.60	0.30	0.10

*[1] Leopold and Langbein (1962).
*[2] Smith (1974).

(Park, J. Hydrol., 1977)



JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 112, F03025, doi:10.1029/2006JF000734, 2007

Predicting downstream hydraulic geometry: A test of rational regime theory

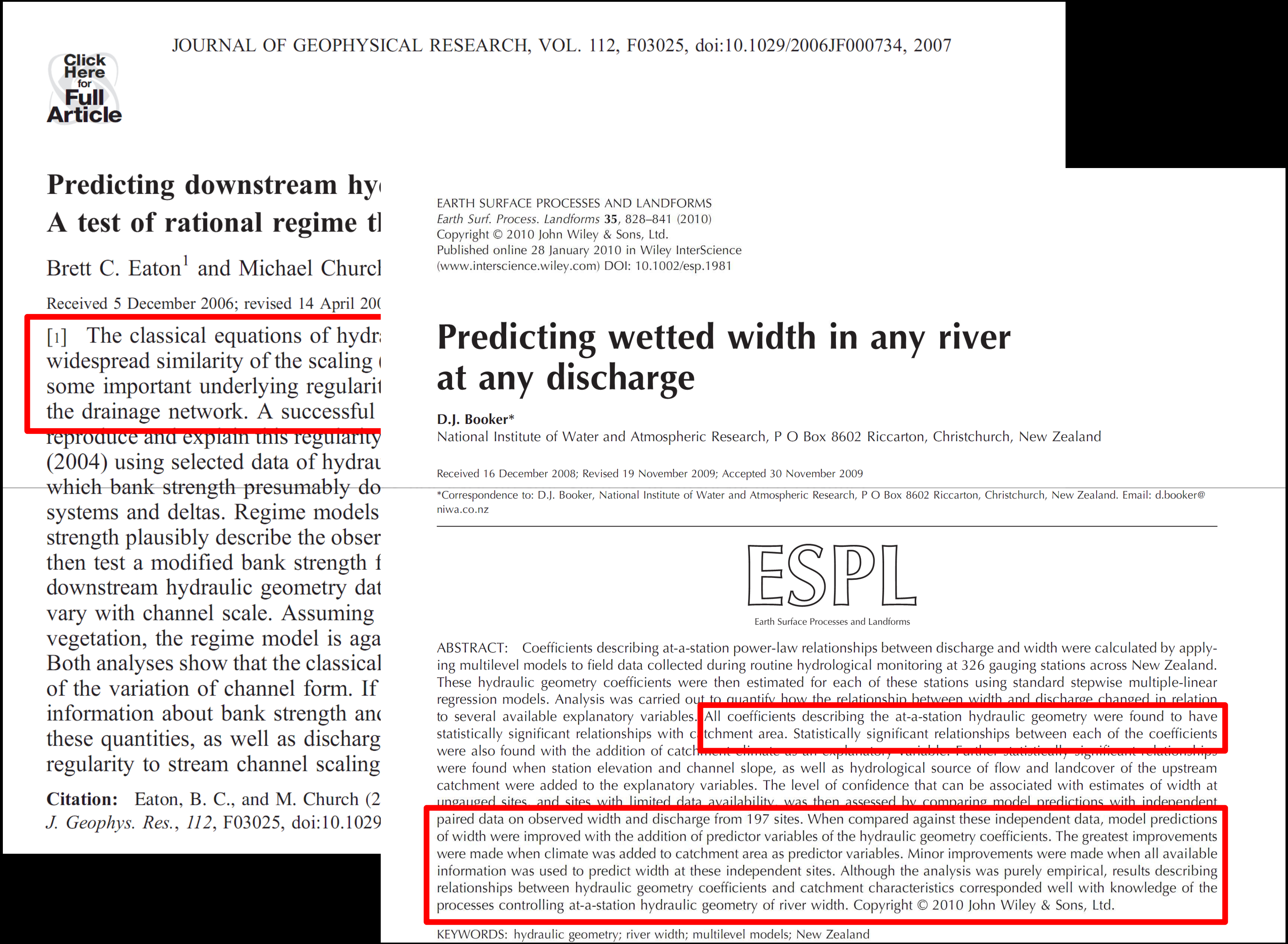
Brett C. Eaton[1] and Michael Church[1]

Received 5 December 2006; revised 14 April 2007; accepted 7 June 2007; published 21 September 2007.

[1] The classical equations of hydraulic geometry are purely empirical, but the widespread similarity of the scaling (downstream) form of them suggests that they express some important underlying regularities in the morphology of stream channels through the drainage network. A successful physical theory of river regime must be able to reproduce and explain this regularity. In this paper we test the regime theory of Eaton et al. (2004) using selected data of hydraulic geometry. We first use data from environments in which bank strength presumably does not vary greatly, such as in anabranched channel systems and deltas. Regime models parameterized by assuming uniform relative bank strength plausibly describe the observed bankfull channel geometries in these systems. We then test a modified bank strength formulation for vegetated gravel bed rivers against downstream hydraulic geometry data sets in which relative bank strength is supposed to vary with channel scale. Assuming a uniform effective cohesion due to riparian vegetation, the regime model is again able to reproduce details of the channel geometry. Both analyses show that the classical hydraulic geometry represents only an approximation of the variation of channel form. If we have confidence in the theory, we may infer information about bank strength and bed material transport. The pattern of variation in these quantities, as well as discharge, through the drainage system lends approximate regularity to stream channel scaling that is summarized in the empirical relations.

Citation: Eaton, B. C., and M. Church (2007), Predicting downstream hydraulic geometry: A test of rational regime theory, *J. Geophys. Res.*, *112*, F03025, doi:10.1029/2006JF000734.

(JGR, 2007)



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EARTH SURFACE PROCESSES AND LANDFORMS
Earth Surf. Process. Landforms **35**, 828–841 (2010)
DOI: 10.1002/esp.1981

Predicting wetted width in any river at any discharge

D.J. Booker*
National Institute of Water and Atmospheric Research, P O Box 8602 Riccarton, Christchurch, New Zealand

Received 16 December 2008; Revised 19 November 2009; Accepted 30 November 2009

*Correspondence to: D.J. Booker, National Institute of Water and Atmospheric Research, P O Box 8602 Riccarton, Christchurch, New Zealand. Email: d.booker@niwa.co.nz

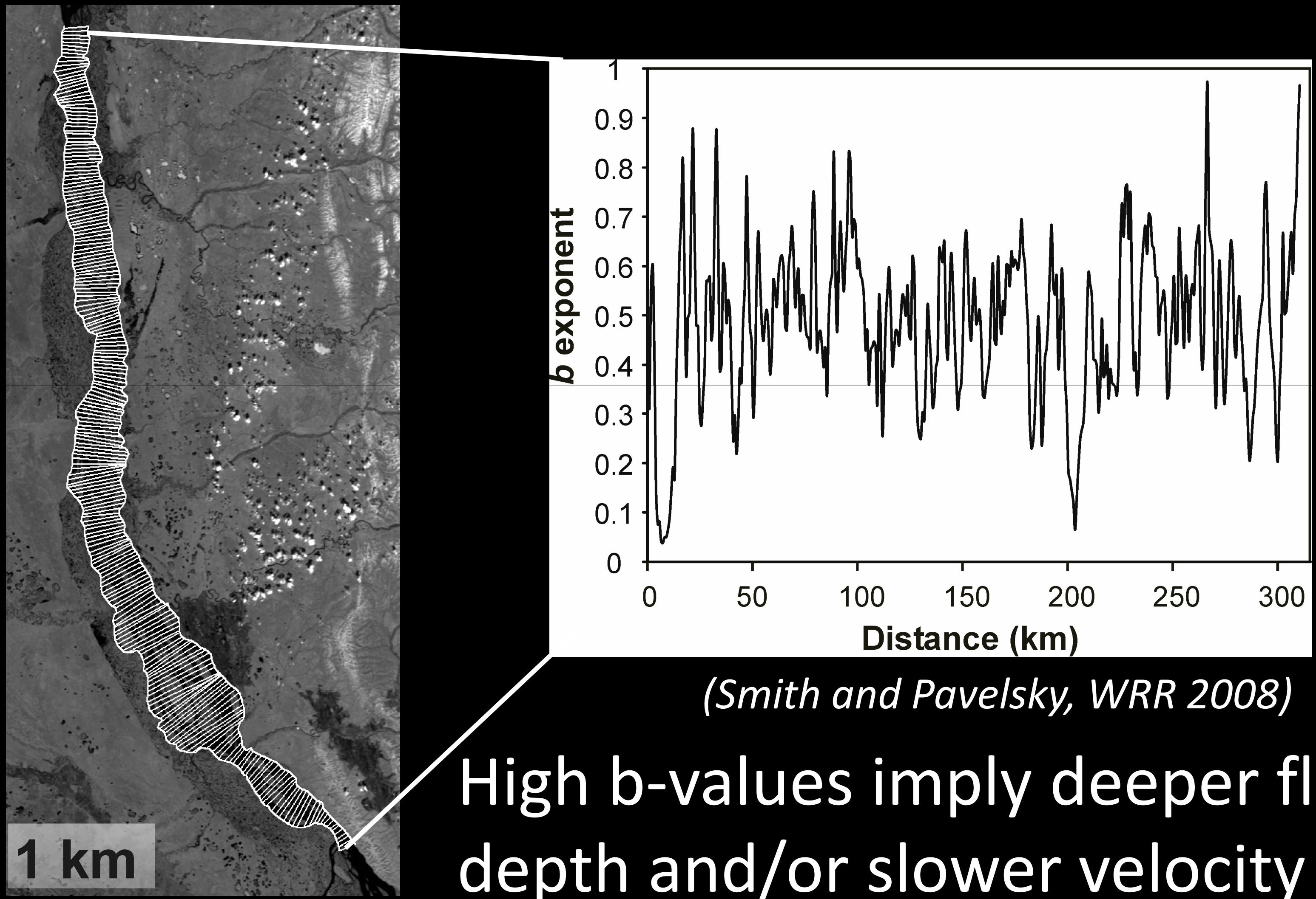
ESPL
Earth Surface Processes and Landforms

