The ocean mesoscale regime of the reduced-gravity quasi-geostrophic model

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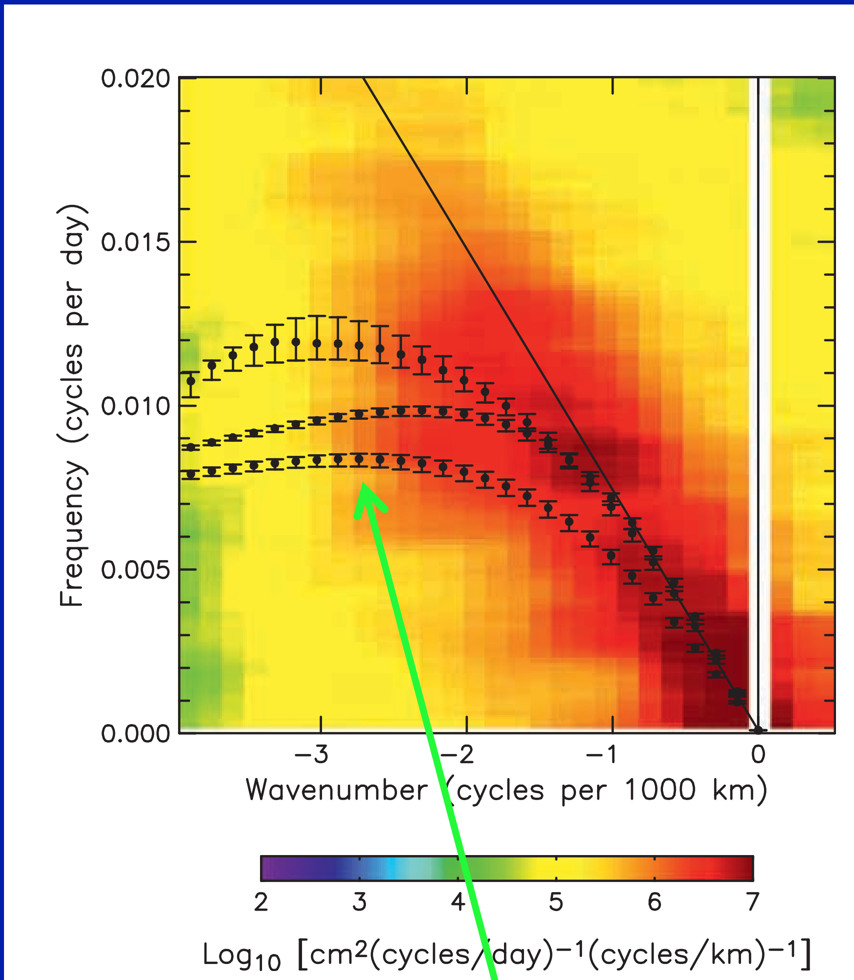
Wavenumber-frequency power spectrum

AVISO gridded altimeter data



Non-dispersive propagation at long-wave speed (approximately), apparently indicating (weakly) nonlinear dynamical balance

Can we detect and track coherent features directly?

Does the nondispersive line end because the resolution limit of the data is reached?

Linear theory (dispersive at short wavelengths)

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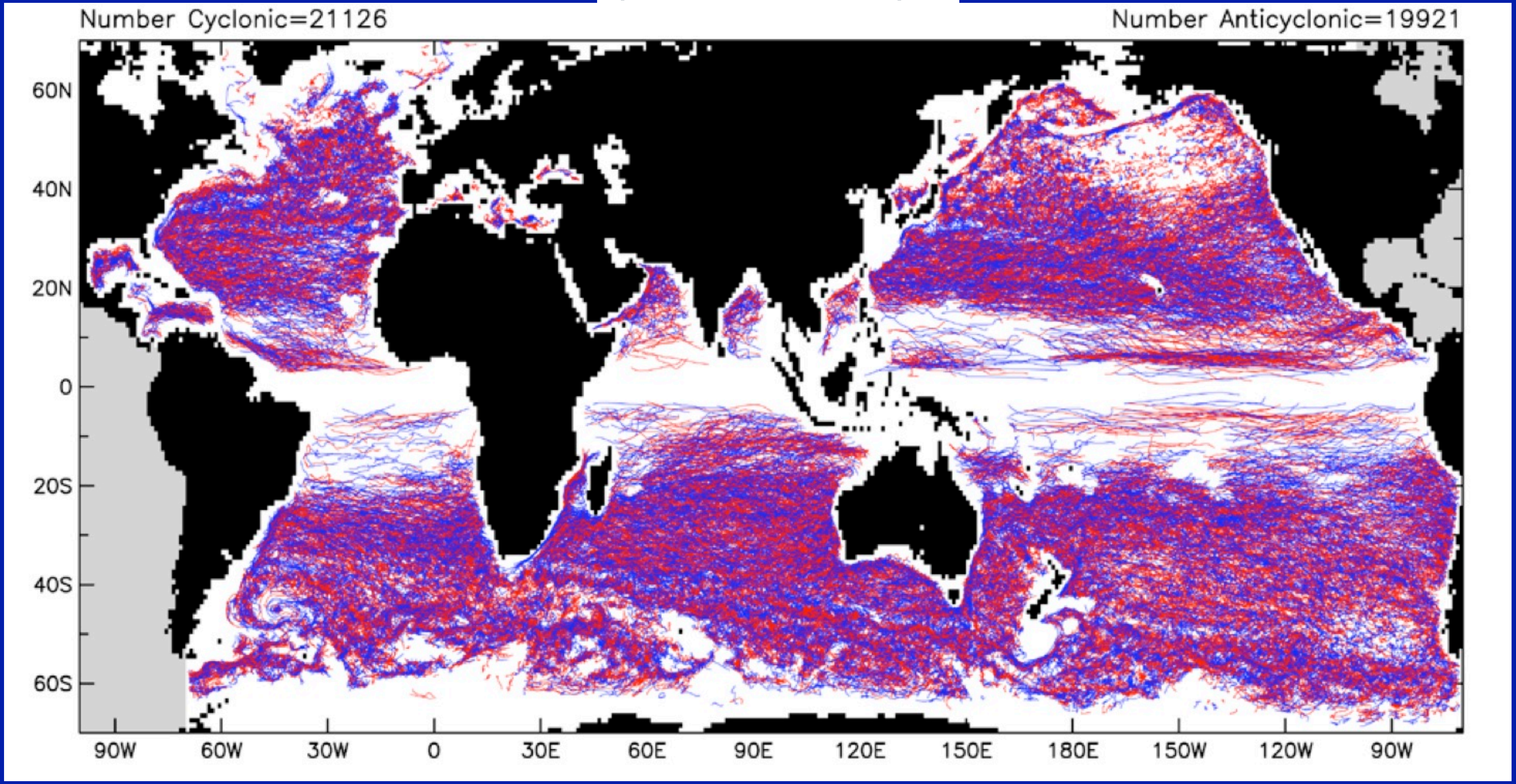
Cyclonic and Anticyclonic Eddies with Lifetimes ≥ 16 weeks

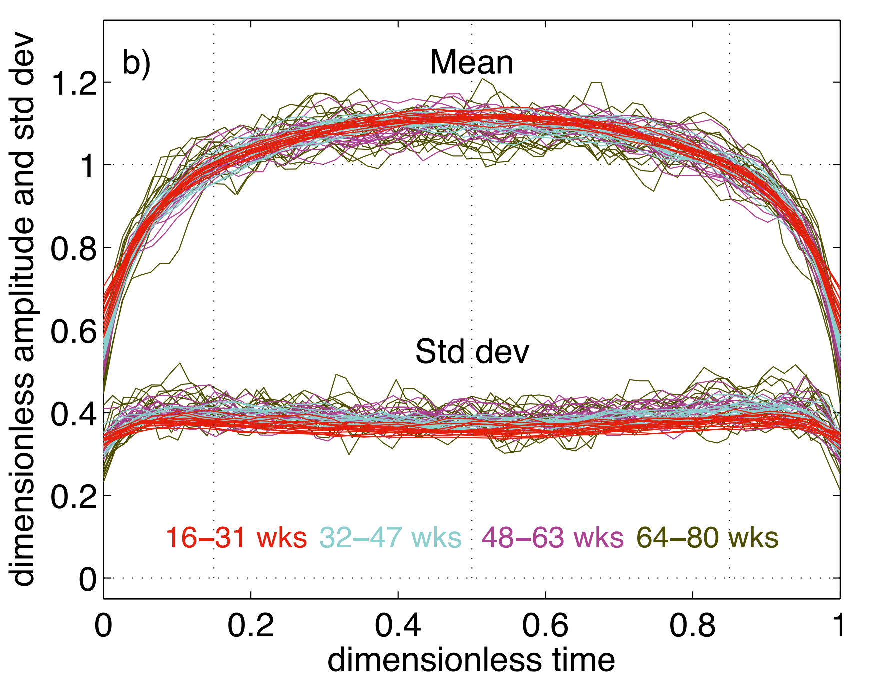
(41,047 total)

