# The ocean mesoscale regime of the reduced-gravity quasi-geostrophic model

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# Wavenumber-frequency power spectrum

#### AVISO gridded altimeter data



Non-dispersive propagation at long-wave speed (approximately), apparently indicating (weakly) nonlinear dynamical balance

Can we detect and track coherent features directly?

Does the nondispersive line end because the resolution limit of the data is reached?



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#### Linear theory (dispersive at short wavelengths)



# Cyclonic and Anticyclonic Eddies with Lifetimes $\geq$ 16 weeks (41,047 total)

Number Cyclonic=21126

Number Anticyclonic=19921



Chelton, D. B., M. G. Schlax, and R. M. Samelson, 2011. Progress in Oceanography, 91, 167-216.

Now available through AVISO: https://www.aviso.altimetry.fr/en/data/products/value-added-products.html





# Normalized mean and std dev life cycles from altimeter data and random-walk model







Random walk with linear damping (first-order autoregressive/AR1 process)

$$A(t_{j+1}) = A(t_j) + \delta_j - rA(t_j)$$
  
=  $\alpha A(t_j) + \delta_j,$   
 $0 < \alpha = 1 - r < 1$ 

The AR1 parameter  $\alpha$  determines autocorrelation structure for A, independent of subsequence ("eddy detection and tracking") analysis.







### Random walk with linear damping

(first-order autoregressive/AR1 process; Markov process)

$$\eta_{j+1} = \alpha \eta_j + \delta_j \qquad 0 < \alpha = 1 - r < 1$$
  
$$< \eta_{j+1}^2 > = \alpha^2 < \eta_j^2 > + \alpha < \eta_j \delta_j > + < \delta_j^2 >$$
  
$$= \alpha^2 < \eta_j^2 > + \sigma_\delta^2$$

 $\{\text{variance from forcing}\} dt' = \int^{t=N\times\Delta t} <$ 

$$dt' = \int <\delta_j^2 > dt'$$
  
=  $N \times \sigma_\delta^2$   
=  $\frac{\sigma_\delta^2}{\Delta t} t$   
=  $\sigma_W^2 t$ ,  $\sigma_W = \frac{\sigma_\delta}{(\Delta t)^{1/2}}$ 

-0

$$\sigma_W \approx 2.5 \times 10^{-3} \mathrm{cm \ s}^{-1/2}$$

Rate of forcing independent of discrete time-step





#### Generalization: a stochastic field model<sup>2</sup>

$$\frac{\partial \eta}{\partial t} + c_R \frac{\partial \eta}{\partial x} = -R\eta + F(x, y, t)$$

Here F(x,y,t) is a stochastic forcing function with:

(1) wavenumber power spectrum chosen to match observed SSH spectrum

(2) random phase for each spectral component at each (weekly) time step

Solve along characteristics:

$$\frac{dX}{dt} = c_R, \quad X(t=0) = x_0,$$

$$\eta(x_0 + c_R t_{p+1}, y_0, t_{p+1}) = \alpha \eta(x_0 + c_R t_p, y_0, t_p) + \delta^F(x_0 + c_R t_p, y_0, t_p),$$

#### Same difference equation as before, but with random increment field.



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<sup>2</sup>(Samelson, R. M., M. G. Schlax, and D. B. Chelton, 2016. A linear stochastic field **niversity** rth.Ocean. Oceanogr., 46, 3103–3120, doi: 10.1175/JPO-D-16-0060.1.)



# Eddy number distribution vs. lifetime



Lifetime (wk)





#### Eddy length scale (radius) and amplitude distributions









Consider simplest nonlinear dynamical theory: reduced-gravity quasi-geostrophic model

$$\frac{\partial}{\partial t}(\nabla^2\psi - \psi) + \beta \frac{\partial\psi}{\partial x} = -J(\psi, \nabla^2\psi) + \frac{\mathcal{F}_{0,\tau}}{\tau^{1/2}} - r_\psi\psi + \mathcal{D}_{ens}$$

 $\mathcal{F}_{0,\tau}$  is a stochastic forcing function with fixed amplitude (unit time-mean spatial standard deviation) and autocorrelation timescale  $\mathcal{T}$ (Morten, Arbic, and Flierl, 2017; Lilly, 1969)

#### Three parameters:

$$\beta = \beta_* L_R^2 / U_F \qquad r_\psi = r_{QG*} L_R / U_F \qquad \tau = \tau_{QG*} U_F / L_R$$

...plus spatial (wavenumber) structure of  $\mathcal{F}_{0,\tau}$ 

Require:

 $\sigma_{\eta} = 0.07 \text{ m at } 35^{\circ}\text{N}$ and for  $L_{R}$  = 40 km





QG simulations: guided initially by Morten, Arbic, Flierl (2017), then by comparison with Chelton et al. (2011) eddy tracking analysis MAF *r* = 0.013 *q*, Run 3  $\tau = 0.034$  $2\pi$  $2\pi$  $\pi$  $k_f = 1/L_R$ *q*, Run 1 *q*, Run 2 *r* = 0.0015 r = 0.0004 $\tau = 0.855$  $\tau = 0.0855$ 2 0  $k_{f} = 6/L_{R}$  $k_f = 2/L_R$ 



