Bayes Linear Retracking

Peter Challenor, Luke West and Paolo Cipollini
National Oceanography Centre, UK

Introduction

Altimeter waveforms are conventionally tracked/retracked using maximum likelihood (MLE) or least squares to fit a 'Brown' model to the received waveform. Each waveform is treated completely separately and we can track them in any order. Even in open ocean we are throwing information away as we know that the sea surface height, wave and wind fields are 'smooth' and there is information about the next altimeter waveform in the previous one. In the coastal zone we may want to include information from previous waveforms, e.g. the presence of 'specular' echos or from external sources, e.g. wavebuoys or models. Or we may want to add less quantitative information such as wanting our estimates to be smooth.

Bayesian Retracking

Before we track an altimeter waveform we have information on what we expect the new waveform to look like. We do not expect any of the waveform parameters to move very far from the values gained from the previous waveform. We might also have some external information; for example we might have information on the wave height and σ^0 value from numerical weather prediction model forecasts or analyses. How can we incorporate such information in our estimates in a rigorous way that still allows to calculate error estimates etc as we can with MLE?

One approach is to use Bayesian statistical methods. If $\pi(w(\theta))$ is the likelihood of our observed waveform for a given set of parameter values, and our prior (pre-existing) information on those parameters expressed as a probability density function is $\pi(\theta)$. Then Bayes theorem states that

$$\pi(\theta|w) \propto \pi(w(\theta))\pi(\theta)$$

where $\pi(\theta|w)$ is the final (posterior) probability density for the retracked parameters. Taking the values of θ that maximise $\pi(\theta|w)$ would give us retracked point estimates. Solving Bayes theorem is non-trivial but modern statistical methods such as Markov Chain Monte Carlo can be used. However this is computationally very expensive (see Severini et al., 2008).

Bayes Linear Retracking

The full Bayes solution gives us the probability density of our parameters (sea surface height, significant wave height, etc) but we do not really need this amount of information. For almost all purposes a simple mean and variance would suffice (in fact most people are happy with a point estimate with no error information!). There is a version of Bayesian statistics that does not worry about the full probability distribution. It is known as Bayes Linear statistics and reformulates Bayes theorem in terms of the first two moments (means and variances/covariances).

Instead of having to solve Bayes theorem we now solve for the Bayes linear update:

$$E(\theta|\boldsymbol{w}) = E(\theta) + Cov(\theta, \boldsymbol{w}(\theta))V(\boldsymbol{w}(\theta))^{-1}(\boldsymbol{w} - E(\boldsymbol{w}(\theta)))$$

$$V(\theta|\boldsymbol{w}) = V(\theta) - Cov(\theta, \boldsymbol{w}(\theta))V(\boldsymbol{w}(\theta))^{-1}Cov(\boldsymbol{w}(\theta), \theta)$$

where θ is the vector of geophysical parameters: normally h, H_S and σ^0 but in the coastal zone we may fit more complex models. $E(w(\theta))$ is the theoretical form of the waveform corresponding to the parameters θ and w is the measured waveform. $E()$ is the expectation and $V()$ and $Cov()$ are the variance and covariance matrices respectively.

The advantages of the Bayes linear retracker compared to MLE or least squares are:-

Fast

There is no non-linear optimisation involved

General

The method works with any parametric form for a waveform including those used in the coastal areas, where specular reflections from `bright' targets may be included.

Gives both estimates and uncertainty on the geophysical parameters

Easy to construct statistical tests to distinguish between different waveform models

This is particularly important in coastal analysis where we will want to test for the presence of `bright' targets.

Sequential

Bayesian form means that it is naturally sequential

An Example

To illustrate the method we use a simplified version of the Brown model given by:

$$E(w(\tau)) = \frac{\sigma^0}{2}\left(1 + erf\left(\frac{\tau - h}{H_s}\right)\right) + t_n$$

where τ is time, h the height of the sea surface above a nominal level, H_s is the significant wave height, σ^0 is the radar backscatter and t_n is the thermal noise on the returned waveform. A example waveform (including simulated noise) is shown here.

Parameter	h	H_S	σ^0	t_n
Truth	1.10	4.10	10.50	1.00 x 10^{-6}
MLE	1.03	4.09	10.59	1.01 x 10^{-2}
Bayes Linear	1.03	4.09	10.56	1.01 x 10^{-2}

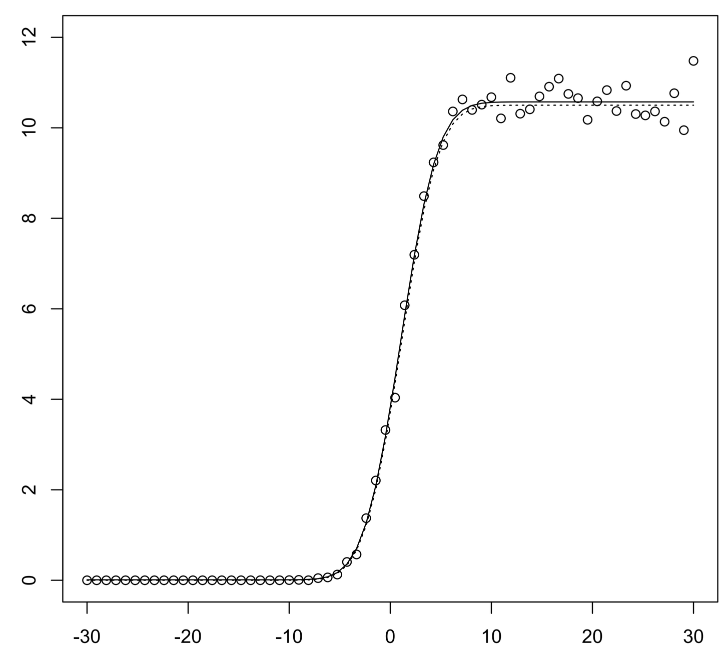

Figure 1 An Example Waveform. The simulated waveform is shown by o's; the true waveform by the solid line and the BL fit by the dashed line

References

M. Goldstein and D.Wooff. Bayes Linear Statistics: Theory and Methods. Wiley, Chichester, 2007.

J Severini, C Mailhes, P Thibaut, and J-Y Tourneret.Bayesian estimation of altimeter echo parameters. Geoscience and Remote Sensing Symposium, 2008. IGARSS 2008. IEEE International, 3:III – 238 – III –241, 2008.

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