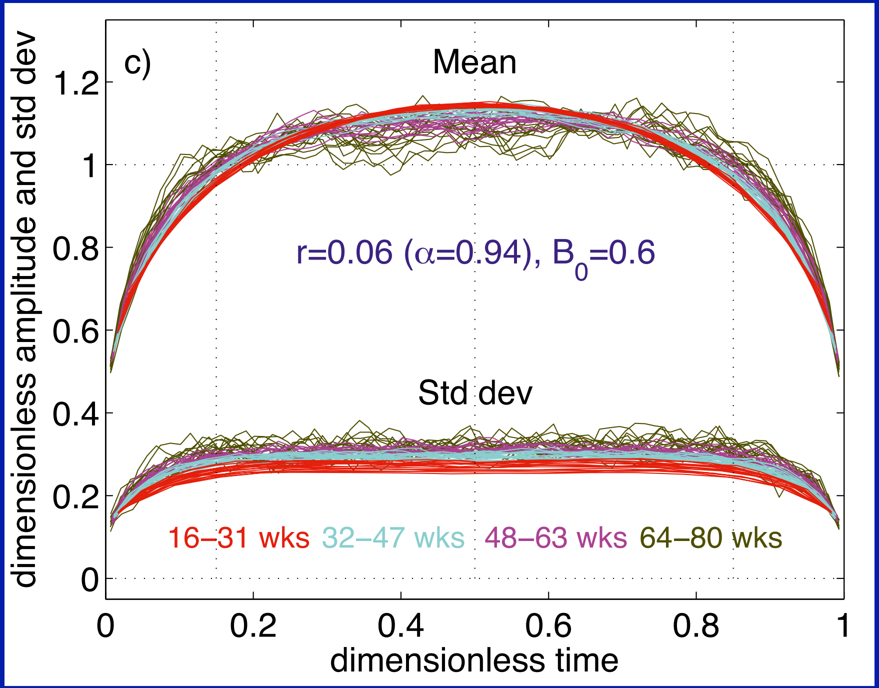


Chelton, D. B., M. G. Schlax, and R. M. Samelson, 2011. *Progress in Oceanography*, 91, 167-216.

Now available through AVISO: https://www.aviso.altimetry.fr/en/data/products/value-added-products.html

Normalized mean and std dev life cycles from altimeter data and random-walk model



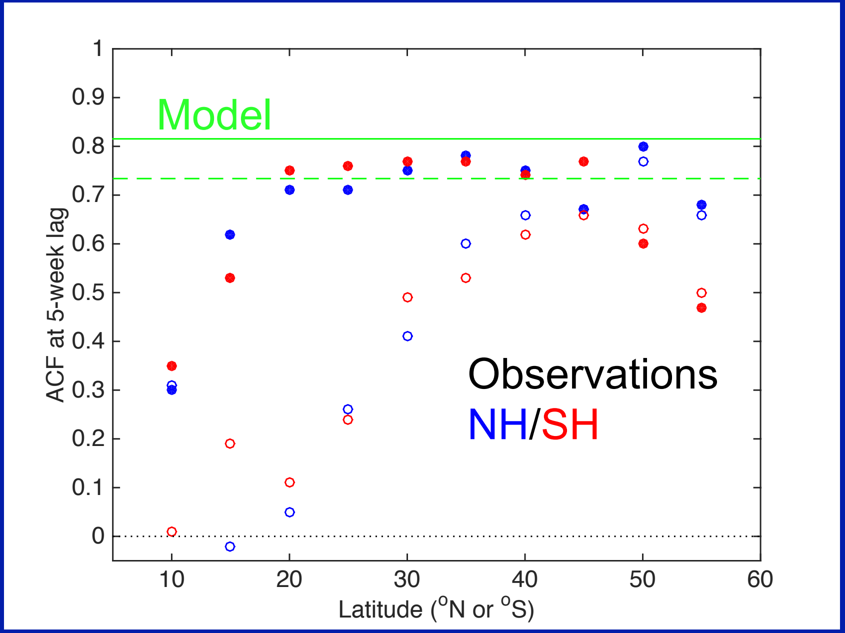


Random walk with linear damping

(first-order autoregressive/AR1 process)

$$
\begin{aligned}
A(t_{j+1}) &= A(t_j) + \delta_j - rA(t_j) \\
&= \alpha A(t_j) + \delta_j, \\
& \quad 0 < \alpha = 1 - r < 1
\end{aligned}
$$

The AR1 parameter α determines autocorrelation structure for A, independent of subsequence ("eddy detection and tracking") analysis.



Random walk with linear damping

(first-order autoregressive/AR1 process; Markov process)

$$
\begin{aligned}
\eta_{j+1} &= \alpha\eta_j + \delta_j \\
< \eta_{j+1}^2 > &= \alpha^2 < \eta_j^2 > + \alpha < \eta_j \delta_j > + < \delta_j^2 > \\
&= \alpha^2 < \eta_j^2 > + \sigma_\delta^2
\end{aligned}
$$

$$0 < \alpha = 1 - r < 1$$

$$
\begin{aligned}
\int^t \{\texttt{variance from forcing}\}\, dt' &= \int^{t=N\times\Delta t} < \delta_j^2 > \, dt' \\
&= N \times \sigma_\delta^2 \\
&= \frac{\sigma_\delta^2}{\Delta t}\, t \\
&= \sigma_W^2\, t, \quad \sigma_W = \frac{\sigma_\delta}{(\Delta t)^{1/2}}
\end{aligned}
$$

$$\sigma_W \approx 2.5 \times 10^{-3} \mathrm{cm\ s}^{-1/2}$$

Rate of forcing independent of discrete time-step

Generalization: a stochastic field model[2]

$$\frac{\partial \eta}{\partial t} + c_R \frac{\partial \eta}{\partial x} = -R\eta + F(x, y, t)$$

Here $F(x,y,t)$ is a stochastic forcing function with:

(1) wavenumber power spectrum chosen to match observed SSH spectrum
(2) random phase for each spectral component at each (weekly) time step

Solve along characteristics:

$$\frac{dX}{dt} = c_R, \quad X(t = 0) = x_0,$$

$$\eta(x_0 + c_R t_{p+1}, y_0, t_{p+1}) = \alpha\eta(x_0 + c_R t_p, y_0, t_p) + \delta^F(x_0 + c_R t_p, y_0, t_p),$$

Same difference equation as before, but with random increment field.

[2](Samelson, R. M., M. G. Schlax, and D. B. Chelton, 2016. A linear stochastic field model of mid-latitude mesoscale sea-surface height variability. J. Phys. Oceanogr., 46, 3103–3120, doi: 10.1175/JPO-D-16-0060.1.)