ABSTRACT: Coefficients describing at-a-station power-law relationships between discharge and width were calculated by applying multilevel models to field data collected during routine hydrological monitoring at 326 gauging stations across New Zealand. These hydraulic geometry coefficients were then estimated for each of these stations using standard stepwise multiple-linear regression models. Analysis was carried out to quantify how the relationship between width and discharge changed in relation to several available explanatory variables. All coefficients describing the at-a-station hydraulic geometry were found to have statistically significant relationships with catchment area. Statistically significant relationships between each of the coefficients were also found with the addition of catchment climate as an explanatory variable. Further statistically significant relationships were found when station elevation and channel slope, as well as hydrological source of flow and landcover of the upstream catchment were added to the explanatory variables. The level of confidence that can be associated with estimates of width at ungauged sites, and sites with limited data availability, was then assessed by comparing model predictions with independent paired data on observed width and discharge from 197 sites. When compared against these independent data, model predictions of width were improved with the addition of predictor variables of the hydraulic geometry coefficients. The greatest improvements were made when climate was added to catchment area as predictor variables. Minor improvements were made when all available information was used to predict width at these independent sites. Although the analysis was purely empirical, results describing relationships between hydraulic geometry coefficients and catchment characteristics corresponded well with knowledge of the processes controlling at-a-station hydraulic geometry of river width. Copyright © 2010 John Wiley & Sons, Ltd.

KEYWORDS: hydraulic geometry; river width; multilevel models; New Zealand

Proof-of-concept: First-ever mapping of AHG from space

(multi-temporal MODIS, b-exponents only, 1-km posting)



(Smith and Pavelsky, WRR 2008)

High b-values imply deeper flow depth and/or slower velocity

Potential value of HG for SWOT-type discharge retrievals

At-a-station AHG *(temporal, local scale)*:

Accumulate empirical relationships over time at each of thousands of point locations along a river course, enabling:

(a) Power-law extrapolation of baseflow depths?

(b) Directly-measured apportionment between w, d, v at each posting for calibration/validation of Data Assimilation methods

(c) Exponent ratios sensitive to channel form (e.g. b/f varies from 0 for rectangular cross-section to 1 for triangular); also frictional resistance (decreased roughness increases m/f)

Downstream DHG *(systematic, landscape scale)*:

Goal: Accumulate empirical relationships over space, collected on same day over large areas (steady flow, e.g. bankfull) required), enabling:

(a) Refined data-assimilation of discharge retrievals upstream and downstream of SWOT retrievals

Can HG aid river discharge retrievals? MAYBE (esp cal/val)

What if it none of it works?

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SWOT will measure: A_t and dH/dt; dH/dx

Lakes: $(A_t)(dH/dt) = dS/dt$

(temporal storage anomaly, m^3/dt)

Rivers: $(A_t/L)(dH/dt)(v_{est}) = dQ/dt$

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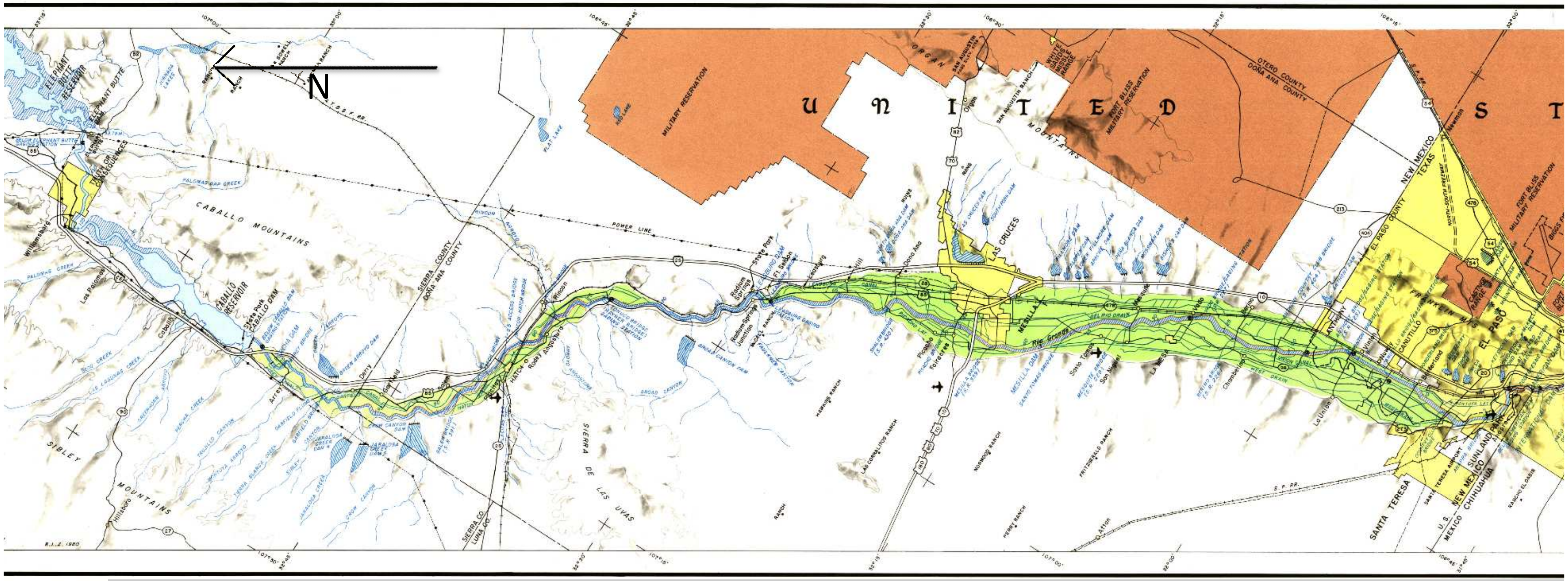
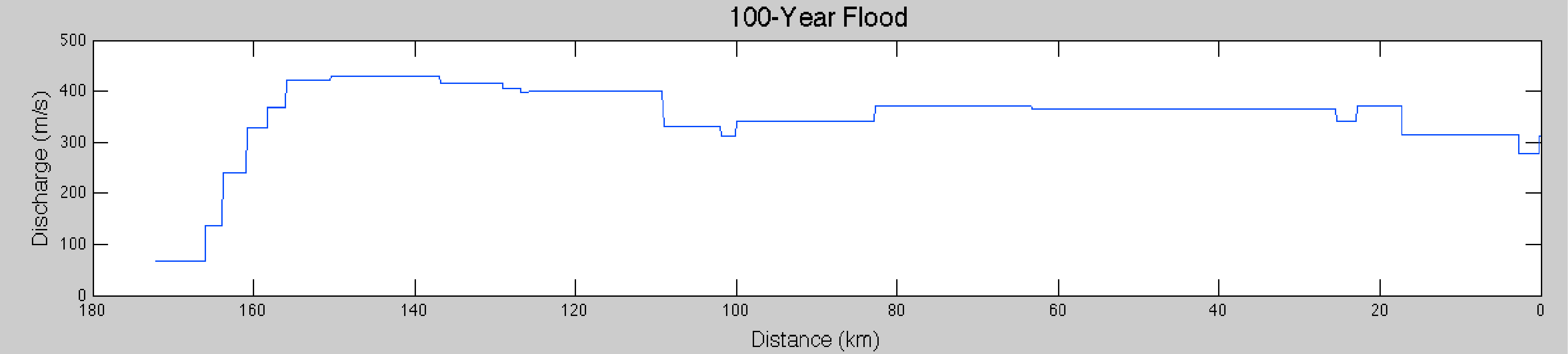
(temporal discharge anomaly, $(m^3/s)/dt$)

ALSO longitudinal discharge anomalies, dQ/dx…
… “DIFFERENTIAL DISCHARGES”

