AN OCEANOGRAPHIC ESTIMATE OF THE GEOID HEIGHT IN THE NORTH ATLANTIC

P. Le Grand (IFREMER, France), H. Mercier (IFREMER, France), and T. Reynaud (LEGI/IMG, France)

The mean dynamic topography of the surface of the North Atlantic is estimated using an inverse model of the ocean circulation constrained by hydrographic and altimetric observations. This estimate is used to compute a geoid height correction over the North Atlantic that is consistent with both the JGM-2 geoid model, oceanographic observations, and ocean dynamics within realistic uncertainties. The correction reduces the uncertainty in the geoid model expanded to spherical harmonic 40 to a level of about 5 cm.

Introduction

The geoid height can be computed by subtracting an oceanographic estimate of the mean dynamic topography from the altimetric mean sea surface height (SSH) [Roemmich and Wunsch, 1982]. The mean SSH above a reference ellipsoid is known from the TOPEX/POSEIDON (T/P) observations within an uncertainty of a few centimeters, most of this uncertainty being due to the natural variability of the ocean rather than the instrumental error. This SSH is combined with an inverse model estimate of the mean dynamic topography to compute the geoid height in the North Atlantic.

Inverse calculation

The mean dynamic topography of the North Atlantic is estimated using the 2.5° longitude by 2° latitude finite difference inverse model of Mercier et al. [1993]. The model finds an ocean circulation, and the associated dynamic topography, that best fits in a least-square sense observational and dynamical constraints.

The ocean circulation and the associated topography are required to satisfy mass, heat, and salt conservation in addition to the thermal wind and geostrophic balances. The solution is also required to be consistent with hydrographic and altimetric observations. The hydrographic observations consist of an estimate of the mean density field and its variance calculated using a compilation of more than 100 000 hydrographic stations collected over the past 70 years [Reynaud, 1996; LeGrand et al., 1997]. The altimetric observations consist of the mean dynamic topography estimated by subtracting the JGM-2 geoid model [Tapley et al., 1994] from the mean of 2 years of T/P SSH observations (cycles 2 to 75) corrected for environmental effects. The topography is computed along the satellite tracks in a latitude band of the North Atlantic from 20°N to 50°N and then averaged onto a 5° longitude by 4° latitude grid. The uncertainty in this topography estimate is calculated using the spatial autocorrelation function of the uncertainty in the JGM-2 geoid expanded to spherical harmonic 40 [Rapp, Le

Traon, personal communication, 1994] and the uncertainty in the 5°x 4° mean SSH calculated with the assumption that the oceanic mesoscale variability has a decorrelation scale of 100 km. The resulting topography uncertainty is dominated by the geoid height uncertainty and is on the order of 20 cm.

An ocean circulation consistent with the observational constraints and the dynamical constraints is found by the inverse model. The mean dynamic topography estimated by the inverse model is consistent with known features of the circulation in the North Atlantic such as the Gulf Stream and its recirculation, and the subtropical gyre. The uncertainty in the mean dynamic topography estimate decreases on average from about 5 cm for an estimation of the topography onto the inverse model grid $(2.5^{\circ} \text{ longitude by } 2^{\circ} \text{ latitude})$ down to about 0.3 cm for an estimation of the topography onto a 37.5° longitude by 30° latitude box [LeGrand et al., 1997].

Geoid correction

The inverse estimate of the mean dynamic topography in the North Atlantic has a smaller uncertainty than the T/P - JGM-2 estimate and one expects it to yield a significant improvement of the JGM-2 geoid height model. It is subtracted from the T/P mean SSH to estimate the geoid height onto the 5° longitude by 4° latitude grid (Figure 1a). The T/P - Inverse geoid height differs from the JGM-2 geoid height by values on the order of 10 cm and is better visualized as a geoid height correction. It is slightly more consistent with the recent EGM96 [Lemoine et al., 1997] geoid height estimate than with the JGM-2 estimate at a scale of 1000 km [LeGrand et al, 1997], which suggests that the inverse geoid correction goes in the right direction. However, the inverse geoid height estimate is still different from the EGM96 estimate. This difference is plausible if one considers that the EGM96 estimate is noisy at scales of 1000 km.

