An analytical model for Doppler altimetry and its estimation algorithm

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Introduction

Proposed analytical model

Retracking

Validation (synthetic data + Cryosat waveforms)

Conclusions and perspectives

Introduction

Contributions

- Elaboration of an analytical model for Doppler altimetry
- Link between conventional and Doppler altimetry

Advantages of Doppler altimetry ?

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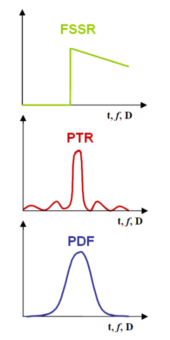
Double convolution model

Conventional altimetry

$$\boldsymbol{P}(t) = \mathrm{FSSR}(t) \otimes \mathrm{PTR}(t) \otimes \mathrm{PDF}(t)$$



Flat Sea Surface Response (FSSR)
Response of the radar to a pulse reflected on a flat surface (sea without waves).

Radar Point Target Response (PTR)

Probability Density Function (PDF)
Probability density of height specular points (assumed Gaussian for the Brown model[1])

1. G. Brown, "The average impulse response of a rough surface and its applications", *IEEE TAP*, Jan. 1977

Double convolution model

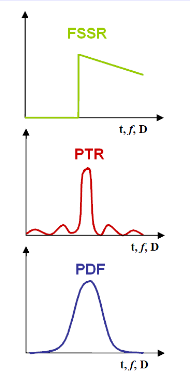
Conventional altimetry

$$\boldsymbol{P}(t) = \mathrm{FSSR}(t) \otimes \mathrm{PTR}(t) \otimes \mathrm{PDF}(t)$$

Doppler altimetry [2 3]

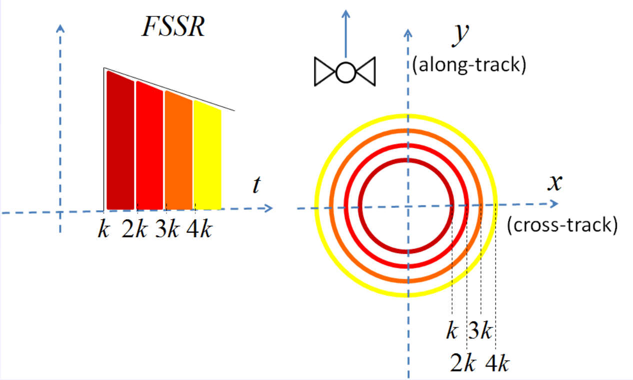
$$\boldsymbol{P}(t,n) = \mathrm{FSSR}(t,n) \otimes \mathrm{PTR}(t,n) \otimes \mathrm{PDF}(t)$$

- n : Doppler band or frequency

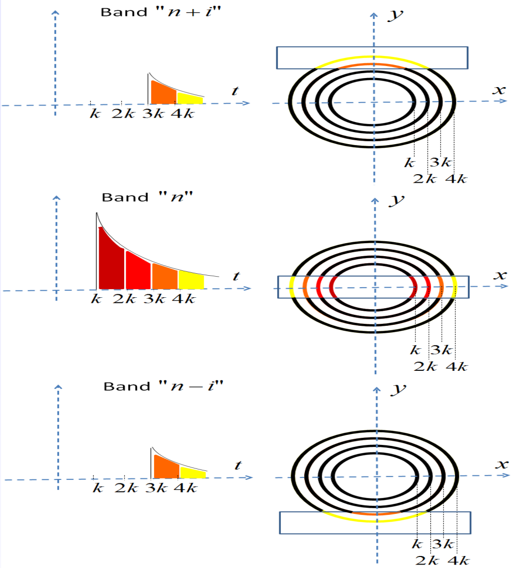


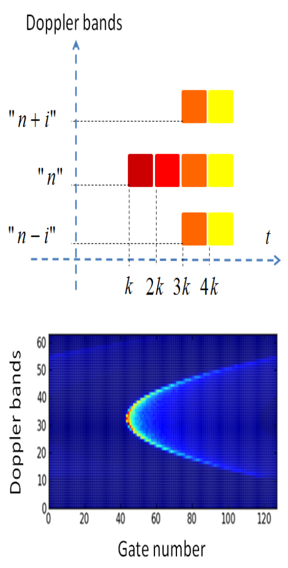
2. Y. Shuang-Bao and al., "The mean echo model and data process of SAR altimeter", *IEEE IGARSS*, Vancouver, Canada, 2011
3. L. Phalippou and al., "Re-tracking of SAR altimeter ocean power-waveforms and related accuracies of the retrieved sea surface height, significant wave height and wind speed ", *IEEE IGARSS*, Barcelona, Spain, 2007

