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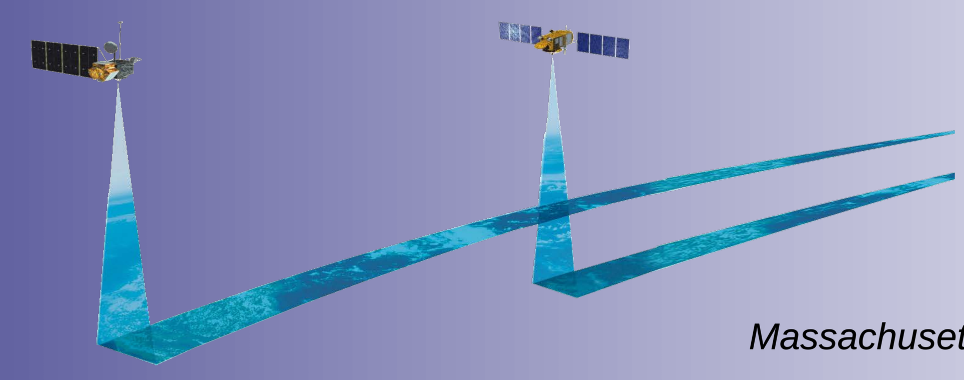
Asymmetries between Along- and Across-Track Velocity Wavenumber Spectra from Tandem-Mission Altimetry



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Abstract

Satellite altimetry has proven to be one of the most useful oceanographic datasets, providing a continuous, near-global record of surface geostrophic currents, among other uses. One limitation of observations from a single satellite is the difficulty of estimating the full velocity field. The three-year Jason-1-TOPEX/POSEIDON tandem-mission, with two satellites flying parallel tracks, promised to overcome this limitation. Velocities estimated from the tandem-mission, however, suffer from three important limitations. **First**, as anticipated, the distance between the tracks limits the resolution and reduces the observed velocity variance. **Second**, there is a fundamental asymmetry between along- and across-track velocity spectra estimated from a tandem-mission, even given the same measurement resolution in the two directions (i.e. along-track sample spacing equal to track separation). The finite sample spacing acts as a low-pass filter in wavenumber for the across-track velocity. The same sample spacing, however, attenuates the along-track velocity at all wavelengths. **Finally**, sampling pattern steepens spectral slopes by a factor of k^2 at wavelengths smaller than the track separation for both velocity components. The effects are a direct consequence of the filtering implied by the sampling pattern (Wortham, Callies, Scharffenberg, submitted manuscript).

Observed Wavenumber Spectra

The JTP tandem-mission spectra are significantly different from the ADCP and mono-mission spectra.

In this poster, we compare along- and across-track velocity spectra in the western sub-tropical Atlantic from three sources: **mono-mission** and

tandem-mission altimetry (Stammer and Dieterich, 1999; Leeuwenburgh and Stammer, 2002; Scharffenberg and Stammer, 2011) as well as

shipboard acoustic doppler current profiler (ADCP) velocity from the Oleander Project (Flagg et al., 1998).