Rio Grande: Caballo Dam to American Dam





Climate Driven Water Resources Model of the Sacramento Basin, California

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Abstract
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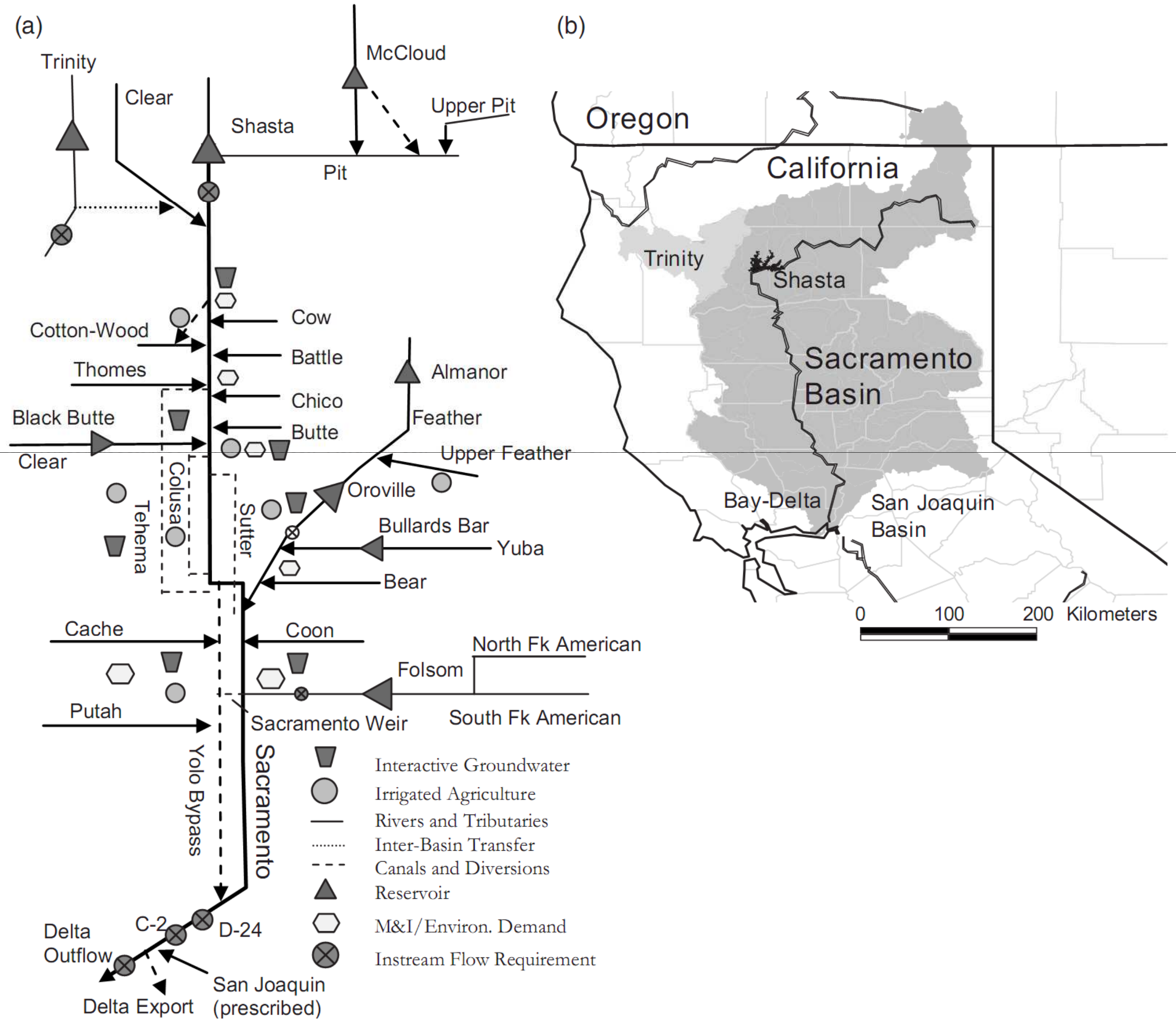

Fig. 3. Simplified schematic of the water resource elements implemented in the (a) WEAP21 model of the SACB; (b) position of the Sacramento Basin in California

Climate Driven Water Resources M
of the Sacramento Basin, Califor

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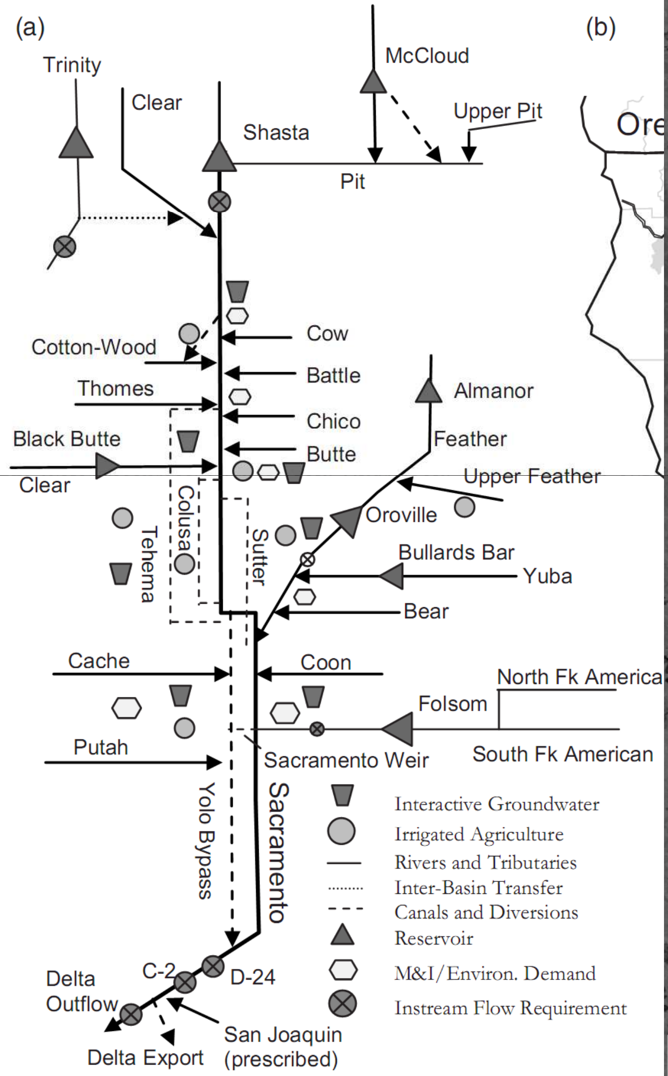

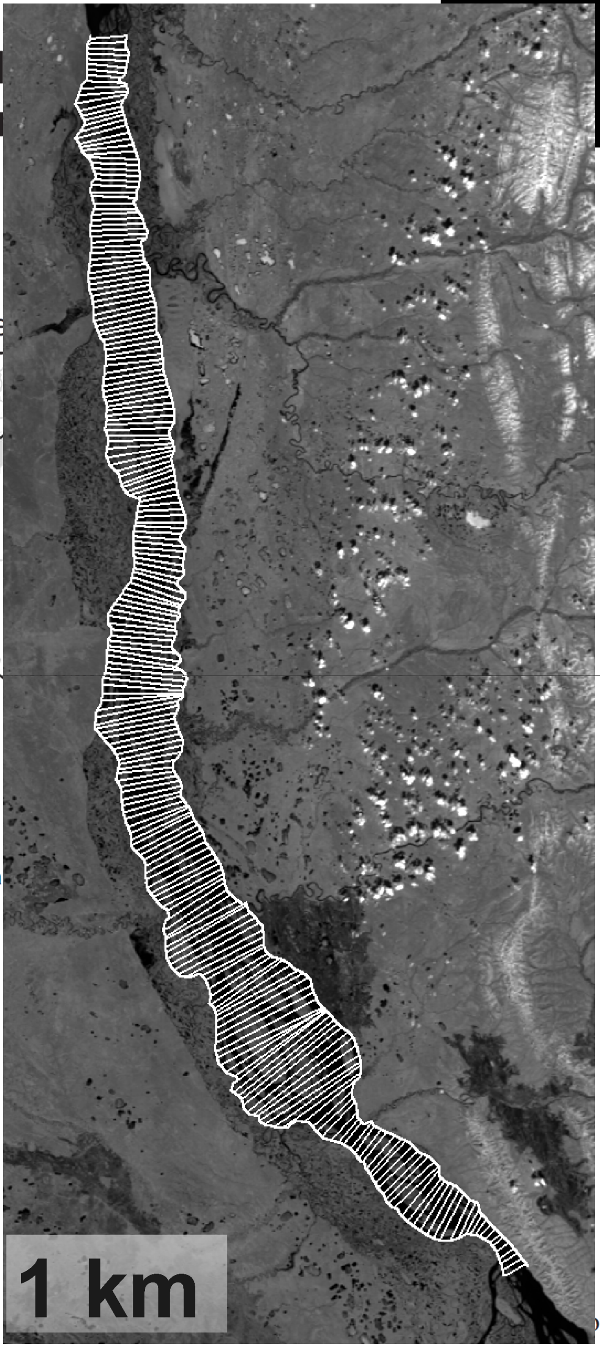

Fig. 3. Simplified schematic of the water resource elements implemented in
of the Sacramento Basin in California

What if it none of it works?