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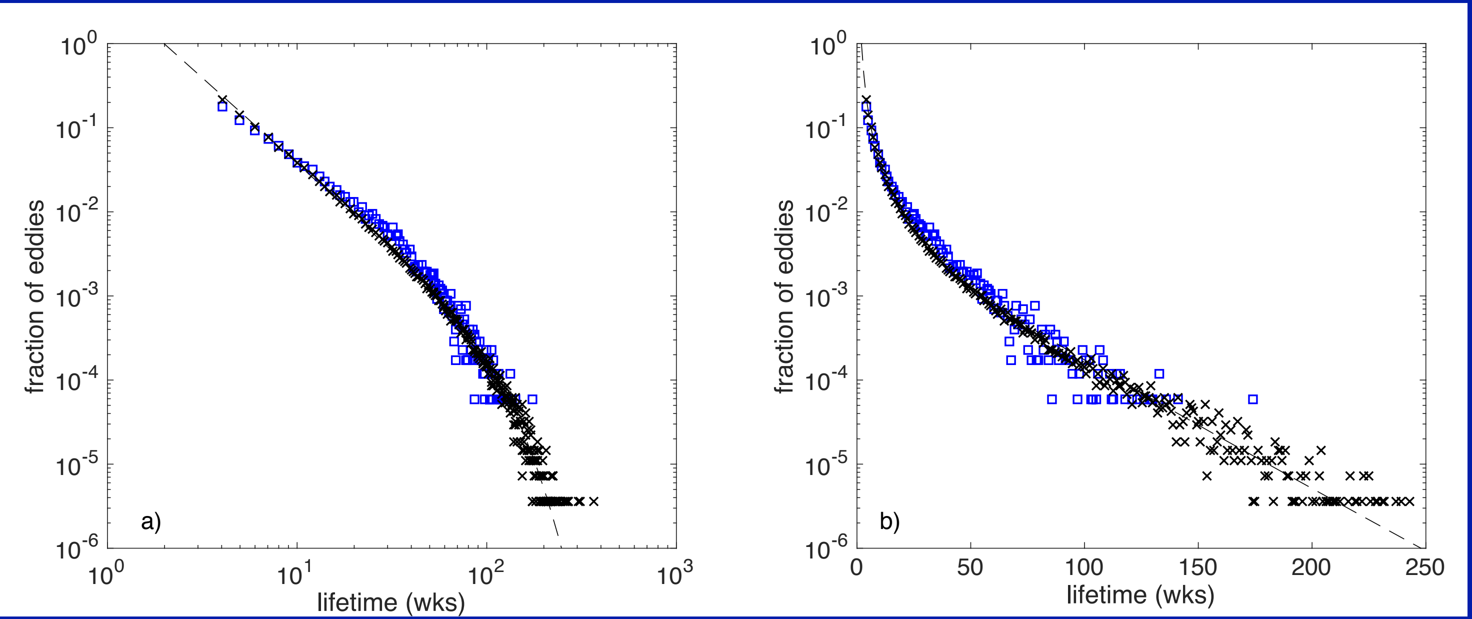
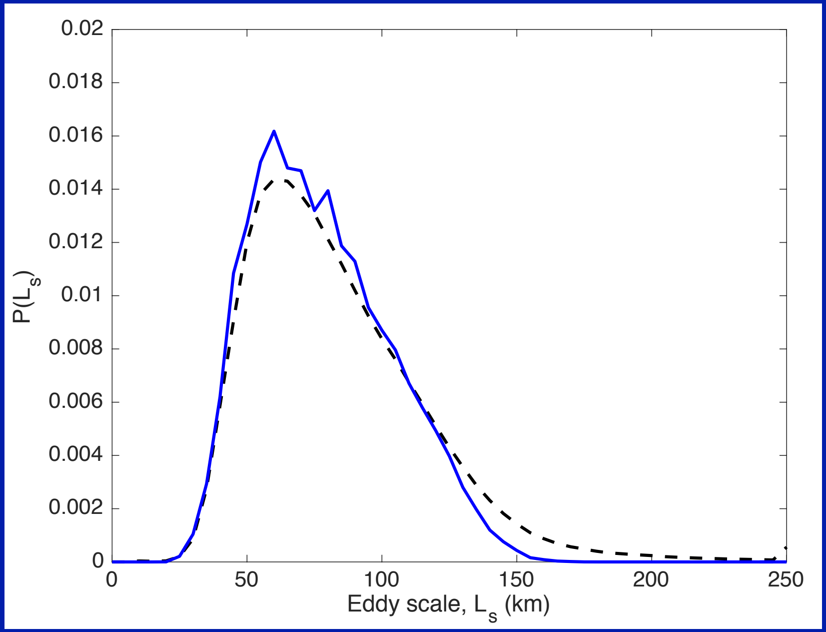
Eddy number distribution vs. lifetime

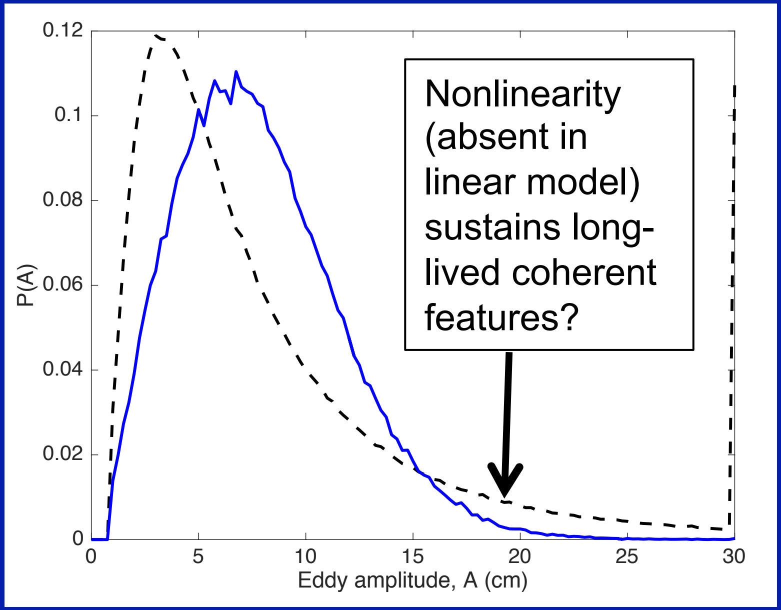
Fraction of eddies



Lifetime (wk)

Eddy length scale (radius) and amplitude distributions





Consider simplest nonlinear dynamical theory: reduced-gravity quasi-geostrophic model

$$\frac{\partial}{\partial t}(\nabla^2\psi - \psi) + \beta\frac{\partial\psi}{\partial x} = -J(\psi, \nabla^2\psi) + \frac{\mathcal{F}_{0,\tau}}{\tau^{1/2}} - r_\psi\psi + \mathcal{D}_{ens}$$

$\mathcal{F}_{0,\tau}$ is a stochastic forcing function with fixed amplitude (unit time-mean spatial standard deviation) and autocorrelation timescale τ
(Morten, Arbic, and Flierl, 2017; Lilly, 1969)

Three parameters:

$\beta = \beta_* L_R^2/U_{\mathcal{F}}$ $r_\psi = r_{QG*}L_R/U_{\mathcal{F}}$ $\tau = \tau_{QG*}U_{\mathcal{F}}/L_R$