$$u_\perp(x,0) = \frac{\phi(x+\Delta,0)-\phi(x,0)}{\Delta} \quad (1)$$

$$u_\parallel(x,0) = \frac{\phi(x,\Delta)-\phi(x,0)}{\Delta} \quad (2)$$

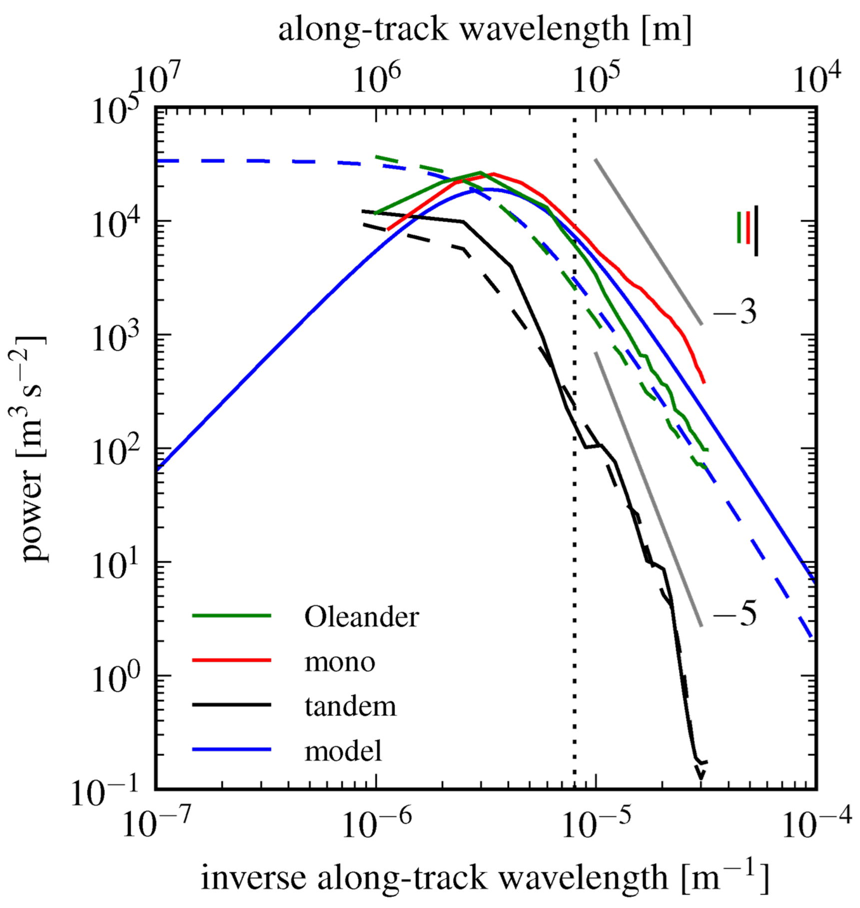
with $\phi(x,y) = g\eta(x,y)/f$

1) Spectral slopes are closer to k^{-5} than the k^{-3} found from the other sources.

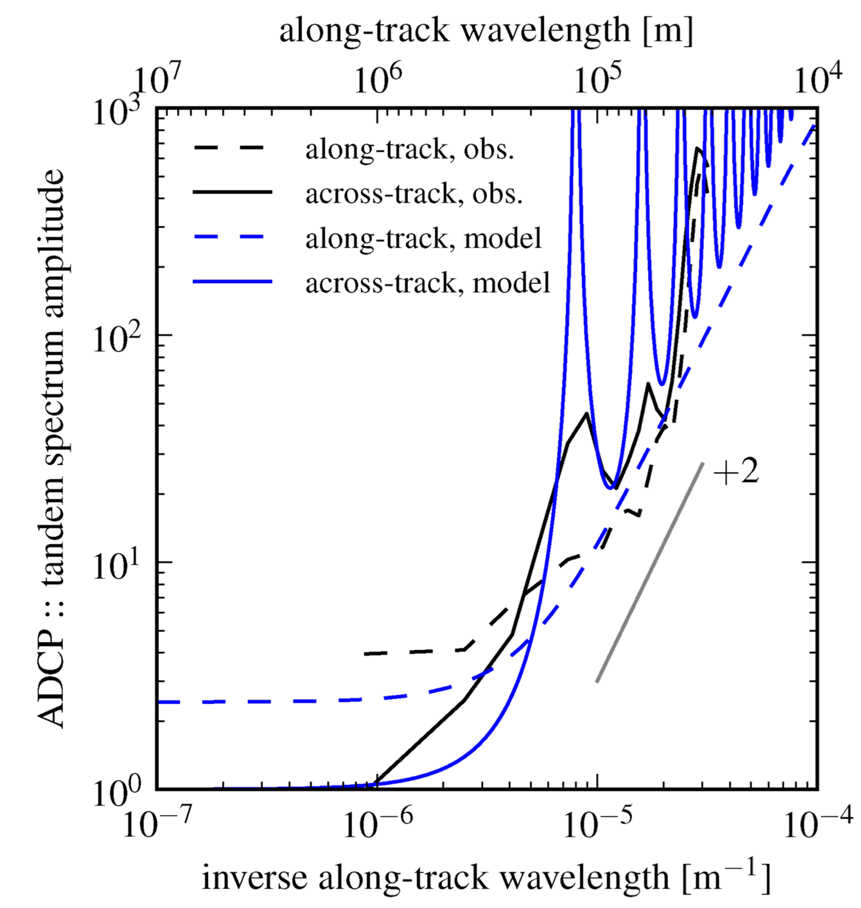
2) JTP tandem-mission spectra have dramatically smaller amplitude than spectra from the other sources.