SWOT will measure: A_t and dH/dt; dH/dx

Lakes: $(A_t)(dH/dt) = dS/dt$

(temporal storage anomaly, m^3/dt)

Rivers: $(A_t/L)(dH/dt)(v_{est}) = dQ/dt$

(temporal discharge anomaly, $(m^3/s)/dt$)

ALSO longitudinal discharge anomalies, dQ/dx…

… "DIFFERENTIAL DISCHARGES"

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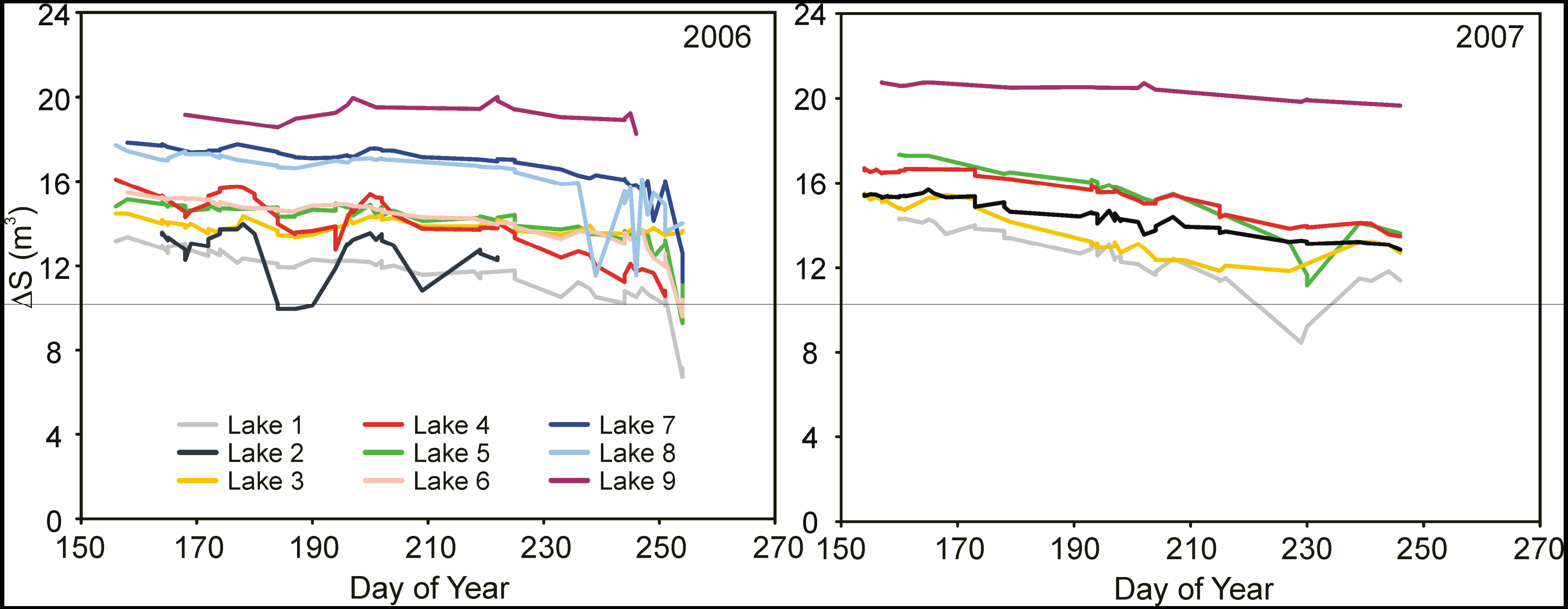
ALSO longitudinal discharge anomalies, dQ/dx…
… “DIFFERENTIAL DISCHARGES”

- will yield *new science* on how water gets in/out of rivers
- unprecedented for water management
- can be done in single overpass (e.g. AirSWOT)

BACK-UP SLIDES

SIMULATING SWOT from ground data:

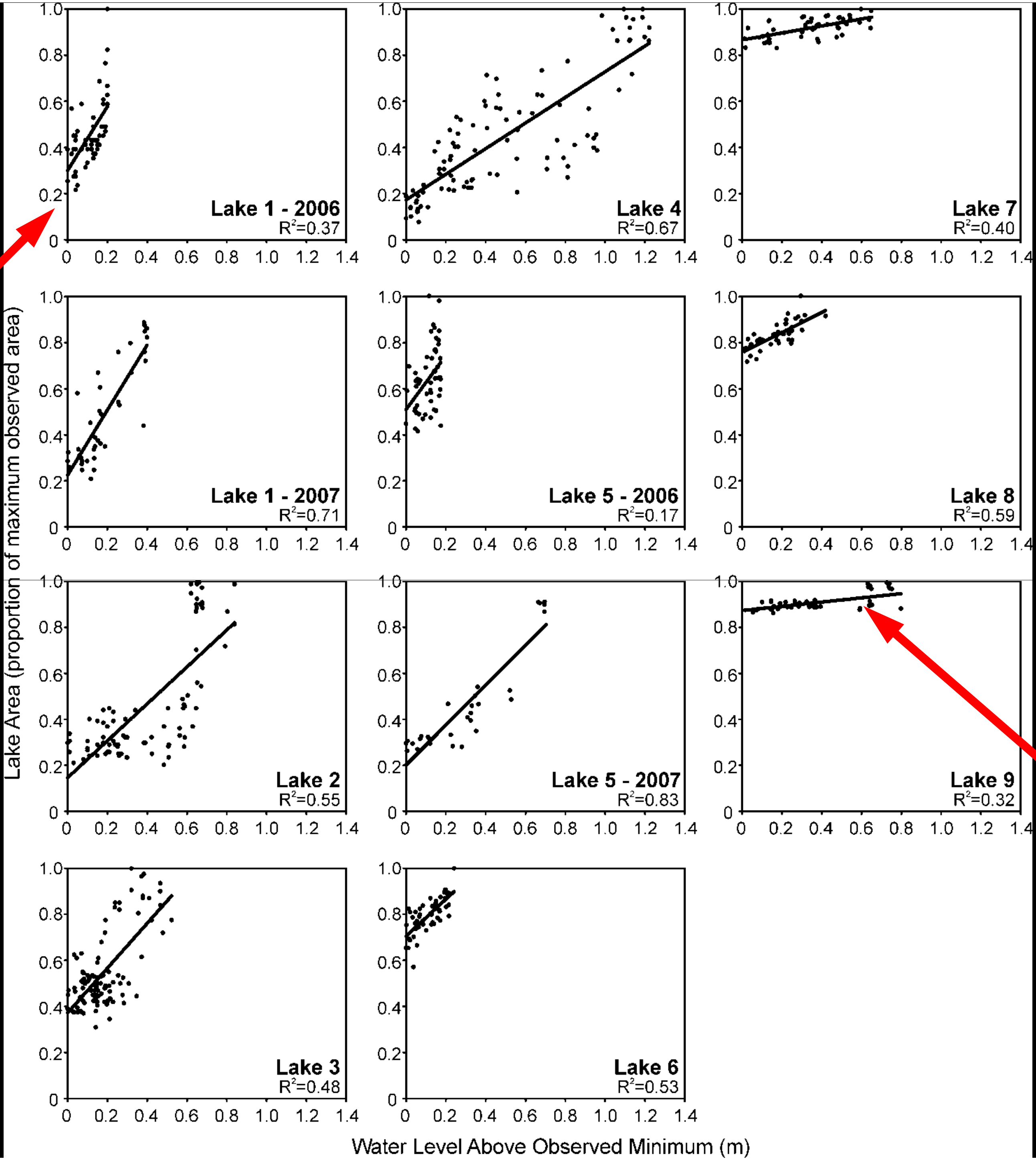
$$(A_t)(dH/dt) = dS/dt$$



(Smith and Pavelsky, ESPL 2009)