The crucial point in an ocean modeling estimate of the geoid height is the calculation of its uncertainty. The present uncertainty calculation assumes that the uncertainties in the mean SSH and in the mean dynamic topography are decorrelated. The uncertainty in the mean dynamic topography is calculated by the inverse procedure taking into account modeling errors and observational errors. Estimating modeling errors is difficult, but not critical because the difference between the topography uncertainty in a calculation with modeling errors multiplied by two and the topography uncertainty in a calculation with modeling errors divided by two is of the order of 20% (difference of 1 cm in the Gulf Stream, and less than 1 cm elsewhere). Observational errors are estimated directly from the data, and despite some crude approximations are surely accurate to within an order of magnitude. All the sources of uncertainty in the geoid height estimate have thus been accounted for in this calculation, and the resulting geoid height uncertainty estimate (Figure 1b) is fairly robust.



Figure 1 (a) Correction to the geoid (cm) estimated on the 5°x 4° grid by the inverse model. (b) Uncertainty (cm) in the 5°x 4° corrected geoid height estimate.

The total uncertainty in the corrected geoid height is on the order of 5 cm in the interior of the ocean at the scale of the $5^{\circ}x \ 4^{\circ}$ grid, and is about one third of the corresponding uncertainty in the JGM-2 estimate. The uncertainties are higher near the continents because the SSH is not precisely know in the $5^{\circ}x \ 4^{\circ}$ boxes corresponding to these regions due to the small number of independent observations.

Conclusion

The 5 cm precision is as good as that expected from intermediate gravimetry missions [Minster, personal communication, 1996]. Higher precision gravimetry missions will not fly before several years, and oceanographic geoid estimates could be used in the meantime. The main difficulty to overcome in order to estimate the geoid height over the global ocean using an inverse model of the ocean circulation is to determine the mean density field and its uncertainty. The hydrographic data base is scarce outside of the North Atlantic, and a global geoid estimate is unlikely to be as precise as the North Atlantic estimate.

Acknowledgments

The inverse calculations presented in this paper were carried out on the IFREMER CONVEX computer. We are grateful to R. Rapp and to P.-Y. Le Traon for providing the spatial autocorrelation function of the JGM-2 geoid and the mean dynamic topography calculated using the T/P observations and the JGM-2 geoid. Discussions with G. Larnicol and J.-F.

Minster were very helpful. Tim Boyer made the NODC hydrographic data available to us and we acknowledge his assistance in compiling the data base used in this study. C. Maillard and M. Fichaut also helped with the preparation of the hydrographic data base. This work was supported in part by the French Programme National d'Etude de la Dynamique du Climat.

References :

- LeGrand, P., H. Mercier, and T. Reynaud, 1997: Combining T/P altimetric data with hydrographic data to estimate the mean dynamic topography of the North Atlantic and improve the geoid (in revision).
- Lemoine et al., 1997: EGM96, The NASA GSFC and NIMA Joint Geopotential Model, *CDDIS web site* (http://cddis.gsfc.nasa.gov/926/egm96/egm96.html).
- Mercier, H., M. Ollitrault, and P.-Y. Le Traon, 1993: An inverse model of the North Atlantic general circulation using Lagrangian float data, *J. Phys. Oc.*, 23, 689-715.
- Reynaud, T., 1996: Modélisation inverse de l'Atlantique Partie 1: Traitement des données hydrographiques, *Technical Report of the Laboratoire de Physique des Océans*, Brest.
- Roemmich, D., and C. Wunsch, 1982: On combining satellite altimetry with hydrographic data, *J.*, 40, 605-619.
- Tapley, B.D., J.C. Ries, G.W. Davis, R.J. Eanes, B.E. Schutz, C.K. Shum, M.M. Watkins, J.A. Marshall, R.S. Nerem, B.H. Putney, S.M. Klosko, S.B. Luthcke, D. Pavlis, R.G. Williamson, and N.P. Zelensky, 1994: Precision orbit determination for TOPEX/POSEIDON, *J. Geophys. Res.*, 99, 24383-24404.