The ratio of spectral amplitudes between ADCP and JTP tandem-mission depends on wavelength. For the across-track component, the JTP tandem-mission spectrum approaches the ADCP spectrum at the longest resolved wavelength, but the amplitude is reduced at shorter wavelengths. For the along-track component, however, the JTP tandem-mission spectrum is weak by a factor of 4 even at 1000 km wavelength, much longer than the filter scale. The large discrepancy even at these wavelengths, illustrates the counter-intuitive impact of the finite tandem-mission resolution.

The different high-wavenumber slopes for ADCP and JTP tandem-mission spectra are apparent, with a ratio generally following a k^2 power law.



(Figure 1) LEFT Along-track wavenumber spectra of across-track (solid lines) and along-track (dashed lines) velocity from Oleander ADCP measurements (green), mono-mission altimetry (red), and JTP tandem-mission altimetry (black). The analytical spectrum-model (blue). The vertical dotted line shows the JTP tandem-mission track separation. Power laws with slope k^{-3} and k^{-5} are shown for reference. (grey lines). Vertical bars show 95% confidence intervals.



(Figure 2) RIGHT Ratio of ADCP to JTP tandem-mission spectral amplitude (black lines). Across-track (solid lines) and along-track (dashed lines) components are shown. Blue lines are for the spectrummodel. A k^2 power law is shown for reference (grey line).

Predicted Tandem-Mission Wavenumber Spectra

Geostrophic velocity is estimated from $\phi(x,y) = g\eta(x,y)/f$

at two points separated by Δ

$$u_{\mathbf{n}}(\mathbf{x}) = \frac{\phi(\mathbf{x}+\Delta\mathbf{n})-\phi(\mathbf{x})}{\Delta} \quad (4)$$

The two dimensional Fourier Transform is

$$\hat{u}_{\mathbf{n}}(k,l) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(\mathbf{x}) e^{i\mathbf{k}\cdot\mathbf{x}}\,\mathrm{d}x\,\mathrm{d}y \quad (5)$$

$$= \frac{1}{\Delta}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} [\phi(\mathbf{x}+\Delta\mathbf{n})-\phi(\mathbf{x})]\, e^{i\mathbf{k}\cdot\mathbf{x}}\,\mathrm{d}x\,\mathrm{d}y \quad (6)$$

$$= \frac{1}{\Delta}\left[e^{-i\Delta\mathbf{n}\cdot\mathbf{k}} - 1\right]\hat{\phi}(\mathbf{k}) \quad (7)$$

With $\mathbf{k} = (k,l)$ the along- and across-track WN the spectrum gets

$$\langle|\hat{u}_{\mathbf{n}}(k,l)|^2\rangle = w(\Delta\mathbf{n}\cdot\mathbf{k}/2)(\mathbf{n}\cdot\mathbf{k})^2\langle|\hat{\phi}(k,l)|^2\rangle \quad (8)$$

The window function introduced by the finite sampling is

$$w(s) = \frac{\sin^2(s)}{s^2} = \mathrm{sinc}^2(s) \quad (9)$$

As the satellites sample only in along-track direction the observable spectra are only in along-track dirrection and therefore the

Along-track spectrum of across-track and of along-track velocity

$$\langle|\hat{u}_\perp(k)|^2\rangle = w(\Delta k/2)k^2\int_{-\infty}^{\infty}\langle|\hat{\phi}(k,l)|^2\rangle\,\mathrm{d}l \quad (11)$$

$$\langle|\hat{u}_\parallel(k)|^2\rangle = \int_{-\infty}^{\infty} w(\Delta l/2)l^2\langle|\hat{\phi}(k,l)|^2\rangle\,\mathrm{d}l \quad (12)$$