FSSR for conventional altimetry



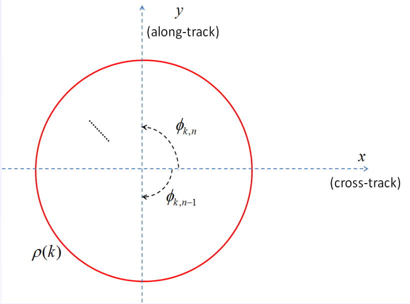
FSSR for Doppler altimetry

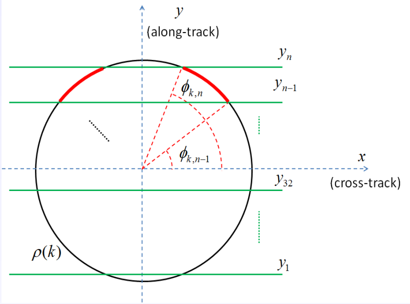




Analytical model for FSSR (3 parameters)

$$\mathrm{FSSR}(k,n) = P_u \left(\frac{2h}{t_k c}\right)^3 \exp\left\{-\frac{4}{\gamma}\left[1-\left(\frac{2h}{t_k c}\right)^2\right]\right\}\left[\frac{\phi(k,n)-\phi(k,n-1)}{\pi}\right]$$





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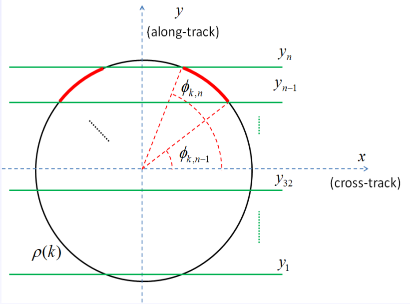
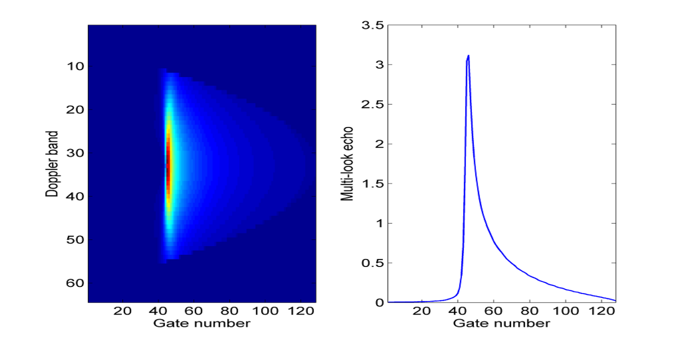
- $k = 1, \cdots, 104$ (gates)
- $n = 1, \cdots, 64$ (frequencies)
- $t_k = \frac{2h}{c} + kT - \tau$ for $k > \frac{\tau}{T}$
- $\phi(k,n) = \arctan\left(\frac{y_n}{\sqrt{\rho^2(k)-y_n^2}}\right)$
- $y_n = \frac{h\lambda}{2v_s} f_n$
- $\rho(k) = \sqrt{\left(\frac{t_k c}{2}\right)^2 - h^2}$



Multi-look echo [4]

$$\boldsymbol{P}(t,n) = \mathrm{FSSR}(t,n) \otimes \mathrm{PTR}(t,n) \otimes \mathrm{PDF}(t)$$

$$s(t) = \sum_{n=1}^{64} P\left[t - \delta t(n), n\right]$$



4. K. Raney, "The delay/Doppler radar altimeter", *IEEE TGRS*, Sep. 1998

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Retracking

Optimization problem (least squares)

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \boldsymbol{F}(\boldsymbol{\theta}) = \underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \frac{1}{2}\sum_{i=1}^{N}\left[y_i - s_i(\boldsymbol{\theta})\right]^2$$

where

- $\boldsymbol{y} = (y_1, \ldots, y_N)^T$ is the observed multi-look echo
- $\boldsymbol{s} = (s_1, \ldots, s_N)^T$ is the proposed analytical SAR model
- $\boldsymbol{\theta} = (P_u, \mathrm{SWH}, \tau)^T$ is the unknown parameter vector
- $N = 104$ is the number of samples

