ON THE DISPERSION RELATION OF WESTWARD PROPAGATING SIGNALS IN THE OCEANS

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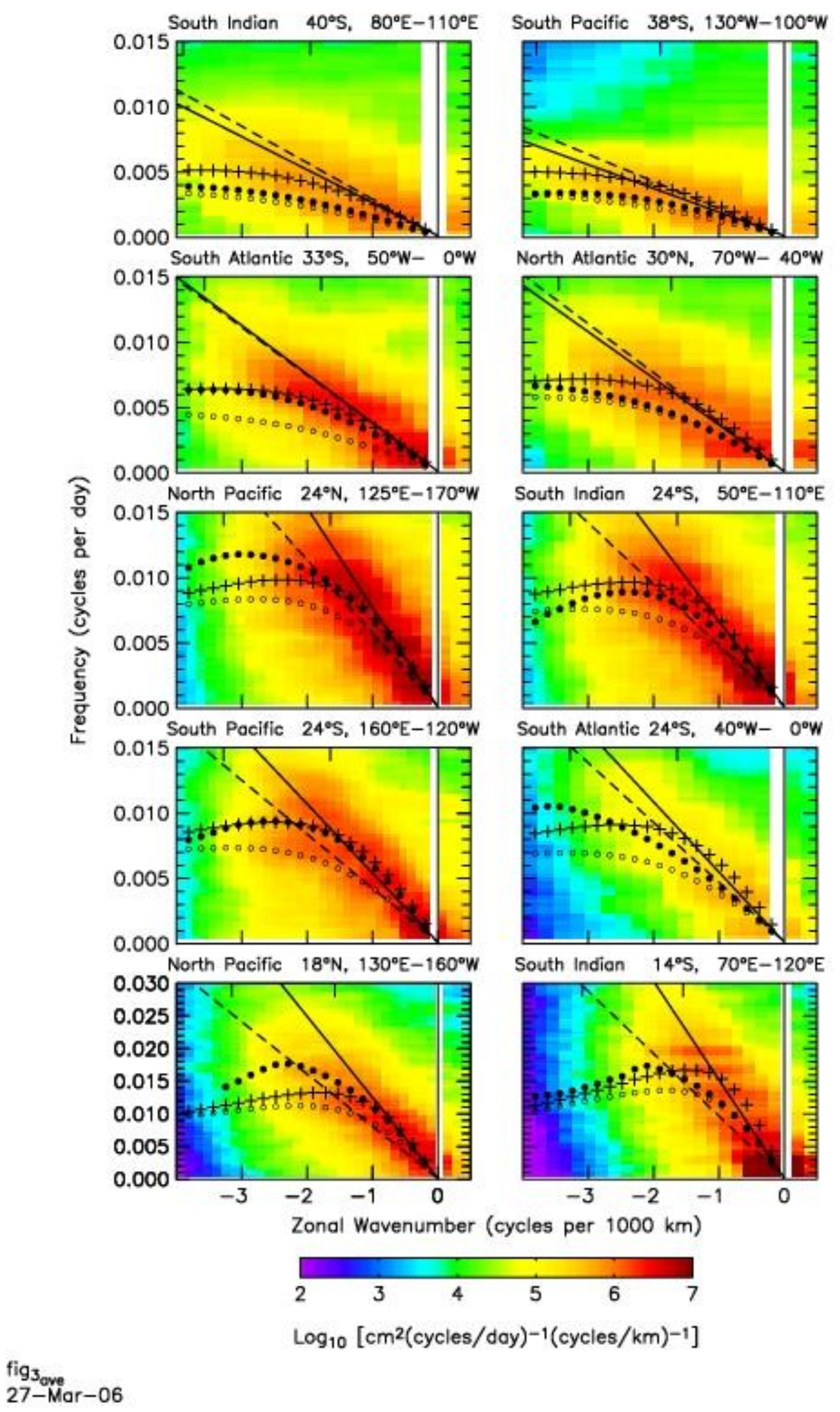
1.INTRODUCTION

An intriguing feature of zonal wavenumber/frequency power spectra of sea surface height (SSH) variability is the apparent quasi-nondispersive character of westward propagation at the highest wavenumbers, which is in contrast with the expectation of the standard linear theory for first mode baroclinic Rossby waves. This feature is illustrated in Fig. 1, which shows the zonal wavenumber/frequency power spectra of SSH variability for several ocean basins at several latitudes, along with various theoretical dispersion relations corresponding to different assumptions. Theoretical dispersion relations tend to exhibit a marked dispersive behaviour at the highest wavenumbers, in contrast to what the observations suggest.