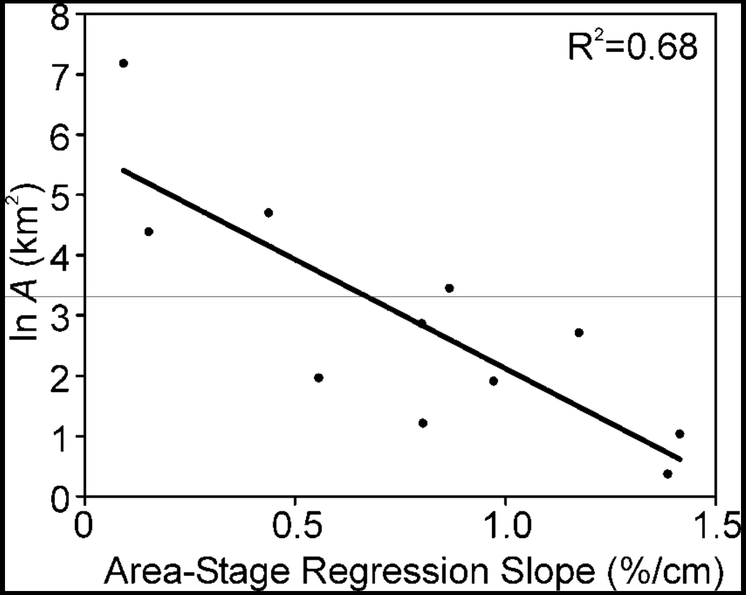
1.5 km²

Small lakes = “steeper” area-stage rating curves



1300 km²

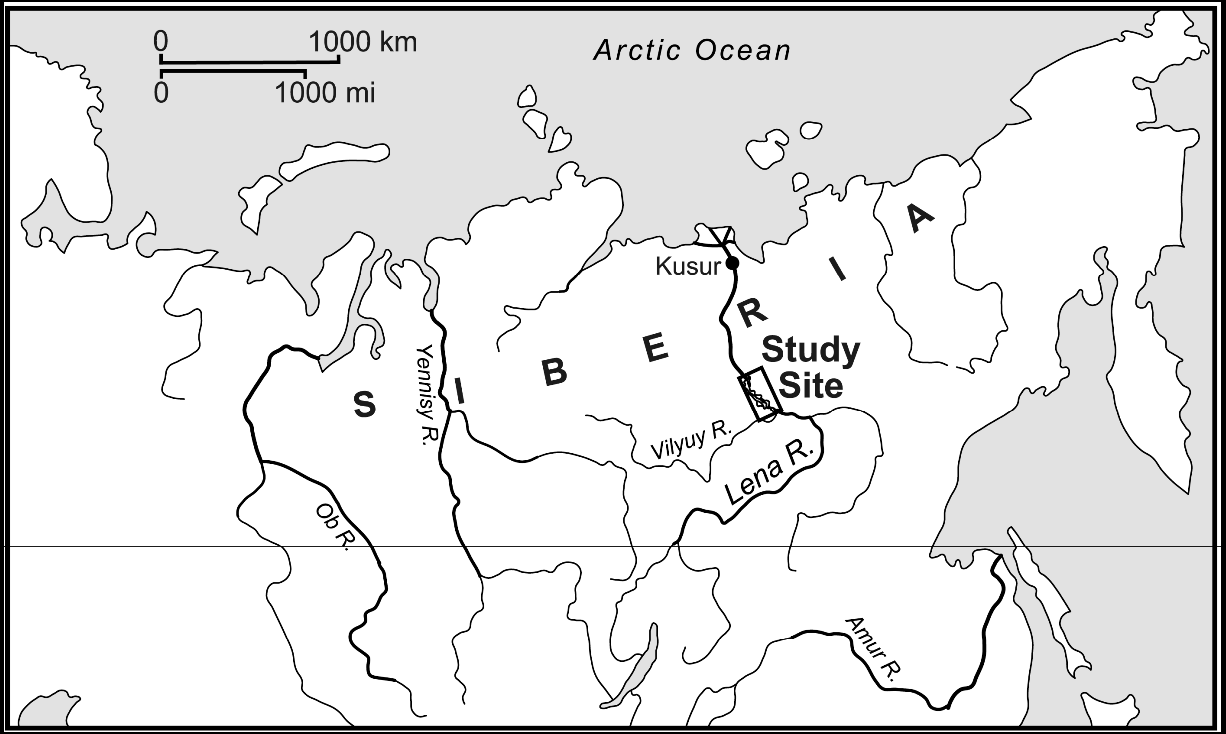
Lake “area-effect”



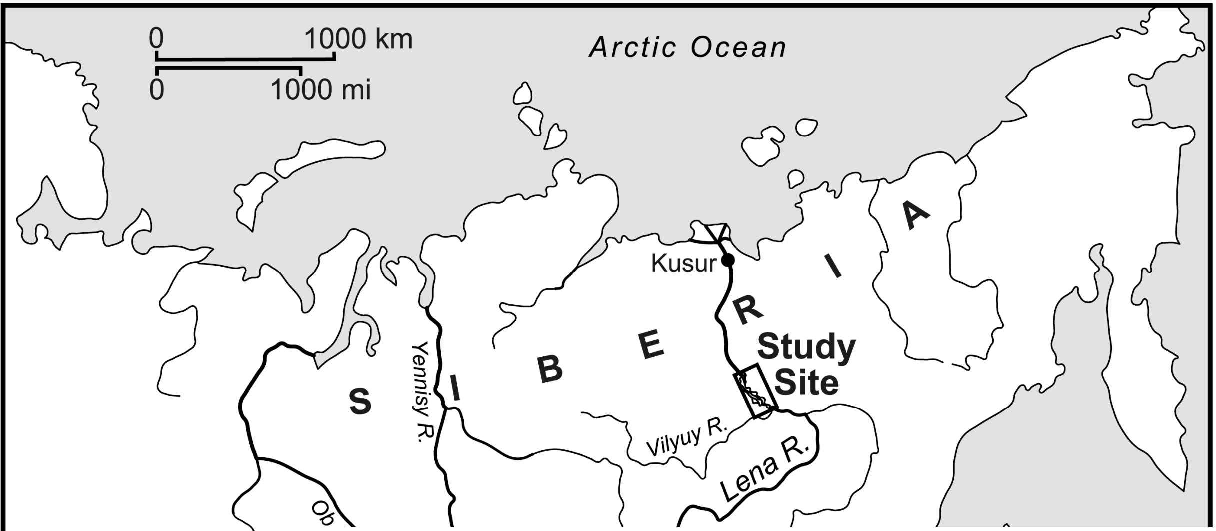
(area changes more important to dS/dt for small lakes, height changes more important for large lakes)

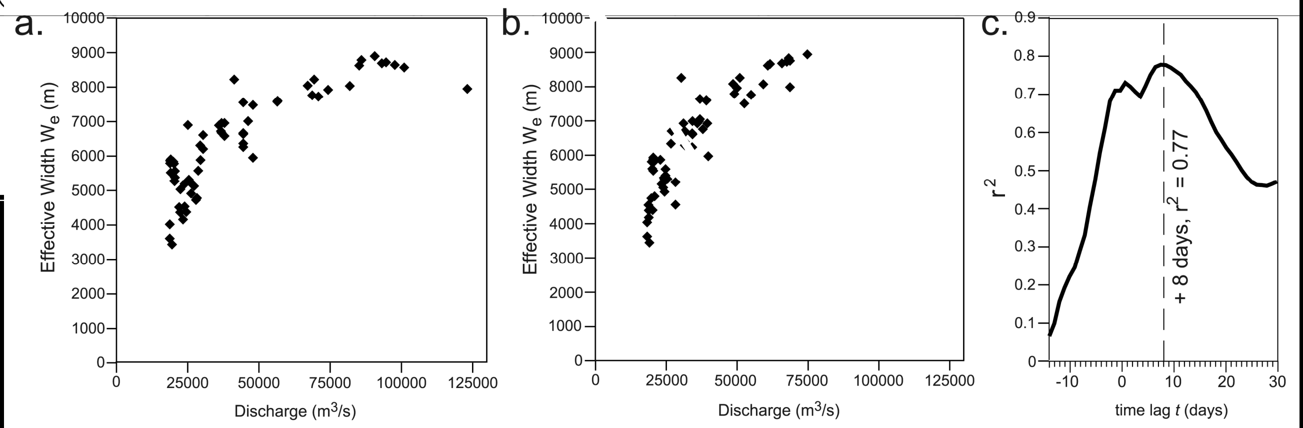
(Smith and Pavelsky, ESPL 2009)