…plus spatial (wavenumber) structure of $\mathcal{F}_{0,\tau}$

Require: $\sigma_\eta = 0.07\ \mathrm{m\ at\ 35^\circ N}$
and for L_R = 40 km

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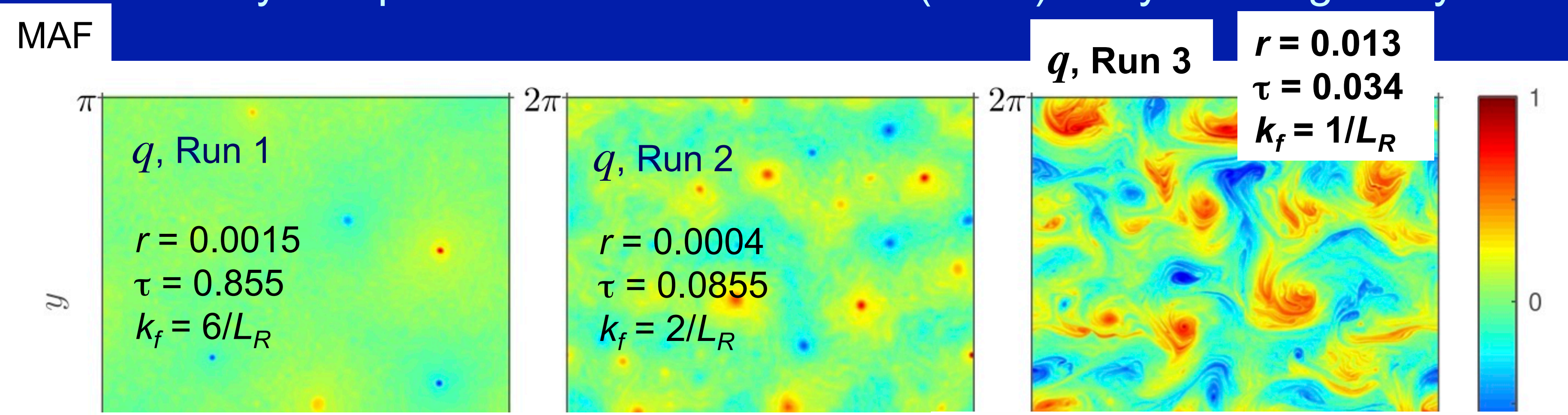
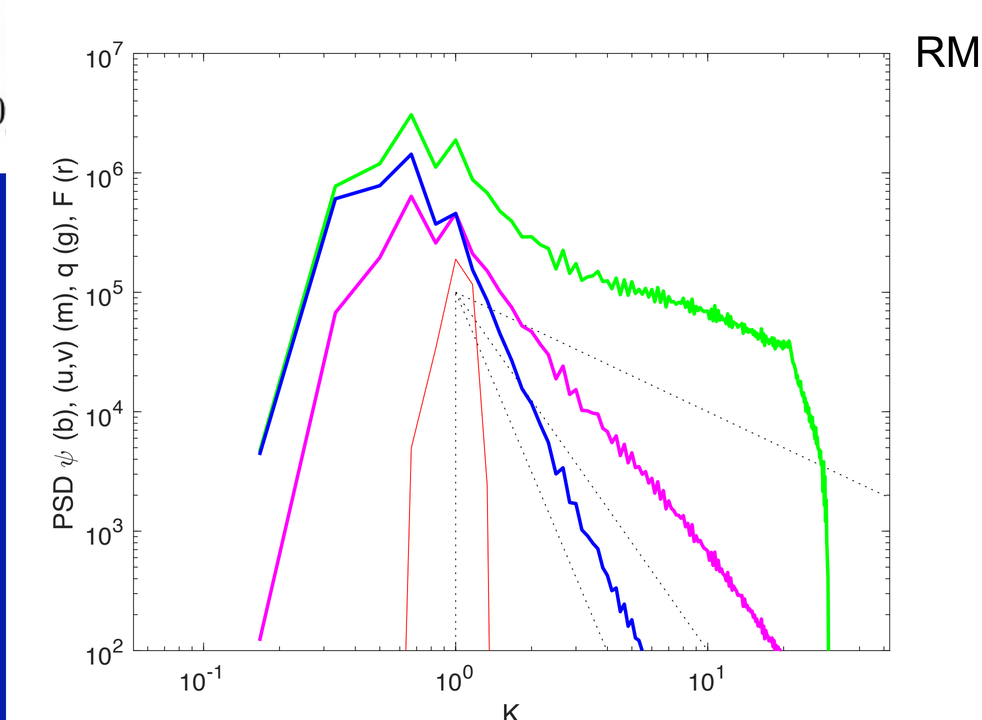
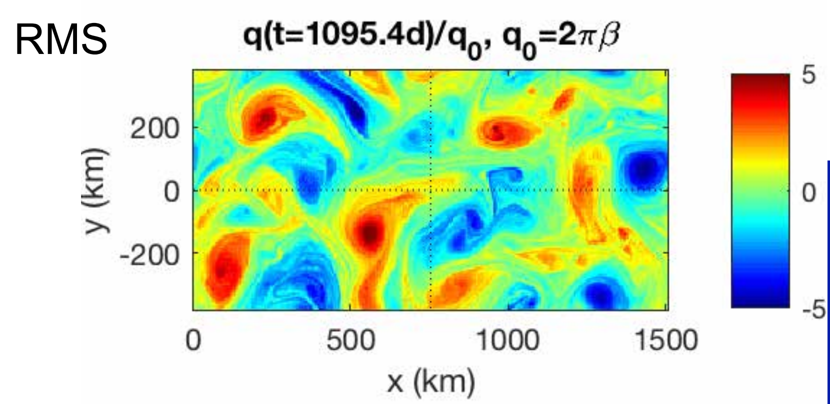
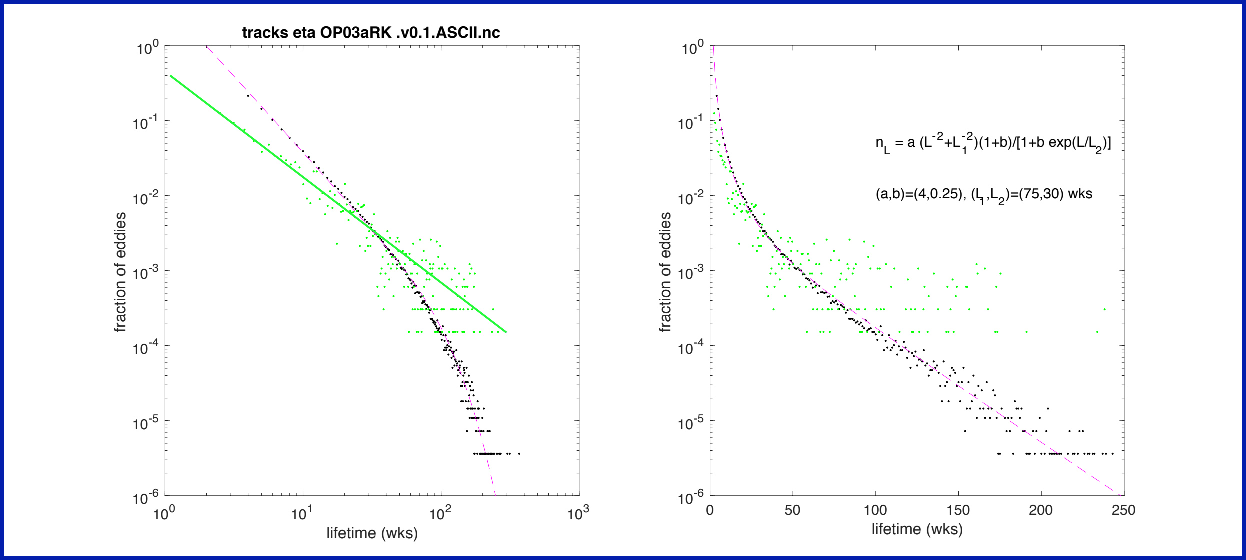
QG simulations: guided initially by Morten, Arbic, Flierl (2017), then by comparison with Chelton et al. (2011) eddy tracking analysis