Two main explanations have been proposed so far. Killworth and Blundell (2004,2005) suggested that this is due to the mean flow, whereas Chelton et al. (2007) suggests that this is due to the presence of quasi-nondispersive eddies. This poster examines the first explanation in more details.

Fig. 1 *Empirical dispersion relations estimated at selected locations in the oceans (color) superimposed with the predictions of theories. The plus signs correspond to the dispersive extension of the bottom pressure compensation theory of Tailleux and McWilliams (2001). The white circles correspond to the classical dispersive theory for Rossby Waves over a flat bottom, and in absence of background mean flow. The filled black circles correspond to a dispersive extension of the zonal mean flow theory over a flat bottom of Killworth and Blundell (2004).*

2. EXTENDED LINEAR ROSSBY WAVE THEORIES

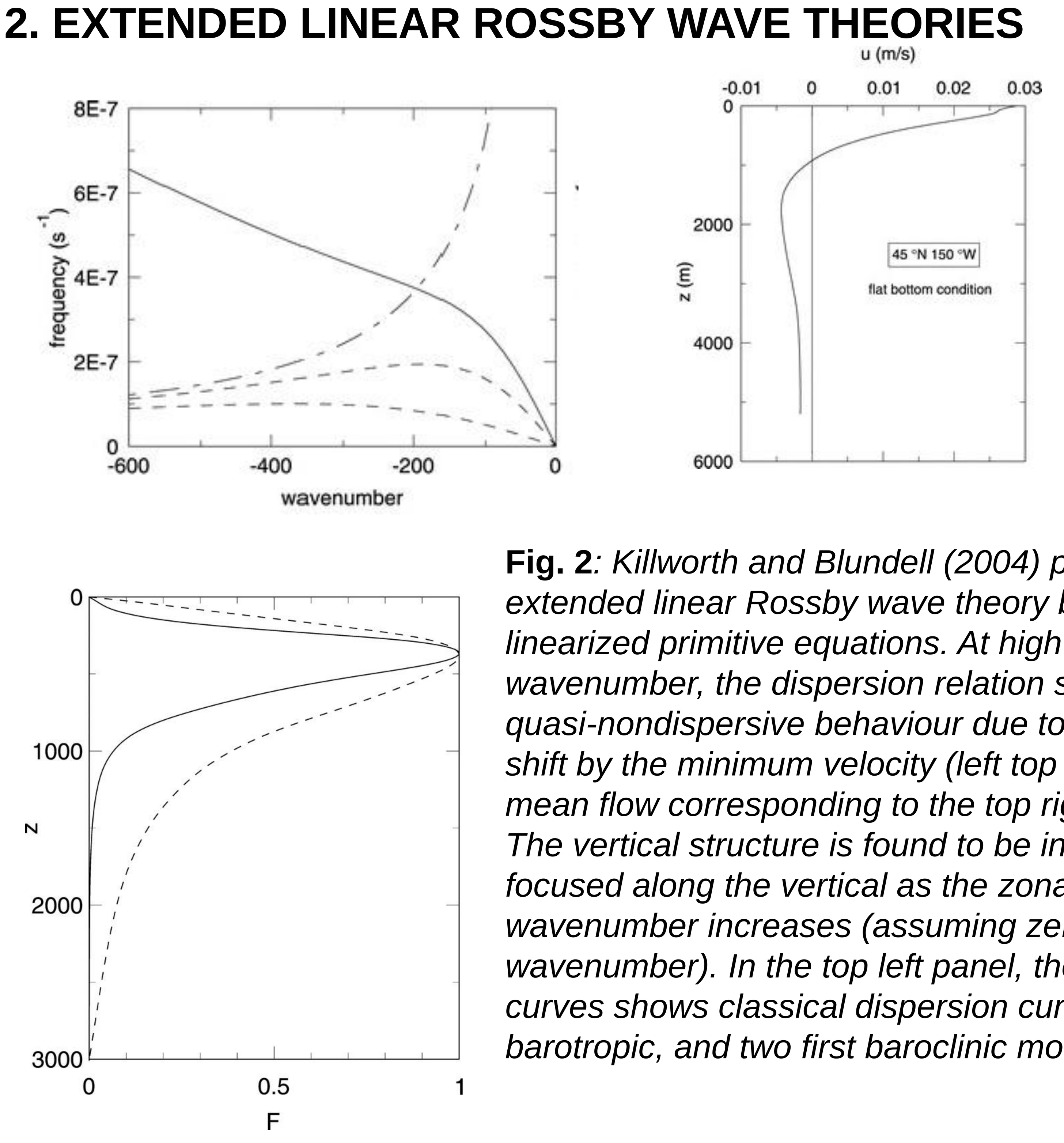


Fig. 2: *Killworth and Blundell (2004) proposed an extended linear Rossby wave theory based on the linearized primitive equations. At high wavenumber, the dispersion relation shows a quasi-nondispersive behaviour due to the Doppler shift by the minimum velocity (left top panel) for a mean flow corresponding to the top right panel. The vertical structure is found to be