SWOT and rivers: Lena River, Siberia



SWOT and rivers: Lena River, Siberia



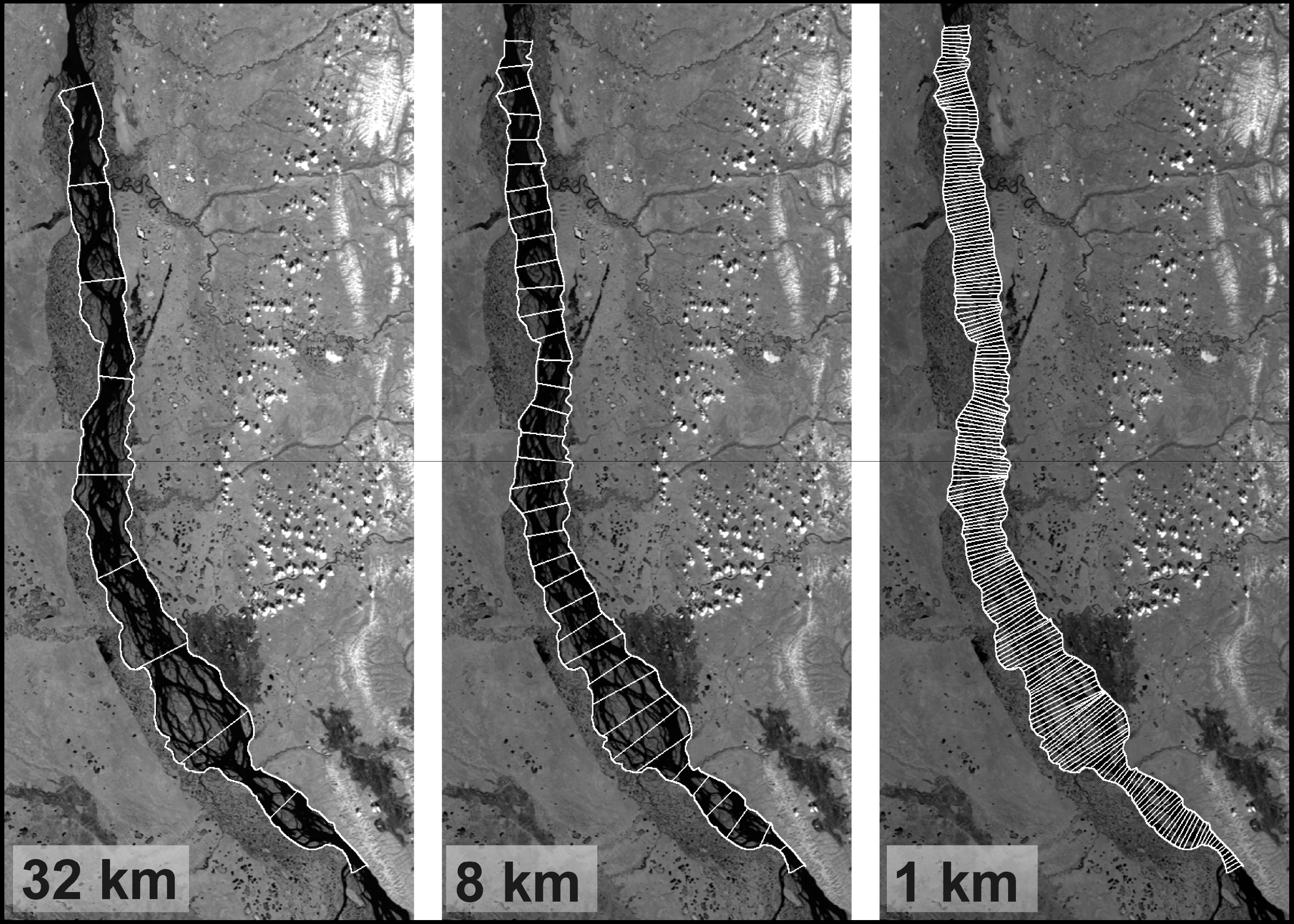


(dQ vs A:
power law)

(TIME LAG = +8 days)

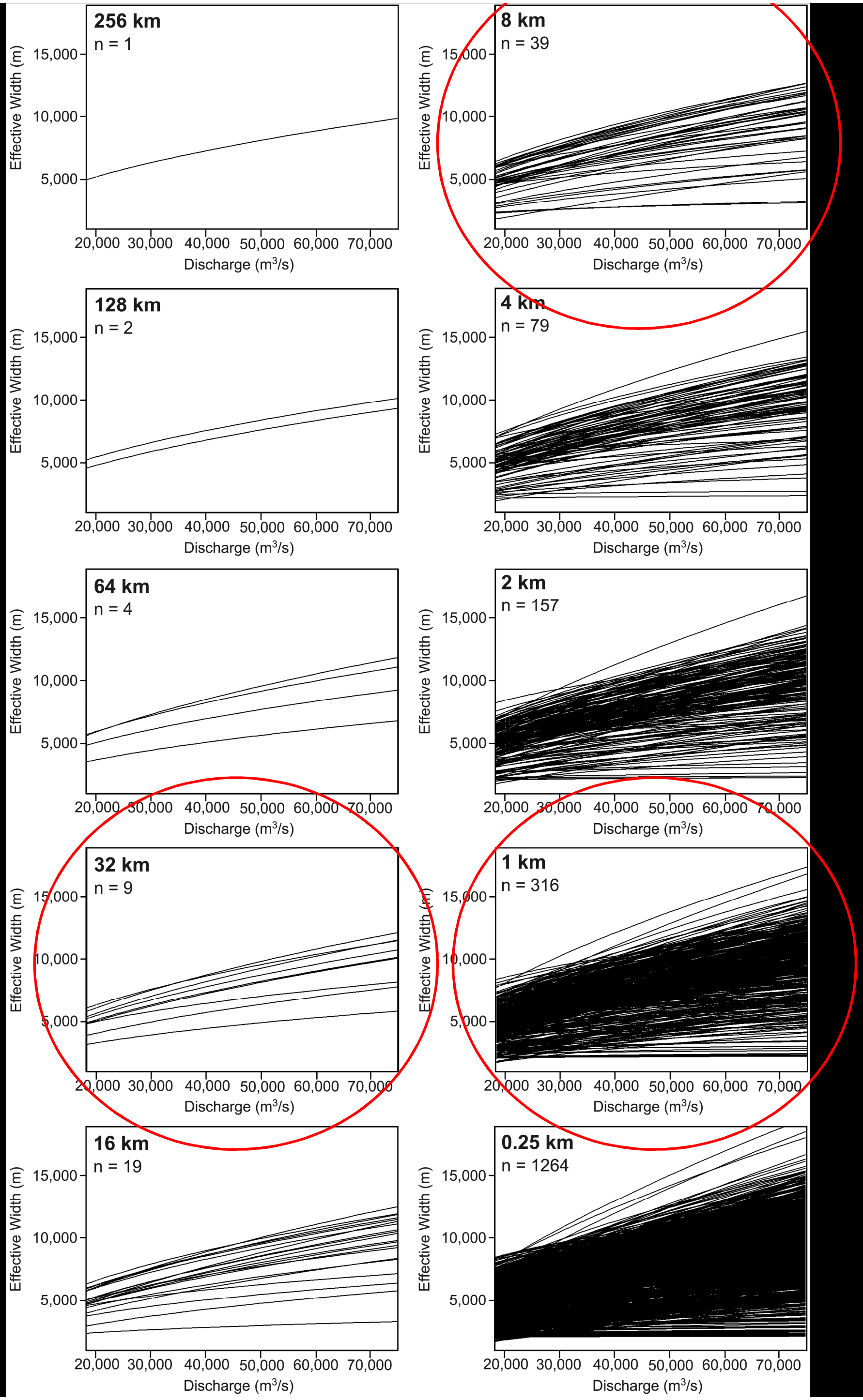
(Smith and Pavelsky, WRR 2008)

Segmenting the Lena River with MODIS:

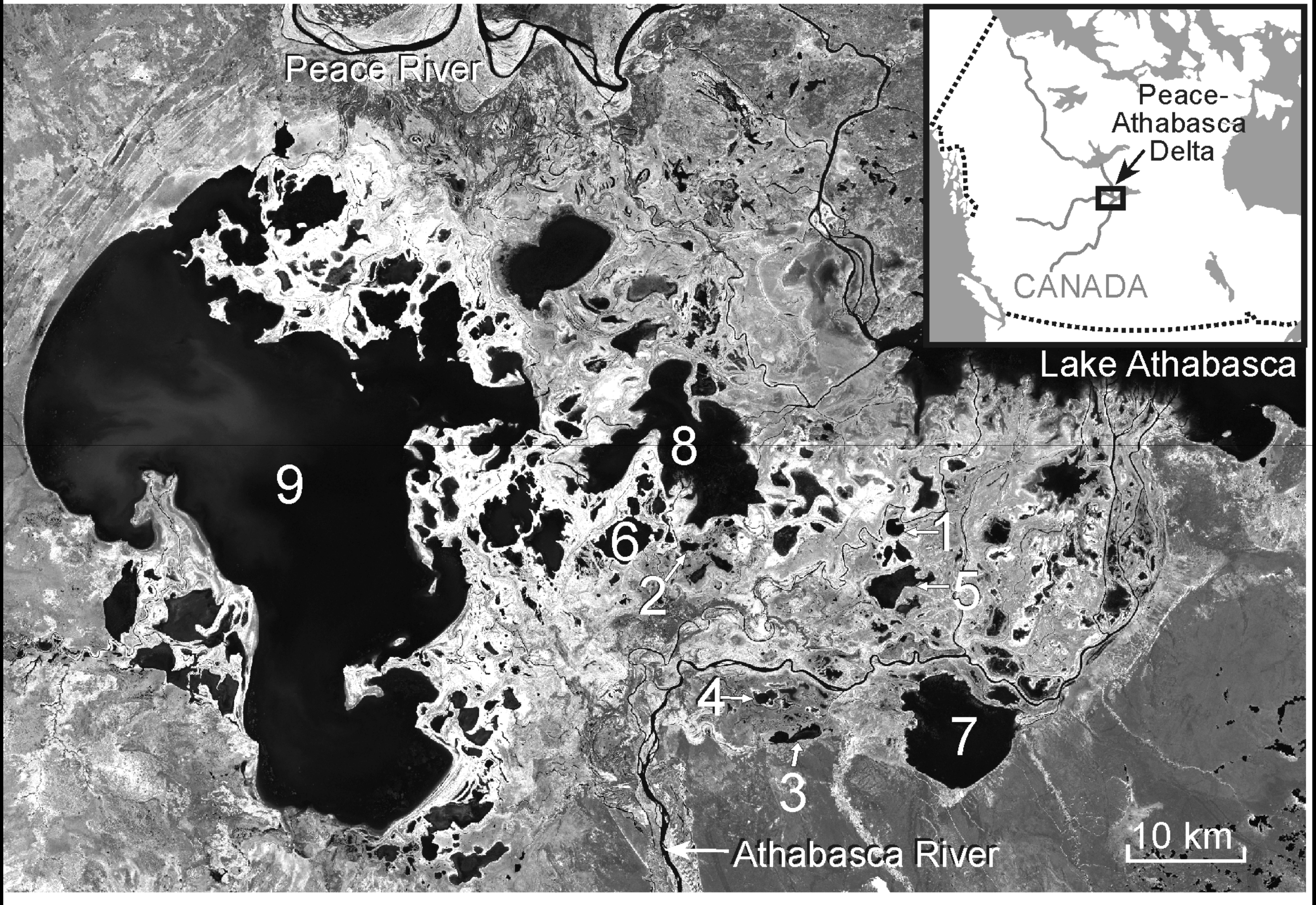


(Smith and Pavelsky, WRR 2008)

Resulting "area-discharge" rating curves converge to stable values at ~2X valley width



Peace-Athabasca Delta

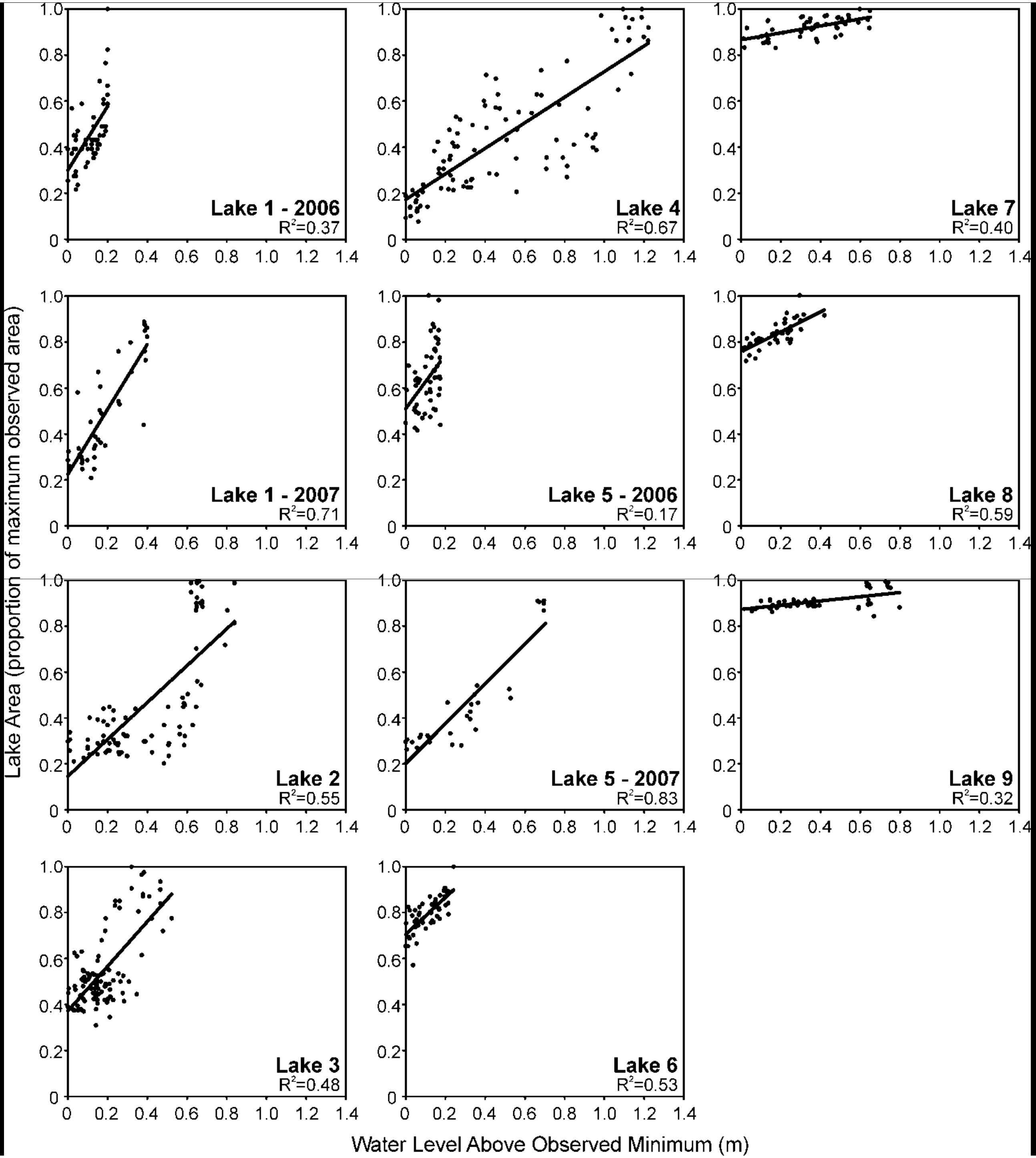


(See Tamlin Pavelsky)