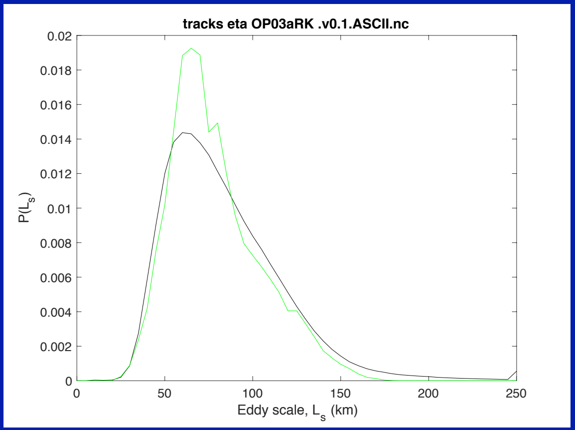


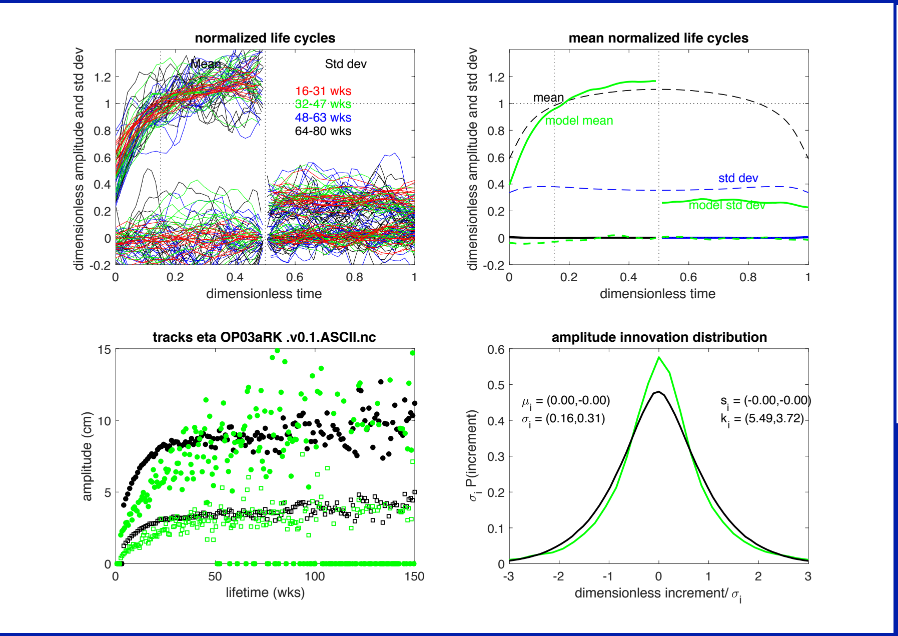


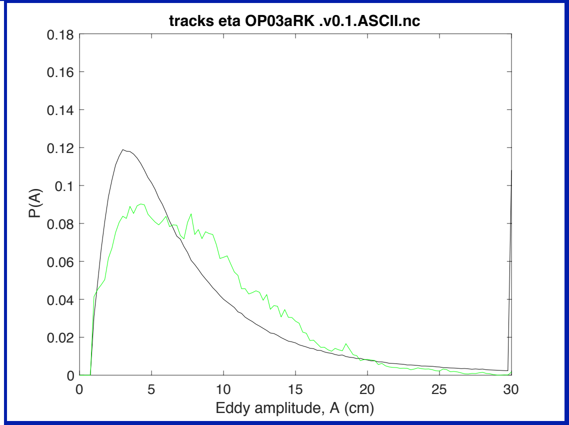


QG eddy identification and tracking

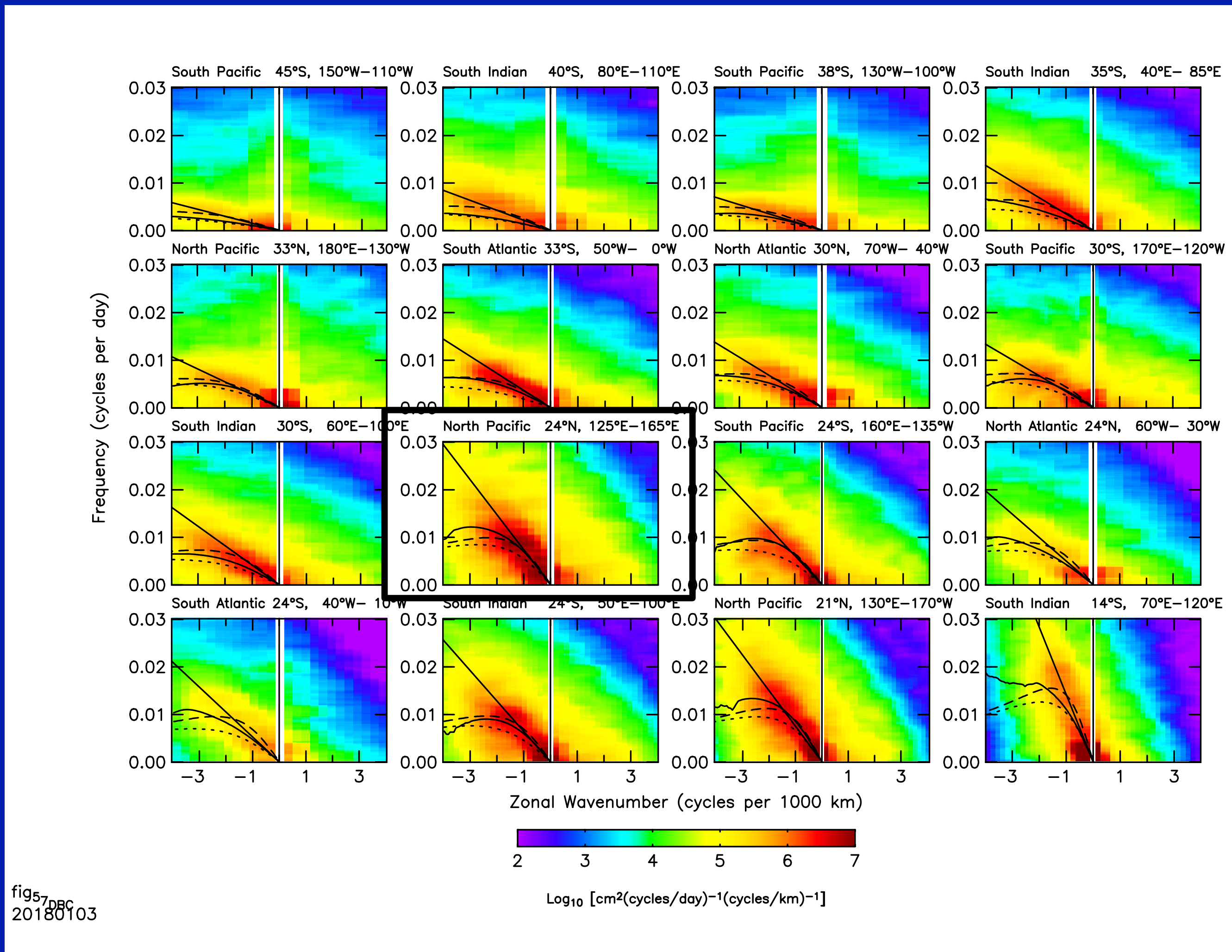




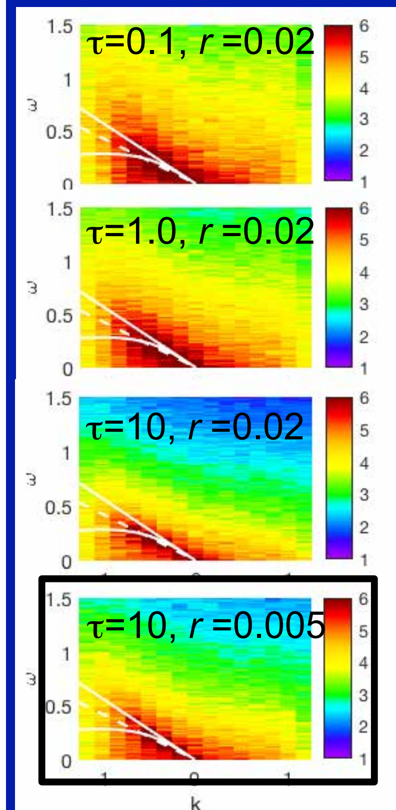




AVISO zonal-wavenumber - frequency spectra (Chelton)