Possible algorithms

- Newton-Raphson algorithm (MLE) : used by CNES
- Levenberg-Marquardt algorithm : used by ESA (in SAMOSA)

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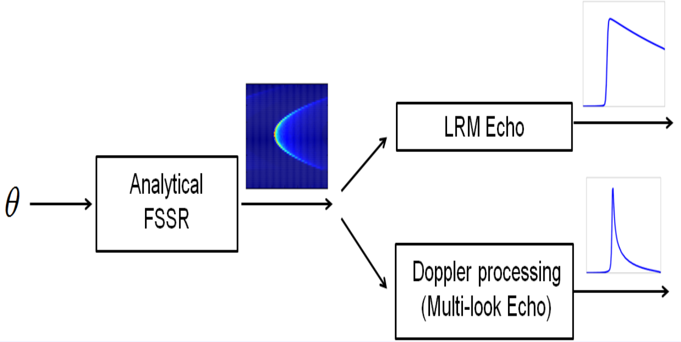
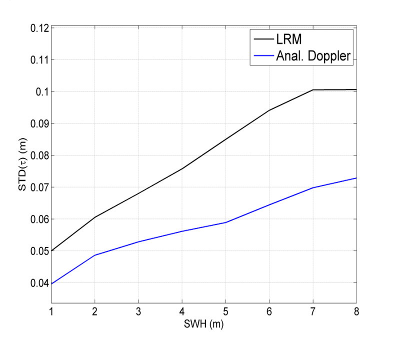
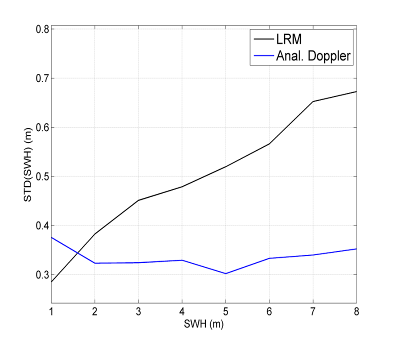
Validation on synthetic data (1)
Conventional and Doppler altimetry



Validation on synthetic data (2)
Conventional and Doppler altimetry

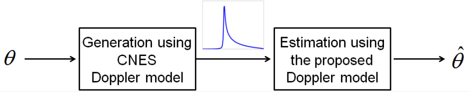


Standard Deviation (Epoch)



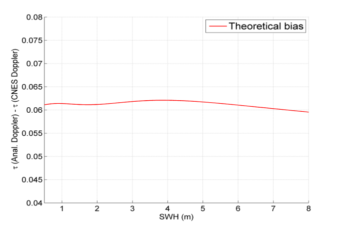
Standard Deviation (SWH)

Validation on synthetic data (3)
Comparison between CNES and analytical models

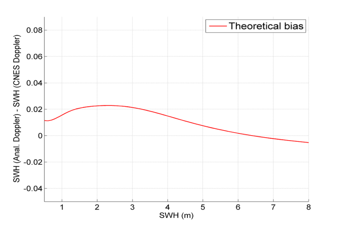


Red curves : $\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$

Epoch bias

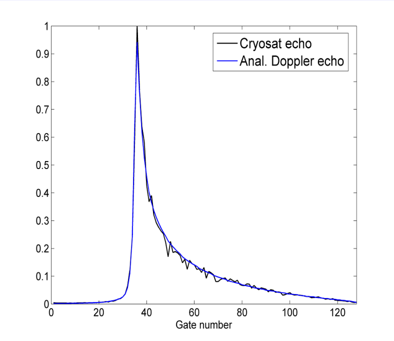


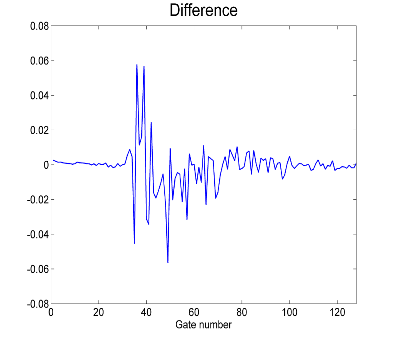
SWH bias



- Constant bias for τ (≈ 6 cm)
- Small bias for SWH

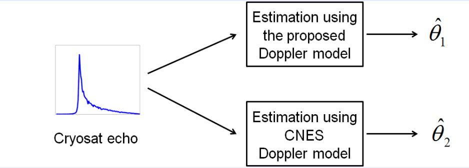
Validation on CRYOSAT Echoes (1)
Example of estimated echo