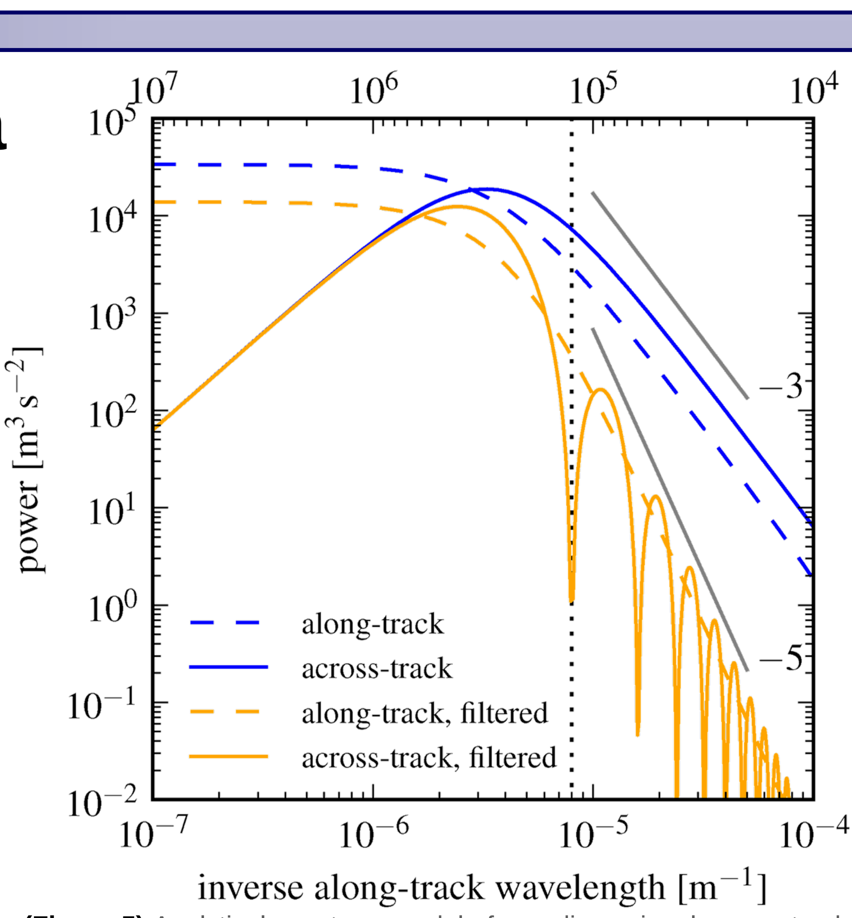
Comparing (11) and (12) clearly shows the distinct behavior of across- and along-track velocity spectra. The across-track velocity spectrum is simply the low-pass filtered full spectrum and approaches the full spectrum for $k \ll \Delta^{-1}$. For the along-track velocity spectrum, on the other hand, the window function affects the estimated spectrum at all wavelengths. The magnitude of the impact of filtering depends on the shape of the SSH spectrum. If the two-dimensional velocity spectrum has significant energy at $l > \Delta$, the along-track velocity spectrum will be reduced, even for $k \ll \Delta^{-1}$. In both cases, the full geostrophic velocity spectrum is recovered as $\Delta \to 0$.

We now consider the simple isotropic wavenumber spectrum for the Oleander Region (32° to 40°N, 288° to 295°E)