Model spectra



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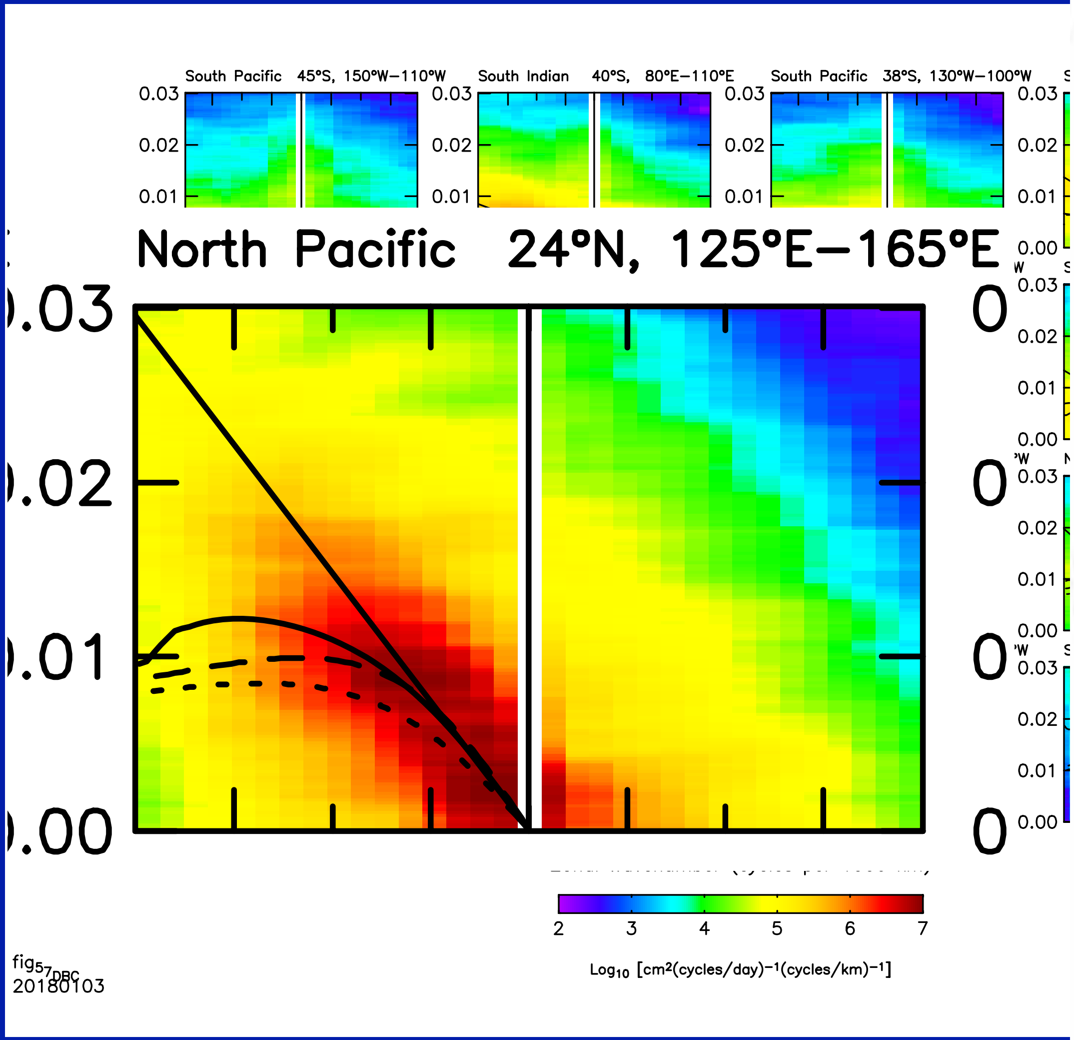

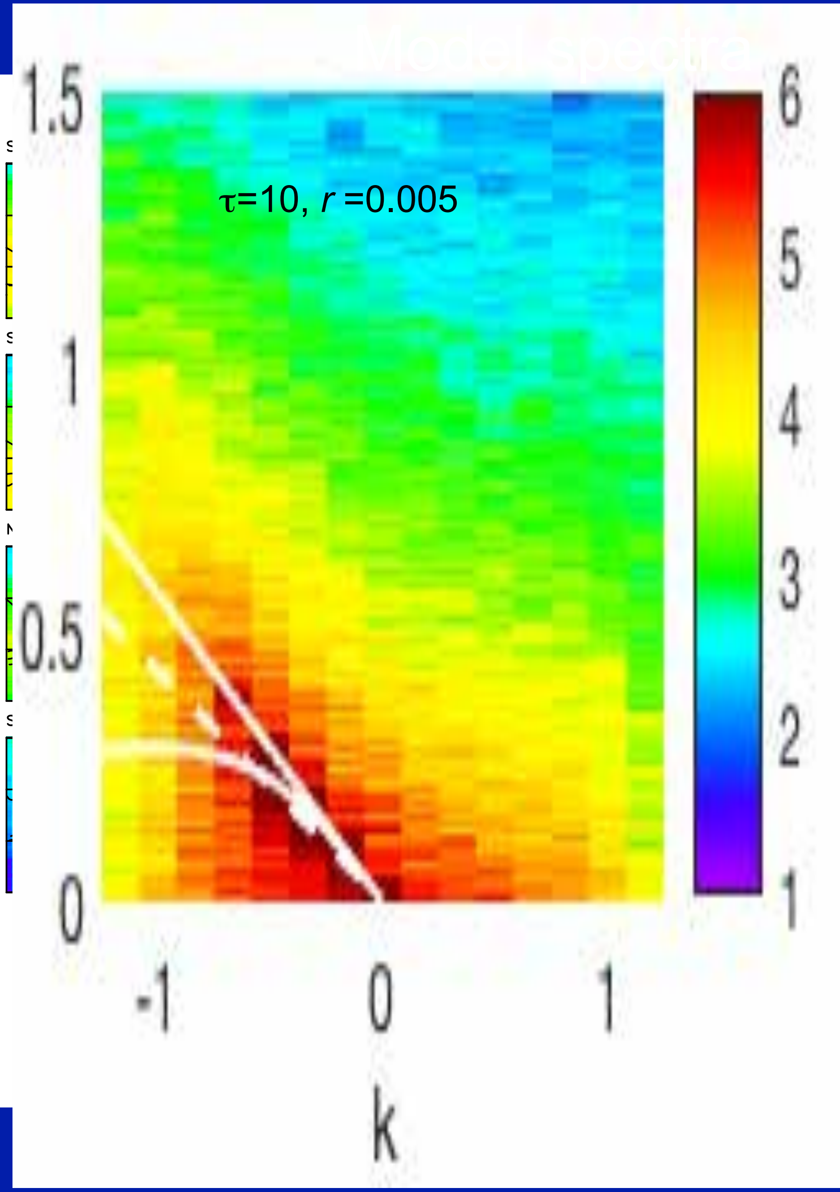

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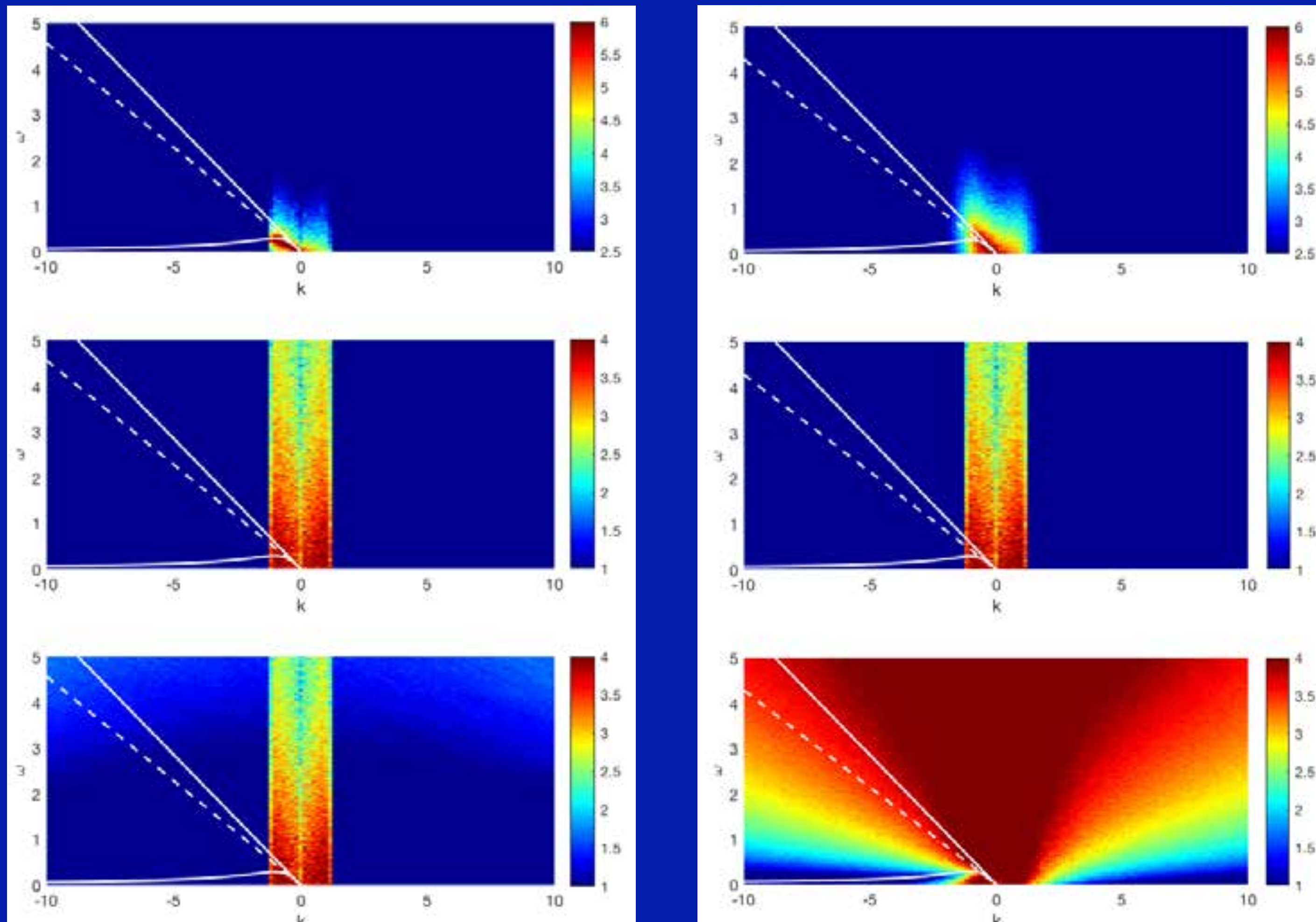
Frequency vs. zonal-wavenumber spectra: Linear inversion of linear and nonlinear models

