Retracking range, SWH, sigma-naught, and attitude in CryoSat conventional ocean data

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CryoSat-2 LRM Mode Products

- LRM (Low Rate Mode) = Operates as a conventional altimeter.
- LRM Products:
  - FDM (Fast Delivery Mode) = short latency, DORIS DIODE or predicted orbit, predicted meteo & ancillary data.
  - “LRM” = Final version, precise orbit, analyzed meteo, etc. (final “GDR”).

Level L1b = Has waveform and geophysical corrections, but no derived quantities (range, SWH, $\sigma^0$) ➞ no sea surface height, wind speed ($U_{10}$), wave height, backscatter, etc.

Level 2 = No waveform; has geophysical corrections and derived quantities.

We build all our results from L1b FDM and LRM waveform products, not Level 2.
CS2 LRM mean waveform

- Amplitude $A$
- Start
- Epoch, $x_0$ (Range)
- Width, $s$ (SWH)
- Noise Level, $N$
- Decay, $\kappa$ (Mispointing)
- Stop
Elliptical antenna pattern

Classical “Brown model” theory for the expected waveform shape assumes a circular antenna pattern.

CryoSat2’s antenna pattern is slightly elliptical.

If we average CS2’s beam width over all azimuths, then the azimuthally averaged half-power beam width is the harmonic mean of the major and minor elliptical HPBWs.

We retrack CS2’s waveforms with a circular beam theory using the azimuthally averaged HPBW.

This is a good approximation for conventional LRM waveforms.

This would be wrong for SAR/SARIN waveforms.

Wingham & Wallace [2010] have developed the full theory for the elliptical antenna. Their paper supports our belief that the circular approximation is a good one in our case.
Retracker options

Retracker allows selection of any or all of these parameters to be fitted:
1) Epoch, $x_0$
2) Width, $s$
3) Amplitude, $A$
4) Mispointing, $\kappa(\xi^2)$, $\xi$ is off-nadir angle
5) Noise level, $N$

Any of these can be free parameters to be fitted, while others are held fixed.

$k(\xi^2)$ can use $0^{th}$, $1^{st}$, or $2^{nd}$ order approximation of Bessel function $I_0(z)$.

- $I_0(z) = 1$, [MacArthur]
- $I_0(z) = \exp(z^2/4)$, [Rodriguez]
- $I_0(z) =$ two terms in $\exp()$ [Amarouche et al.]

Example:
“MLE3” would be first 3 parameters, and $1^{st}$ order approximation.
“MLE4” would be first 4 parameters, and $2^{nd}$ order approximation.
“RED3” would be first 3 parameters, $1^{st}$ order approximation, and fewer gates fitted.
Retracker iterative search requires these steps:

1) Initialization (by given or default values)
2) Interactive update using Modified Gauss-Newton steps
3) Stopping criteria for success and failure

Initialization can supply “known” values (e.g., off-nadir angle given from star trackers, or along-track smoothed prior estimates, as in Sandwell & Smith two-pass method).

Iteration is solved by QR decomposition of column-balanced Jacobian (MLE3&4 use QR with column pivoting).

Stopping criteria for success/failure (in unweighted case) are MLE-like ($A^2$-normalized change in Mean Quadratic Error < $5 \times 10^{-4}$ for 3 iterations = success).

Example:
To behave as MLE3 or MLE4, initialize with default values and proceed as above.
Initialization of search may default to these values:

1) Epoch, \( x_0 \): set to normal track point.
2) Width, \( s \): set equivalent to SWH = 2 m.
3) Amplitude, \( A \): set to Max(waveform)
4) Mispointing, \( \kappa(\xi^2) \), \( \xi \) is off-nadir angle: set to \( \xi = 0 \).
5) Noise level, \( N \): set to average of first five gates used.

For CryoSat2, we fit the middle 104 of the 128 gates, as is done also for Jason. Laurent P. said in Coastal meeting we should make this 106 and 126, I think.