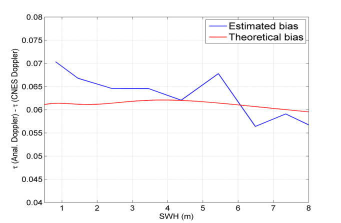
Validation on CRYOSAT Echoes (2)
Comparison between CNES and analytical models (Bias)

- August 2011 data
- Processed by CPP-CNES

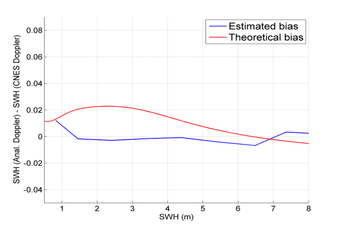


blue curves : $\hat{\boldsymbol{\theta}}_1 - \hat{\boldsymbol{\theta}}_2$

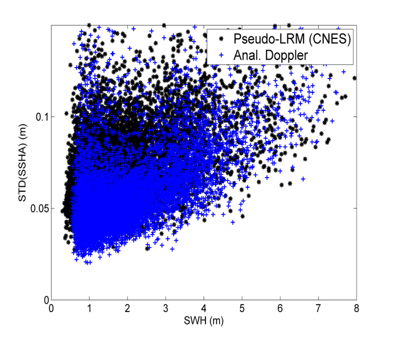
Epoch bias



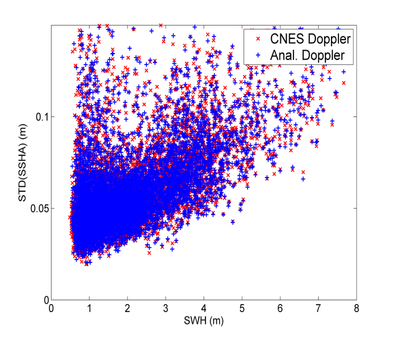
SWH bias



Validation on CRYOSAT Echoes (3)
Standard deviations for the estimated SSHA parameter.



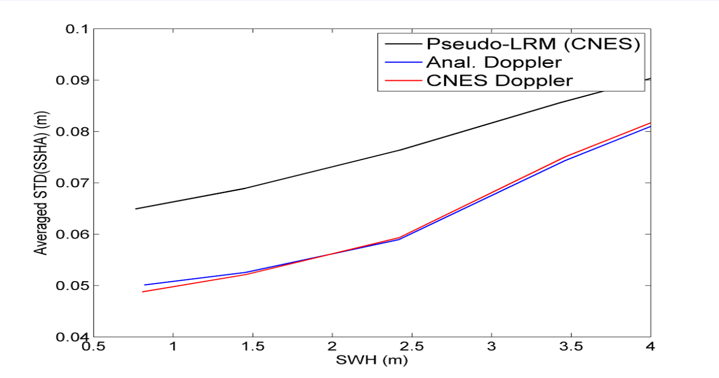
Pseudo-LRM vs Anal. Doppler



CNES Doppler vs Anal. Doppler

- The Doppler cloud is below the LRM cloud (gain ≈ 1.27)
- Similar results for analytical and CNES models
- $> 94\%$ of the data have SWH < 4 m

Validation on CRYOSAT Echoes (4)
Averaged standard deviations for the estimated SSHA parameter.



Pseudo-LRM (CNES), Anal. Doppler and CNES Doppler

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Conclusions and perspectives

Conclusions

- An analytical model based on a geometrical approach
- Good agreement between the analytical and CNES Doppler models
- Fast waveform generation and parameter estimation

Perspectives

- Use the proposed model and estimation algorithm to analyze the performance of Doppler altimetry (versus the number of Doppler bands, ...)
- Validate the proposed model by using additional real datasets

End

Thank you for your attention

REFERENCES

[1] G. Brown, "The average impulse response of a rough surface and its applications", *IEEE Trans. Antennas and Propagation,* vol. 25, no. 1, pp. 67- 74, Jan 1977

[2] Yang Shuang-Bao, Liu He-Guang, Xu Ke, and Xu Xi-Yu, "The mean echo model and data process of SAR altimeter", in *IGARSS-11*, Vancouver, Canada, August 2011.