increasingly focused along the vertical as the zonal wavenumber increases (assuming zero meridional wavenumber). In the top left panel, the other curves shows classical dispersion curves for the barotropic, and two first baroclinic modes.*

3. NEW RESULTS

3.1 Correction of an error in Killworth and Blundell (2004)'s theory

Although KB04's theory claims to be based on the linearized primitive equations, its scaling seems to be consistent with that underlying quasi-geostrophic (QG) theory, so that it should be recovered directly from linearized QG theory. Doing so yields a generalized eigenvalue problem that is naturally in terms of the pressure anomaly, whereas KB04's theory was naturally formulated in terms of the vertical velocity anomaly. It turns out, however, that while the QG eigenvalue problem is linear in pressure, it is nonlinear in vertical velocity! The fact that KB's eigenvalue problem for the vertical velocity is linear is due to an unjustified assumption.

Comparison of the different eigenvalue problems: the two eigenvalue problems differ by the nonlinear term in red

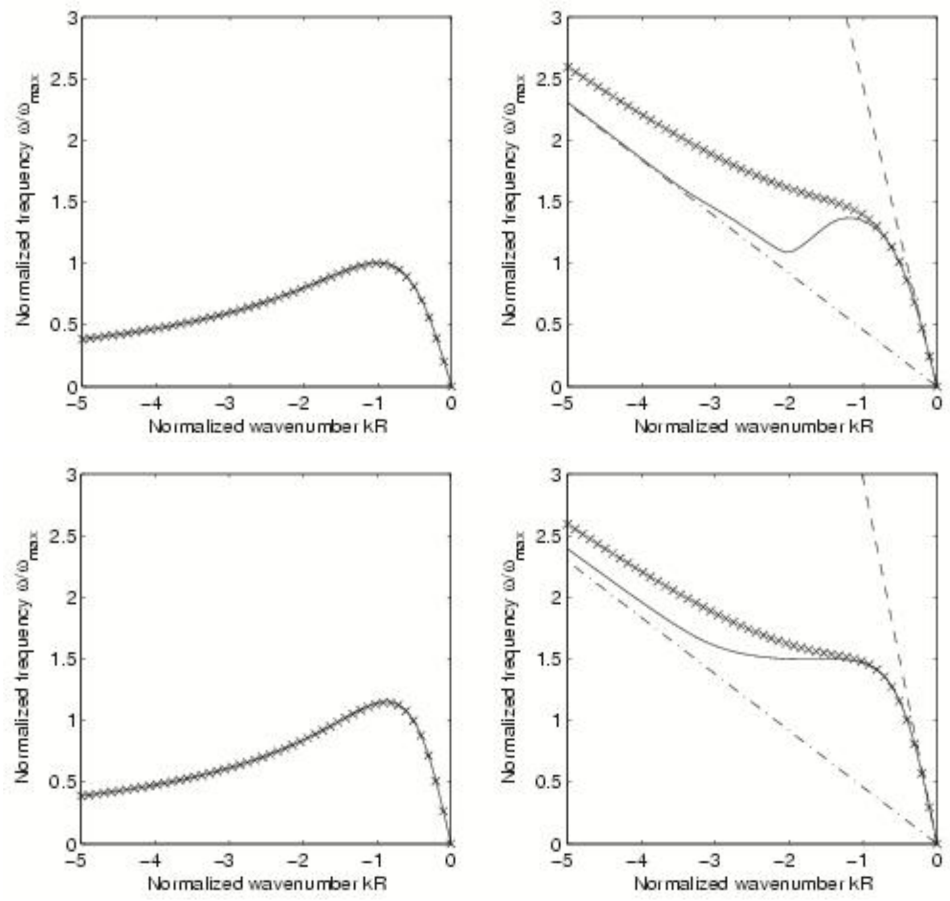
QG Theory
$$(\bar{u}-c)\frac{d^2W}{dz^2}-\left[1-\frac{K^2(u-c)}{\beta-K^2(u-c)}\right]\frac{d\bar{u}}{dz}\frac{dW}{dz}+\frac{\left[\beta-K^2(\bar{u}-c)\right]N^2}{f_0^2}W=0$$