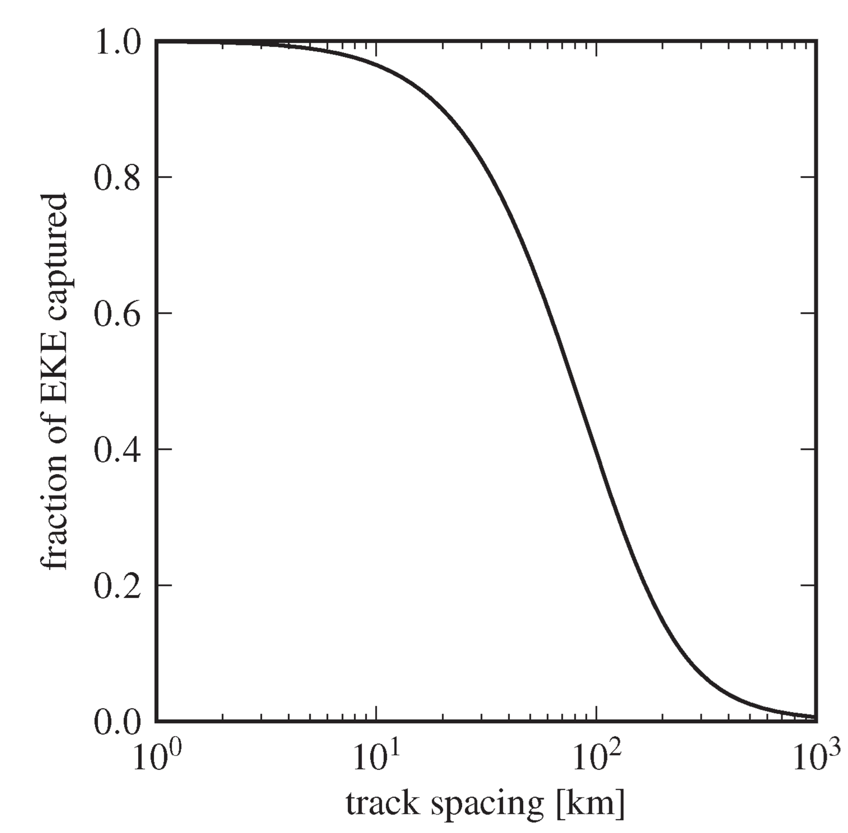
$$\langle|\hat{\phi}(k,l)|^2\rangle = I\left(k^2+l^2+k_0{}^2\right)^{-3} \quad (13)$$

where $I = 8.6\times10^{-9}\ s^{-2}$ sets the amplitude of the spectrum and $k_0 = 2\pi/250\ km$ sets the dominant wavelength. This isotropic spectrum is flat at wavenumbers $k^2 + l^2 \ll k_0^2$ and has a power-law roll-off at wavenumbers $k^2 + l^2 \gg k_0^2$. One-dimensional spectra deduced from this have a k^{-5} roll-off in the SSH spectrum and a k^{-3} roll-off in the velocity spectra. The across-track velocity spectrum peaks near k_0. The parameters I and k_0 and the roll-off slope are chosen to match the Oleander ADCP velocity spectrum and would change in different locations. Equation (13) is a simplified version of the spectrum proposed by Wortham (2013). The spectrum-model is shown in **Figure 3**.

Figure 4 shows the two-dimensional wavenumber spectrum of along- and across-track velocity from the spectrum model. The shading of regions where $\mathbf{n}\cdot\mathbf{k} > \Delta^{-1}$ in Fig. 4 illustrates the impact of the window function in (11) and (12): unshaded areas are passed by the window function, while shaded areas are attenuated.