[3] L. Phalippou, and V. Enjolras, "Re-tracking of SAR altimeter ocean power-waveforms and related accuracies of the retrieved sea surface height, significant wave height and wind speed ", in *IGARSS-07*, Barcelona, Spain, July 2007.

[4] K. Raney, "The delay/Doppler radar altimeter", *IEEE Trans. Geosci. and Remote Sensing*, vol. 36, no. 5, pp. 1578-1588, Sep. 1998.

[5] A. Garcia and al., *"Study of the origins of the σ^0 Blooms"*, M.S. in Electrical and Computer Engineering, Virginia Polytechnic Institute, June 1999.

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- $\phi(k,n) = \arctan\left(\frac{y_n}{\sqrt{\rho^2(k) - y_n^2}}\right)$
- $y_n = \frac{h\lambda}{2v_s} f_n$
- $\rho(k) = \sqrt{\left(\frac{t_k c}{2}\right)^2 - h^2}$
- τ : is the epoch
- P_u : is the amplitude of the waveform
- h : is the altitude of the satellite
- c : is the speed of light
- γ : is the antenna beamwidth parameter
- v_s : is the satellite velocity
- f_n : is the Doppler frequency
- λ : is the wavelength

Levenberg-Marquardt algorithm

Optimization problem

$$\underset{\boldsymbol{\theta}}{\operatorname{argmin}}\, \boldsymbol{F}(\boldsymbol{\theta}) = \frac{1}{2}\sum_{i=1}^{N} {f_i}^2(\boldsymbol{\theta}), \text{ with } f_i(\boldsymbol{\theta}) = [y_i - s_i(\boldsymbol{\theta})].$$

Levenberg-Marquardt algorithm

- Descent algorithm in a direction $\boldsymbol{h}$, i.e., $\boldsymbol{\theta}^{(k+1)} = \boldsymbol{\theta}^{(k)} + \boldsymbol{h}$;
- Elimination of the nonlinearity by the use of a first order Taylor expansion of $\mathbf{f}$ leading to

$$\boldsymbol{F}(\boldsymbol{\theta} + \boldsymbol{h}) \simeq \boldsymbol{L}(\boldsymbol{h}) = \boldsymbol{F}(\boldsymbol{\theta}) + \boldsymbol{h}^T \boldsymbol{J}(\boldsymbol{\theta})^T \mathbf{f} + \frac{1}{2}\boldsymbol{h}^T \boldsymbol{J}(\boldsymbol{\theta})^T \boldsymbol{J}(\boldsymbol{\theta})\boldsymbol{h}$$

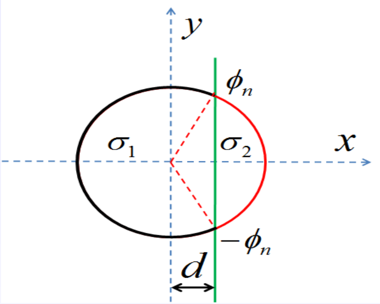
  where $\boldsymbol{J}$ is the gradient of $\mathbf{f}$ according to $\boldsymbol{\theta}$.
- $\boldsymbol{h}$ is obtained by minimizing $\boldsymbol{L}(\boldsymbol{h})$

$$(\boldsymbol{J}(\boldsymbol{\theta})^T \boldsymbol{J}(\boldsymbol{\theta}) + \mu \boldsymbol{I})\boldsymbol{h} = -\boldsymbol{J}(\boldsymbol{\theta})^T \mathbf{f}$$

  where μ is a regularization factor.

Analytical model for σ_0 blooms [5]

$$\mathrm{FSSR}(t) = A(\sigma_1)\exp\left[f(t,\xi)\right] I_0\left[g(t,\xi)\right]$$
$$+B(\xi,\sigma_1,\sigma_2)\left\{ I_0\left[g(t,\xi)\right] \arccos\left(\frac{d}{\sqrt{cht}}\right) + \sum_{k=1}^{\infty}\frac{1}{k}\cdots\right\}$$



5. A. Garcia, G. Brown and al., *Study of the origins of the σ^0 Blooms*, Master of science, Virginia Polytechnic Institute, June 1999.