RMS  $q(t=1095.4d)/q_0, q_0=2\pi\beta$  $\int_{-200}^{200} \int_{-200}^{0} \int_{-500}^{0} \int_{1000}^{0} \int_{1500}^{0} \int_{-500}^{0} \int_{-500}^{0} \int_{1000}^{0} \int_{1500}^{0} \int_{-500}^{0} \int_{1000}^{0} \int_{1000}^{0} \int_{1500}^{0} \int_{-500}^{0} \int_{1000}^{0} \int_{1000}^{0} \int_{1500}^{0} \int_{-500}^{0} \int_{1000}^{0} \int_{1000$ 



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0

#### QG eddy identification and tracking













50

100

lifetime (wks)

normalized life cycles







#### AVISO zonal-wavenumber - frequency spectra (Chelton)

#### South Pacific 45°S, 150°W-110°W South Indian 40°S, 80°E-110°E South Pacific 38°S, 130°W-100°W South Indian 35°S, 40°E- 85°E 0.03 0.03 1.5 0.03 0.03 $\tau = 0.1, r = 0.02$ 0.02 0.02 0.02 0.02 Э 0.01 0.01 0.01 0.5 0.01 0.00 0.00 0.00 0.00 North Pacific 33°N, 180°E-130°W South Atlantic 33°S, 50°W-0°W North Atlantic 30°N, 70°W- 40°W South Pacific 30°S, 170°E-120°W 1.5 0.03 =1.0, r=0.020.03 0.03 0.03 Frequency (cycles per day) 0.02 0.02 0.02 0.02 З 0.5 0.01 0.01 0.01 0.01 0 0.00 0.00 30°S, 60°E-100°E North Pacific 24°N, 125°E-165°E South Pacific 24°S, 160°E-135°W Atlantic 24°N. 60°W- 30°W South Indian 1.5 North 0.03 0.03 0.03 0 $\tau = 10, r = 0.02$ 0.02 0.02 0 0.02 З 0.5 0.01 0.01 0. 0.01 0 0.00 0.00 0 0.00 South Atlantic 24°S. 40°W-North Pacific 21°N, 130°E-170°W South Indian 14°S, 70°E-120°E .5 0.03 0.03 0.03 0.03 <mark>τ=10</mark>, *r* =0.005 0.02 0.02 0.02 0.02 0.01 0.01 0.01 0.01 0.00 0.00 0.00 0.00 0 -3 3 -1 3 -3 3 -3 -3 3 1 -1 -11 -1 1 Zonal Wavenumber (cycles per 1000 km) 2 3 5 6 fig<sub>57DBC</sub> 20180103 Log<sub>10</sub> [cm<sup>2</sup>(cycles/day)<sup>-1</sup>(cycles/km)<sup>-1</sup>]



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Model spectra

#### AVISO zonal-wavenumber - frequency spectra (Chelton)







#### Frequency vs. zonal-wavenumber spectra: Linear inversion of linear and nonlinear models



Linear



Nonlinear





# Linear-inverted AVISO and model spectra



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#### Linear-inverted AVISO spectra:

Evidence of propagating objective analysis filter visible => loss of useful information?



Model:







#### Linear-inverted model spectra: unsmoothed and smoothed

Model PSD(n,F,F<sub>inv</sub>) :OP08aRK Smoothed model PSD(n,F,F<sub>inv</sub>) :OP08aRK Model PSD(η,F,F<sub>inv</sub>) :OP08aRK Smoothed model PSD(η,F,F<sub>inv</sub>) :OP08aRK 5 5 4 4 Э 3 3 0.5 3 0.5 2 2 0 0 o -10 10 -10 10 0 0 -2 -1 0 1 2 -2 -1 0 1 2 k k k k 5 5 3 0.5 3 0.5 Э 3 0 0 0 0 0 -2 -1 0 2 -2 0 2 1 -1 1 10 -10 0 10 -10 0 k k 5 5 5 Э 0.5 3 0.5 З 0 2 0 0 0 -2 -1 0 -2 -1 0 2 1 1 0 10 -10 0 10 k k -10 k k

τ=10, *r* =0.005

**τ=10**, *r* **=**0.005

Gaussian smoothing, 200 km in *k*, 30 d in  $\omega$ , centered on  $\omega = c_R k$ .

NB:

$$\hat{W}_{Xt}(k,\omega) = \hat{W}_X(k,\omega)\,\hat{W}_t(k,\omega) = \frac{1}{\pi LT} \exp\left\{-\frac{1}{4}L^2(k-c_0^{-1}\omega)^2 - \frac{1}{4}T^2\omega^2\right\}$$

L = 37.4 km and T = 5.6 d





#### Original and linear-inverted AVISO spectra





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### Conclusions

- 1. A connection is made between linear stochastic and nonlinear quasigeostrophic turbulence models of SSH variability.
- 2. Observed autocorrelation and spectral structures are broadly reproduced by the nonlinear simulations when the model is forced by stochastic fluctuations near the deformation radius.
- 3. The flux of energy into the gravest mode of the ocean mesoscale can be represented as a stochastic forcing with  $\sigma_W \approx 2 \times 10^{-5} \mathrm{m \ s}^{-1/2}$ .
- 4. The ocean mesoscale is nonlinear: nonlinearity removes energy along the linear dispersion relation and deposits it elsewhere.
- 5. There appears to be a visible signature of signal propagation characteristics assumed by the objective analysis procedure in the AVISO altimeter SSH dataset.
- 6. Much remains to be learned SWOT will help!