Our first attempt was to replicate MLE3 assuming \( \xi = 0 \).
CS2 SWH (MLE3, $\xi = 0$)

$\text{swh (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18}$
CS2 $\sigma^0$ (MLE3, $\xi = 0$)

sig0 (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

Note Asc/Dsc Asymmetry

Needs to shift by a constant
Making a guess at $\sigma^0$

From Orbit

$$\ell\left(\frac{P_R}{P_T}\right) = \ell(\sigma_0) + C - \ell(L) - 30 \log_{10}\left(\frac{h'}{h_N}\right) + \frac{10}{\ln(10)} \left[ -\frac{4}{\gamma} \sin^2 \xi + \frac{k_s}{2} \right] + \ell\left(\frac{A}{A_N}\right)$$

Constant, unless raining

From Retracker

Target Amplitude Maintained by AGC

in which

$$C = \ell\left(\frac{c\pi\lambda^2 G_0^2 A_N T}{(4\pi h_N)^3 \eta}\right)$$

$\sigma^0$ is mainly given by a constant, $C$, (+/-?) AGC (is AGC an amplification or an attenuation?) (AGC or 62 – AGC ?). Retracking yields small (< 1 dB) corrections to apply to refine $\sigma^0$. We had to guess the system constant, $C$, and get the sign right, in order that the histogram would not be inverted.
CS2 AGC is not asymmetric

agc (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

No Asc/Dsc Asymmetry
Chasing down the asymmetry

We introduced the ascending / descending asymmetry in $\sigma^0$ either through the orbit height, or by neglecting to correct for mispointing, since we initially assumed $\xi = 0$.

So next we tried fitting the off-nadir angle as a free parameter (MLE4-like).

You will see that this fixes the asymmetry problem.
CS2 $\sigma^0$ (MLE4, $\xi = \text{free}$)

Asymmetry goes away

Constant now almost right
xi2 (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18
But freedom in $\xi$ has a price!

The 1-Hz averaged SSH is found by fitting a straight line to twenty values of 20-Hz (orbit – range). The standard deviation of the residuals around the line we call here $\sigma_{SSH}$. This number represents the random noise in range.

Range noise goes up, and is more dependent on SWH, with freedom in $\xi$. 
CS2 $\sigma_{SSH}$ from MLE4 ($\xi = \text{free}$)

$\sigma_{ssh}$ (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

~6.5 cm, corr w. SWH
J-1&J-2 $\sigma_{SSH}$ from MLE4

sigssh (j1j2) – cycles 344/105 – 2011/05/04 – 2011/05/19

~7 cm, corr w. SWH
CS2 $\sigma_{SSH}$ from MLE3

$\sigma_{ssh}$ (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

~6 cm, less corr. w. SWH
Solution: a priori $\xi$

We next tried using MLE3, holding $\xi$ fixed to an *a priori* value determined from the platform attitude found from star trackers.

This data is given in the FDM and LRM L1b product.
CS2 $\xi^2$ from retracker (MLE4)

$\xi^2$ (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

![Map with color scale](image)

![Histogram](image)
CS2 $\xi^2$ from star trackers

$\xi^2p (fdm1r)$ – subcycle 014 – 2011/04/19 – 2011/05/18

Ascending

Descending

deg$^2$
Platform attitude biases

We fit a model for platform bias of the form
\[ \xi^2 = (\text{pitch} – \text{bias}_p)^2 + (\text{roll} – \text{bias}_R)^2. \]

Assumptions: negligible thermal flexures; negligible biases between star trackers.

We found we had to do this using the LRM L1b, not the FDM L1b, because of FDM orbit problems that noticeably affect pitch estimates.

We estimate:

Pitch bias: +0.0962 degrees.
Roll bias: +0.0848 degrees.
CS2 $\xi^2$ from adj. stars, LRM

xi2 (lrn1r) – subcycle 014 – 2011/04/19 – 2011/05/18
CS2 $\xi^2$ from adj. stars, FDM

xi2 (fdm1r) – subcycle 014 – 2011/04/19 – 2011/05/18

[Map of ocean surface topography with color scale indicating deg^2]

Histograms showing distribution of data points.
The FDM uses a DORIS orbit if it exists when the pass segment is processed. If not, FDM uses a predicted orbit.

The predicted orbit has height errors of meters but is interpolated at the correct time.

The DORIS orbit has accurate enough (for NRT) height, but is interpolated at the wrong time; delta_UT1 should be \( \sim 0.266 \) sec but is set to 0.

This huge timing bias causes an apparent pitch of

\[
\frac{360^\circ}{6000 \text{s}} \times 0.266 \text{s} = 0.016^\circ.
\]
Conclusions

We are producing SWH, $\sigma_0$, $U_{10}$, and SSH by retracking FDM and LRM L1b waveforms.

Our product compares well with J1, J2, E, though there are *ad hoc* values that could be tuned.

Range precision of CS2 appears superior to J1&2 when both are retracked with MLE4.

With a good orbit 1 to 3 days behind real time we could make an I-GDR.

Our product depends only on the L1b data at ESRIN ftp; we are not proposing upstream changes.