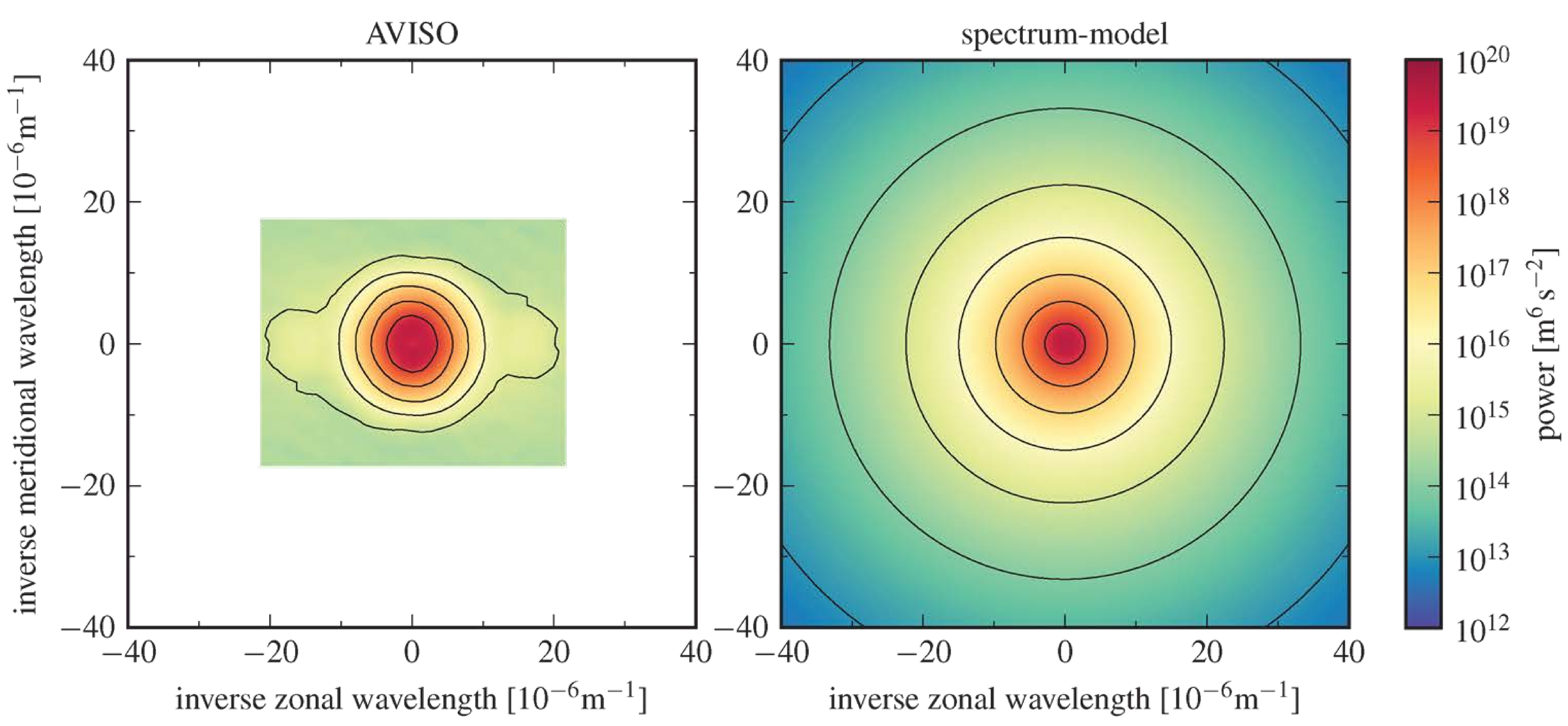
(Figure 5) Analytical spectrum-model of one-dimensional across-track (solid lines) and along-track (dashed lines) velocity. Full (Δ=0, blue) and filtered (Δ=125km, orange) spectra are shown. The dotted line shows the approximate tandem-mission sample spacing of 125 km.



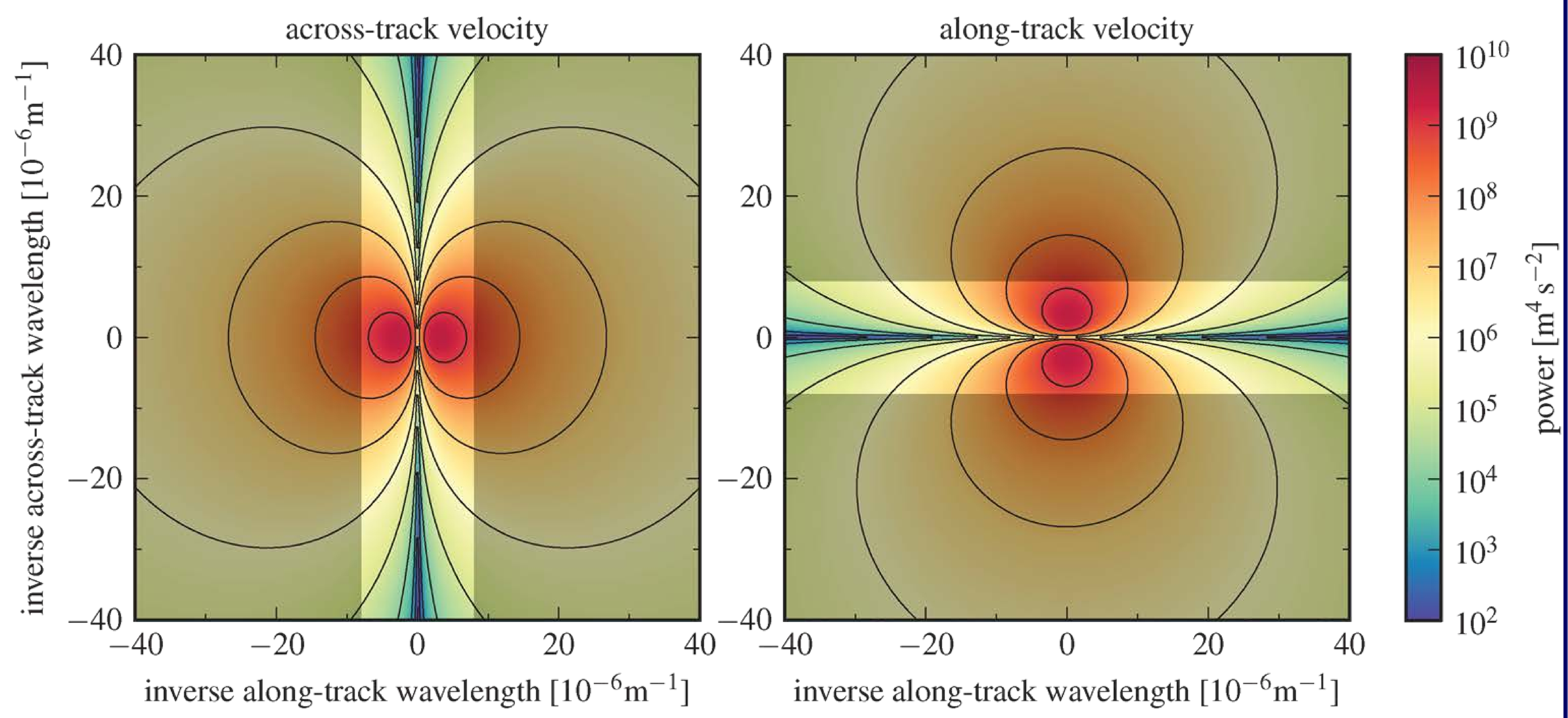
(Figure 6) The fraction of EKE captured by tandem-mission sampling as a function of Δ. This curve is specific to the region 32° to 40°N, 288° to 295°E and depends on the dominant wavelength through k_0 in (13).