HYDRAULIC GEOMETRY

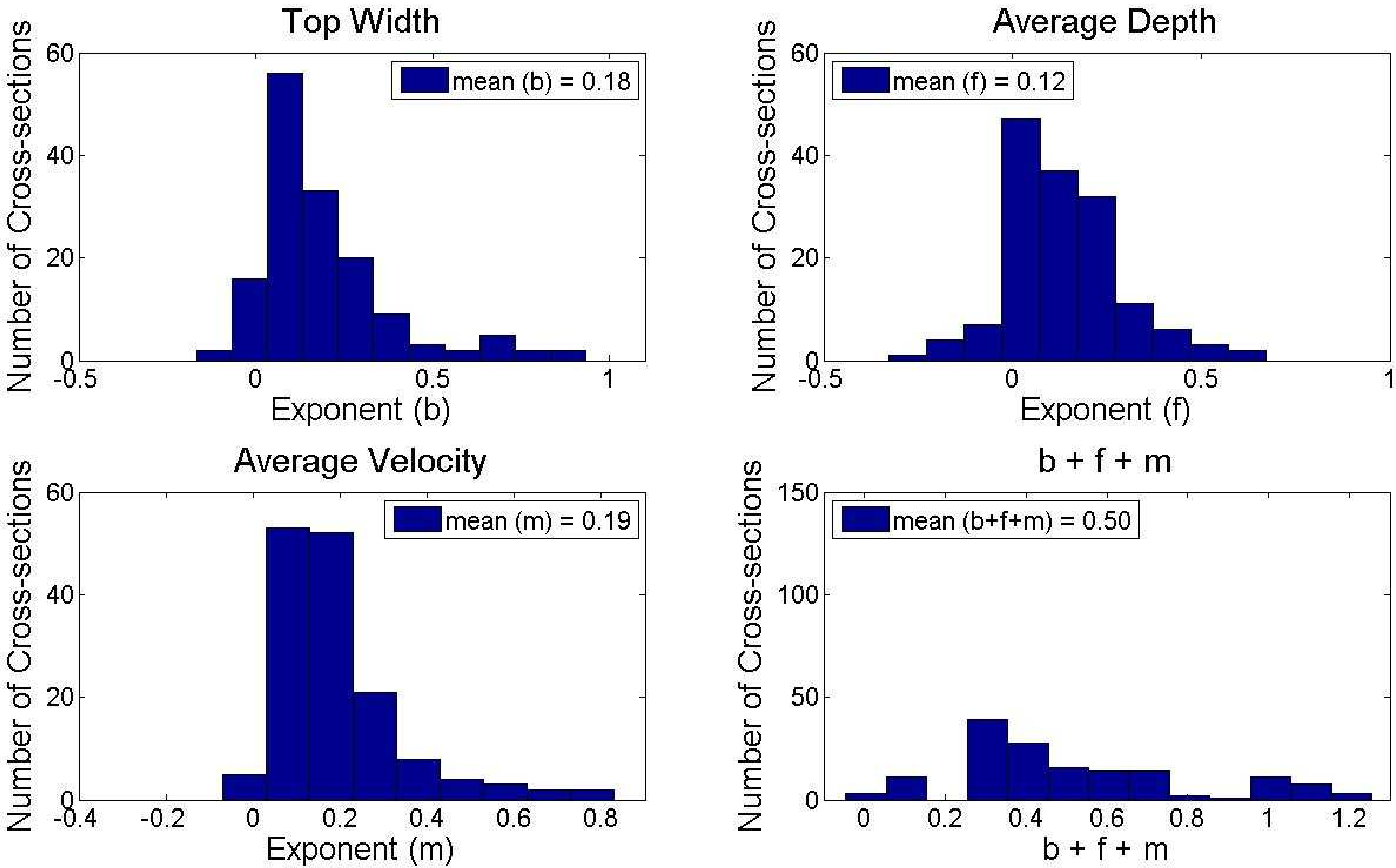
A_t vs. dH/dt

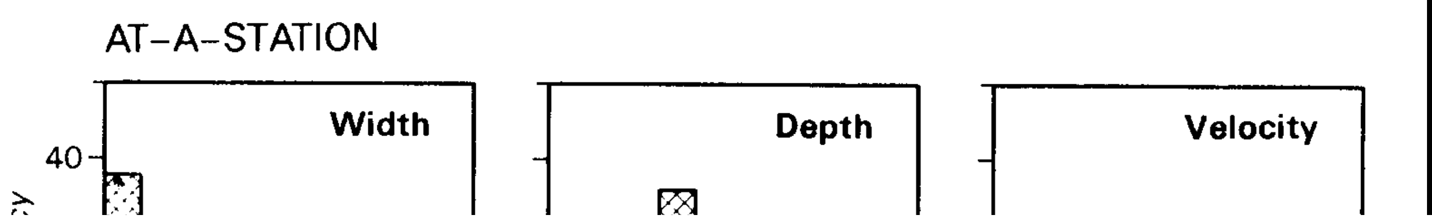
("Area-stage relationship")



(Smith and Pavelsky, ESPL 2009)

[redacted]


Top Width
mean (b) = 0.18
Number of Cross-sections
60
40
20
0
-0.5
0
0.5
1
Exponent (b)
Average Depth
mean (f) = 0.12
Number of Cross-sections
60
40
20
0
-0.5
0
0.5
1
Exponent (f)
Average Velocity
mean (m) = 0.19
Number of Cross-sections
60
40
20
0
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Exponent (m)
b + f + m
mean (b+f+m) = 0.50
Number of Cross-sections
150
100
50
0
0
0.2
0.4
0.6
0.8
1
1.2
b + f + m



Summary of distribution characteristics of hydraulic geometry exponent data

	Width	Depth	Velocity
At-a-station (n = 139):			
Range	0.00—0.59	0.06—0.73	0.07—0.71
Modal class	0.0 —0.1	0.3 —0.4	0.4 —0.5
Theoretical*[1]	0.23	0.42	0.35
Downstream (n = 72):			
Range	0.03—0.89	0.09—0.70	−0.51—0.75
Modal class	0.4 —0.5	0.3 —0.4	0.1 —0.2
Theoretical*[1]	0.55	0.36	0.09
Theoretical*[2]	0.60	0.30	0.10

*[1] Leopold and Langbein (1962).

*[2] Smith (1974).

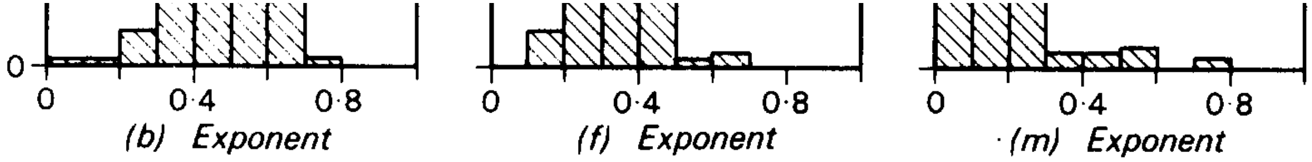
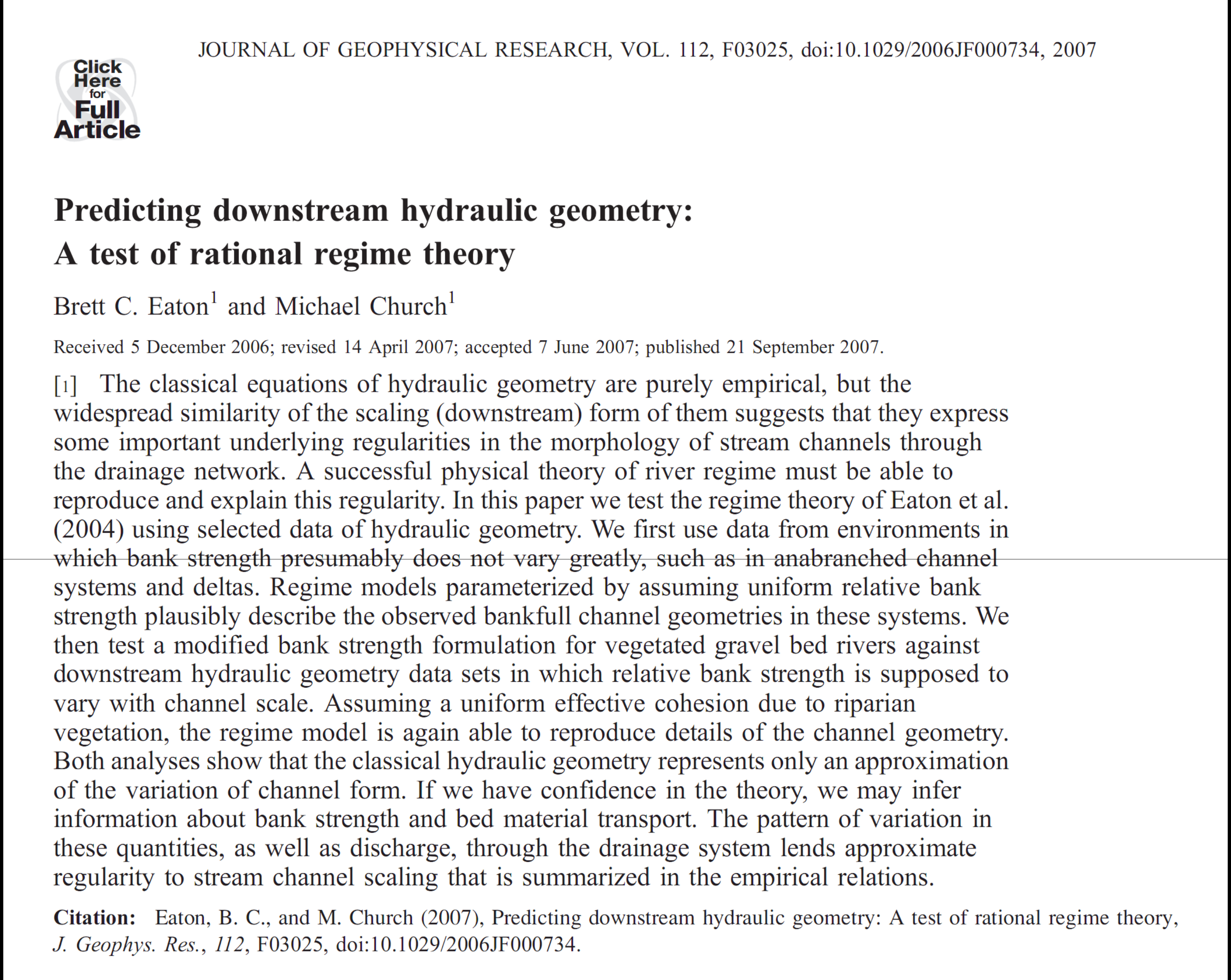


Fig. 1. Histograms of at-a-station and downstream hydraulic geometry data.



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Predicting downstream hydraulic geometry: A test of rational regime theory

Brett C. Eaton[1] and Michael Church[1]

Received 5 December 2006; revised 14 April 2007; accepted 7 June 2007; published 21 September 2007.

[1] The classical equations of hydraulic geometry are purely empirical, but the widespread similarity of the scaling (downstream) form of them suggests that they express some important underlying regularities in the morphology of stream channels through the drainage network. A successful physical theory of river regime must be able to reproduce and explain this regularity. In this paper we test the regime theory of Eaton et al. (2004) using selected data of hydraulic geometry. We first use data from environments in which bank strength presumably does not vary greatly, such as in anabranched channel systems and deltas. Regime models parameterized by assuming uniform relative bank strength plausibly describe the observed bankfull channel geometries in these systems. We then test a modified bank strength formulation for vegetated gravel bed rivers against downstream hydraulic geometry data sets in which relative bank strength is supposed to vary with channel scale. Assuming a uniform effective cohesion due to riparian vegetation, the regime model is again able to reproduce details of the channel geometry. Both analyses show that the classical hydraulic geometry represents only an approximation of the variation of channel form. If we have confidence in the theory, we may infer information about bank strength and bed material transport. The pattern of variation in these quantities, as well as discharge, through the drainage system lends approximate regularity to stream channel scaling that is summarized in the empirical relations.

Citation: Eaton, B. C., and M. Church (2007), Predicting downstream hydraulic geometry: A test of rational regime theory, *J. Geophys. Res.*, *112*, F03025, doi:10.1029/2006JF000734.

(JGR, 2007)

AHG b,f,m exponents, n=145 (overbank flows)