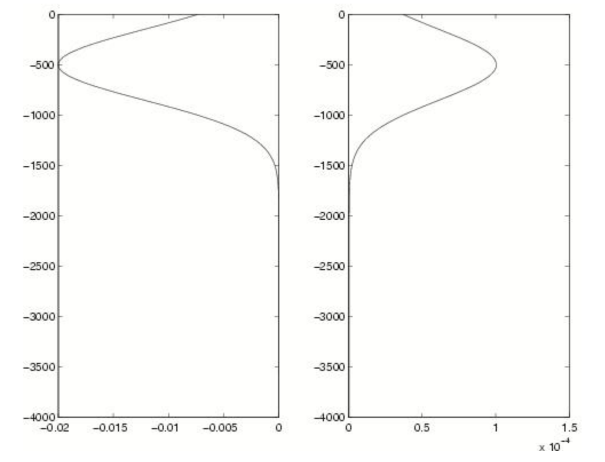
KB theory
$$(\bar{u}-c)\frac{d^2W}{dz^2}-\frac{d\bar{u}}{dz}\frac{dW}{dz}+\frac{\left[\beta-K^2(\bar{u}-c)\right]N^2}{f_0^2}W=0$$

3.2 Consequences

Fig 3. illustrates the consequences of the error for a particular buoyancy frequency and velocity profile (left), for two different bottom boundary conditions (right panels). In absence of mean flow, both QG and KB theories yield the familiar Rossby wave dispersion curve. In presence of mean flow, however, the two theories diverge for the highest wavenumbers.

Fig 3: *Right panels:*
Crosses = KB04 theory
Thin line = New QG theory
Left: Flat-bottom case
Right: Mean flow and buoyancy Frequency depicted below





3.3 Asymptotics

Considerations of the behaviour of the dispersion relation at high wavenumbers reveals that the dispersion relation can be very generally expressed as follows. This result shows that the quasi-nondispersive behaviour at high wavenumbers is explained by the Doppler shift by the minimum velocity.

$$c=\frac{\omega}{k_x}=\bar{u}_{\min}-\frac{\beta+\beta_{mf,\min}}{\lambda^2+K^2}$$

In this expression, $\beta_{mf,min}$ is the modification to the background planetary vorticity gradient by the mean flow at the depth where U reaches its minimum, while λ is the inverse of a pseudo Rossby radius of deformation that depends on the wavenumbers. C is the zonal phase speed. In absence of mean flow, λ becomes a constant, while u= $\beta_{mf,min}$ =0, and the classical dispersion relation is recovered.

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References:
Chelton, D.B., M.G. Schlax, R.M. Samelson, and R.A. De Szoeke (2007): Global observations of large oceanic eddies. Gophys. Res. Letters, 34.
Killworth, P.D. And J.R. Blundell (2004): The dispersion relation for planetary waves in the presence of mean flow and topography. Part I: Analytical theory and one-dimensional examples. J. Phys. Oceanogr., **34,** 2692-2711.
Tailleux, R. And A.M.Maharaj (2010): On the zonal wavenumber/frequency spectrum of westward propagating signals. To be submitted to J. Phys. Oceanogr.

5. CONCLUSIONS The extended theory of Killwortn and Blundell (2004) contains an error that affect the dispersion relation at the highest wavenumbers. The present work shows how this error can be corrected in the context of quasi-geostrophic theory. Interestly, whereas in the long wave limit, generalized eigenvalue problems can be indifferently formulated in terms of vertical velocity or pressure, yielding in both cases a linear eigenvalue problem, this is only the case for the eigenvalue for pressure when dispersive effects are retained. Although both the incorrect KB theory and the new corrected QG theory appears to yield a quasi-nondispersive behaviour at high wavenumbers, both theories appear to be doing so in a quite unrealistic fashion. Moreover, the analysis of the vertical structure of the modes suggests that the latter becomes more and more focused around the depth at which U reaches its minimum, raising questions as to the possibility of observing such modes in the sea surface height. Nonlinearities are therefore likely to be needed to some extent to account for the observed quasi-nondispersive behaviour.