Integrating each two-dimensional spectrum over l results in the along-track wavenumber spectrum of the corresponding velocity component. Figure 5 illustrates the impact of the window function on one-dimensional along- and across-track velocity spectra. For the across-track velocity, the filtered spectrum (Δ=125km) approaches the full spectrum (Δ=0) for $k \ll \Delta^{-1}$. However, for the along-track velocity, the filtered spectrum is attenuated at all wavelengths. The cusps in the filtered across-track velocity spectrum are a result of the sinc window function.

For spectra with an isotropic roll-off, the filter described above steepens spectral slopes at wavelengths much smaller than the sample spacing Δ by a factor k^{-2}, with cusps added to the spectrum of the across-track velocity (Figure 5). The filtered across-track spectrum is the full spectrum multiplied by the filter, which explains the steepening, because $sinc2(\Delta k/2)$ has an envelope proportional to k^{-2}. The filter also steepens the spectrum of the longitudinal velocity, essentially because the envelope of the filter is proportional to l^{-2}. The range in which this steepening occurs is observed by tandem-mission altimetry, because the along-track sample spacing is much smaller than the track spacing.



(Figure 3) Two-dimensional wavenumber spectra of the normalized SSH f from the AVISO gridded product (left) and the analytical spectrum-model (right) in the Oleander region.



(Figure 4) Two-dimensional wavenumber spectra of across-track velocity (left), and along-track velocity (right) from the spectrum-model. Shaded areas illustrate the impact of the window function in (11) and (12), unshaded regions are passed by the window function.

Conclusion

The most important outcome of this is an understanding of the relationship between kinetic energy sampled by tandem-mission altimetry and the satellite track spacing. Along- and across-track velocity spectra from widely-separated satellite tracks behave very differently from each other. While the spectrum of across-track velocity from a tandem-mission is a low-pass filtered version of the full spectrum, the observed along-track velocity spectrum is attenuated at all wavelengths. Also, at wavelengths smaller than the track separation, the observed spectral slopes of both velocity components are steeper than the full spectrum by a factor of k^{-2}. Knowledge of the shape of the filter implied by the sampling allows for isotropy tests and may even allow for a reconstruction of the full spectrum. The best method for reducing the impact of the filtering remains to increase the spatial resolution of the observations, in particular in the across-track direction. The proposed Surface Water and Ocean Topography (SWOT) swath altimeter (Alsford et al. 2007) will address this issue and allow accurate estimation of both velocity components and their wavenumber spectra.

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