ψ

$\mathcal{F}_t$

$\mathcal{L}^{-1}\psi$

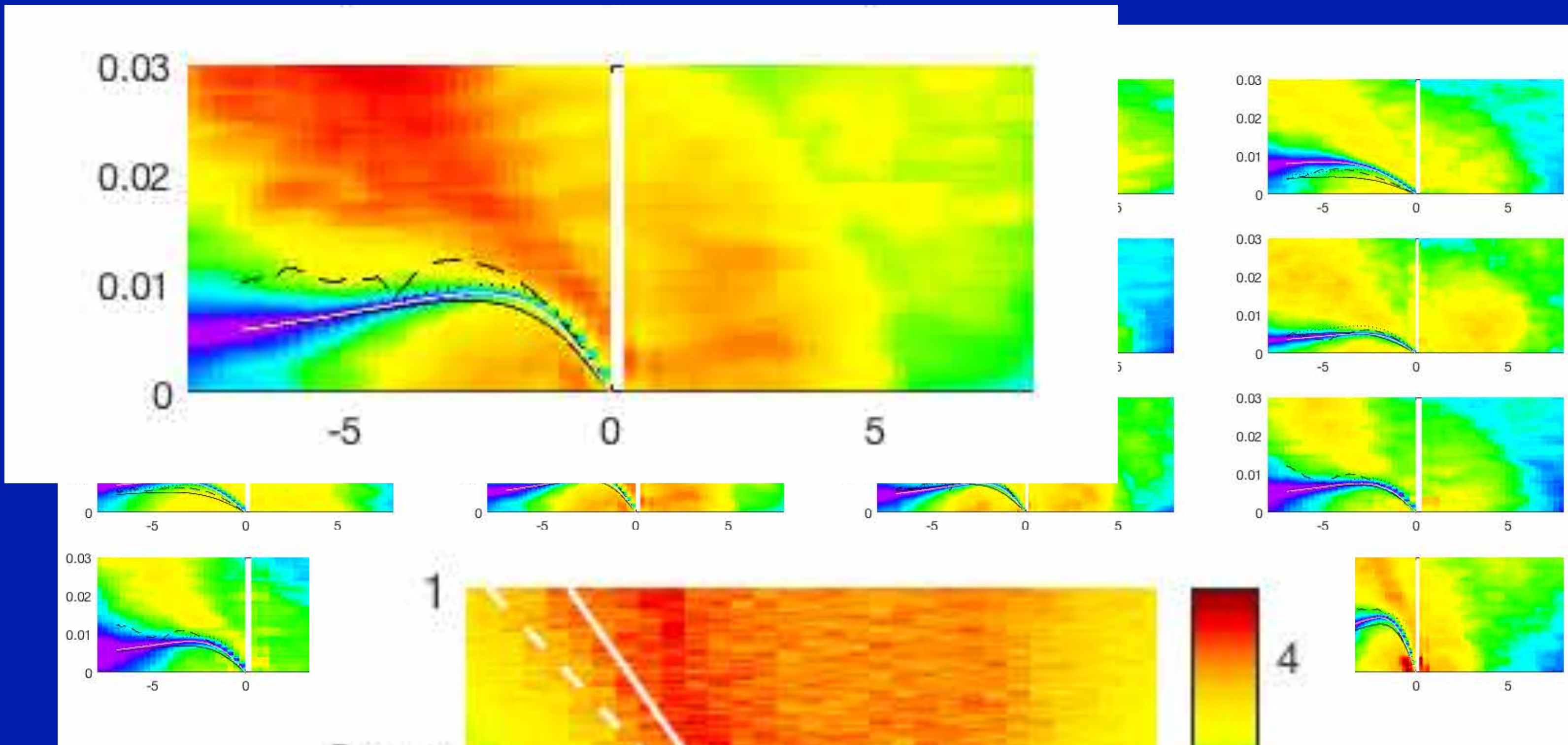


Linear

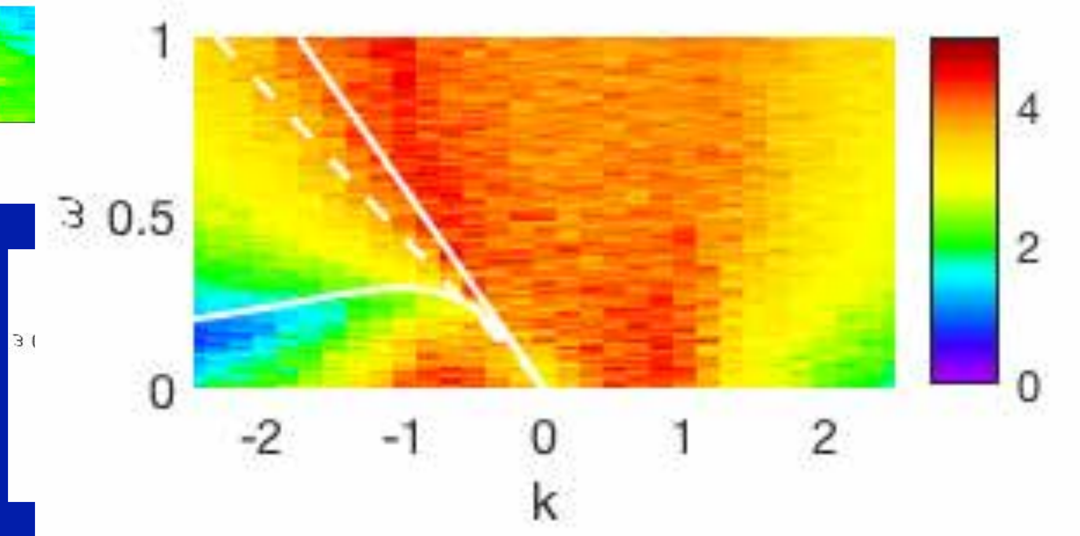
Nonlinear

Linear-inverted AVISO and model spectra

$\mathcal{L}^{-1}\psi$



Model:



τ=10, r =0.005

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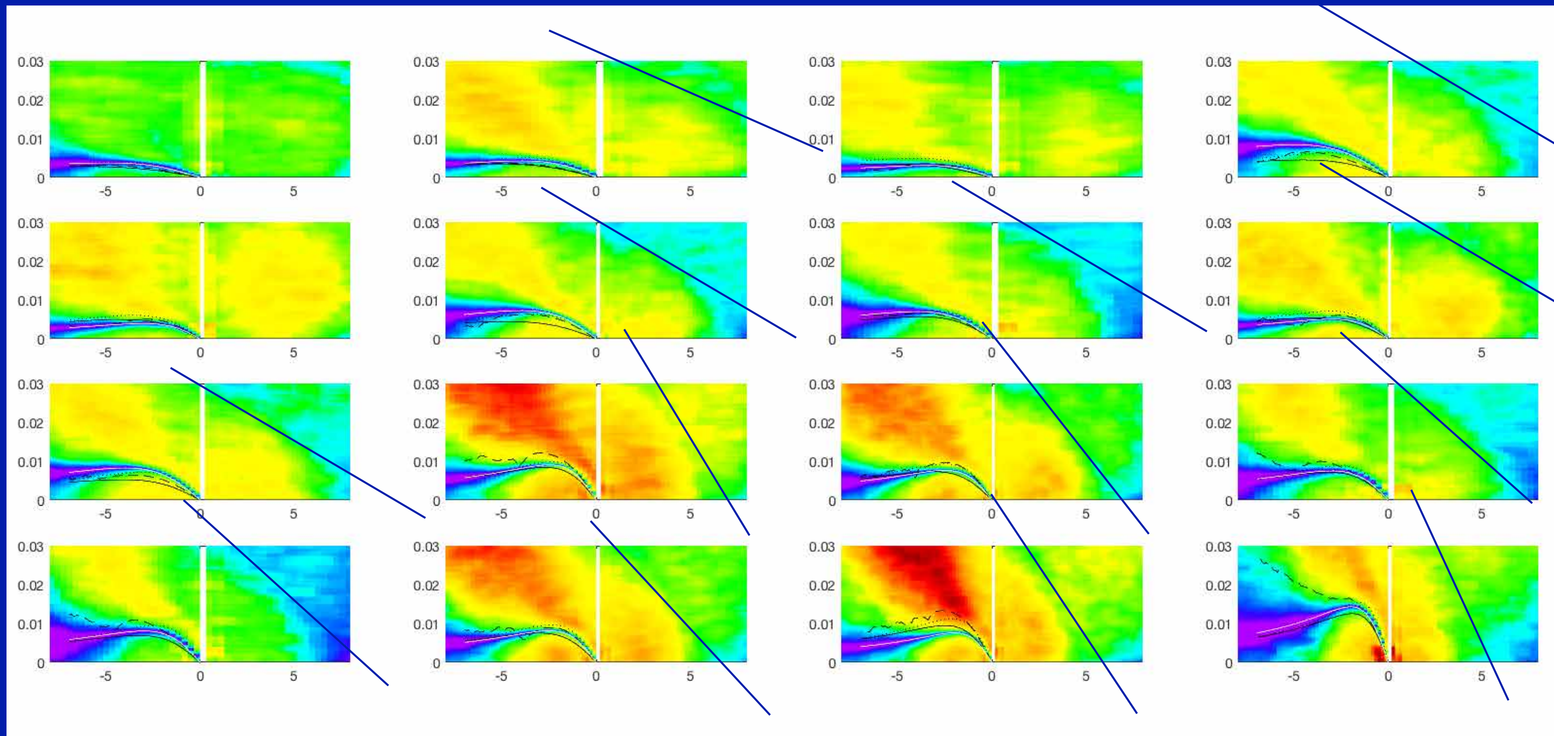
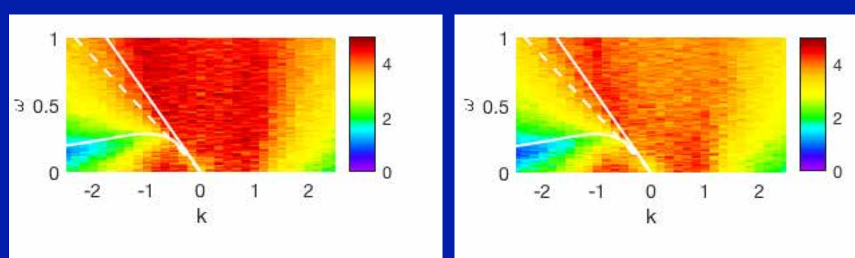
Linear-inverted AVISO spectra:

Evidence of propagating objective analysis filter visible => loss of useful information?

$\mathcal{L}^{-1}\psi$



Model:



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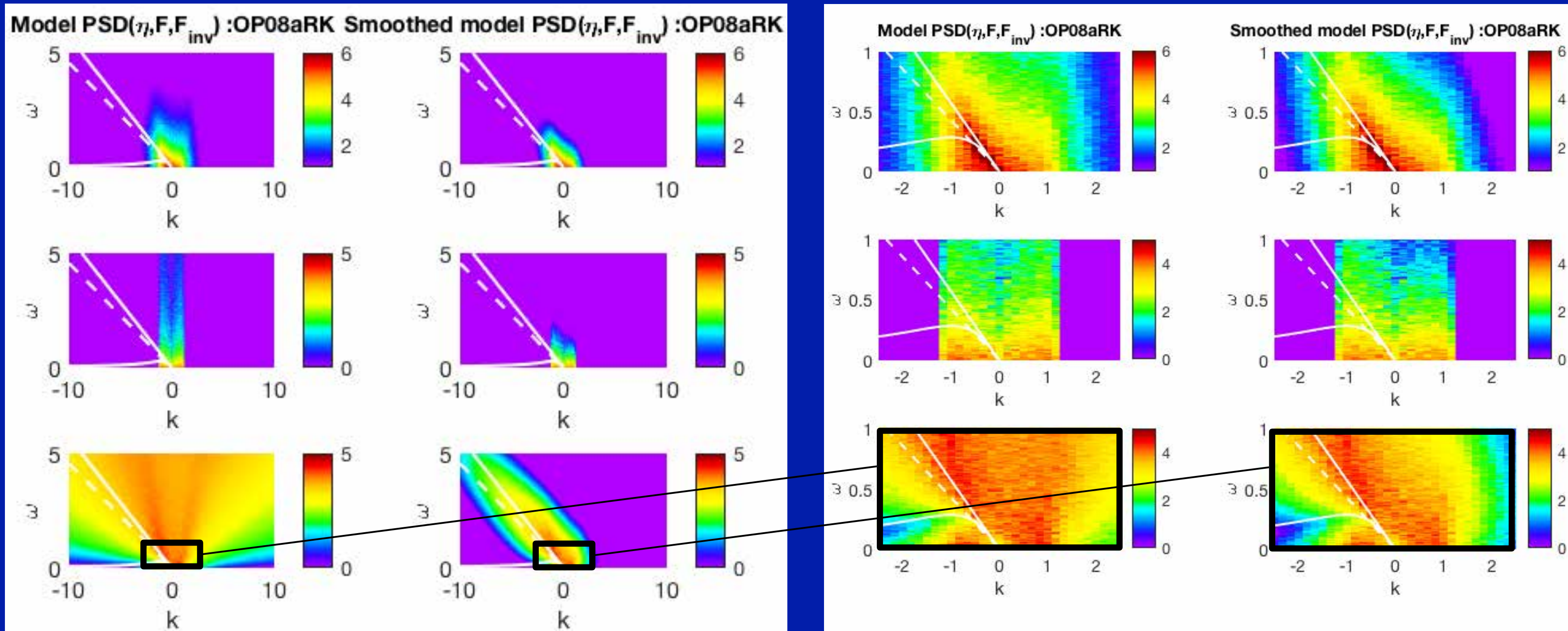
Linear-inverted model spectra: unsmoothed and smoothed

τ=10, *r* =0.005

τ=10, *r* =0.005



Gaussian smoothing, 200 km in *k*, 30 d in ω, centered on $\omega = c_R\, k$.

NB:

$$\hat{W}_{Xt}(k,\omega) = \hat{W}_X(k,\omega)\,\hat{W}_t(k,\omega) = \frac{1}{\pi L T}\exp\left\{-\frac{1}{4}L^2(k - c_0^{-1}\omega)^2 - \frac{1}{4}T^2\omega^2\right\}$$

$L = 37.4$ km and $T = 5.6$ d

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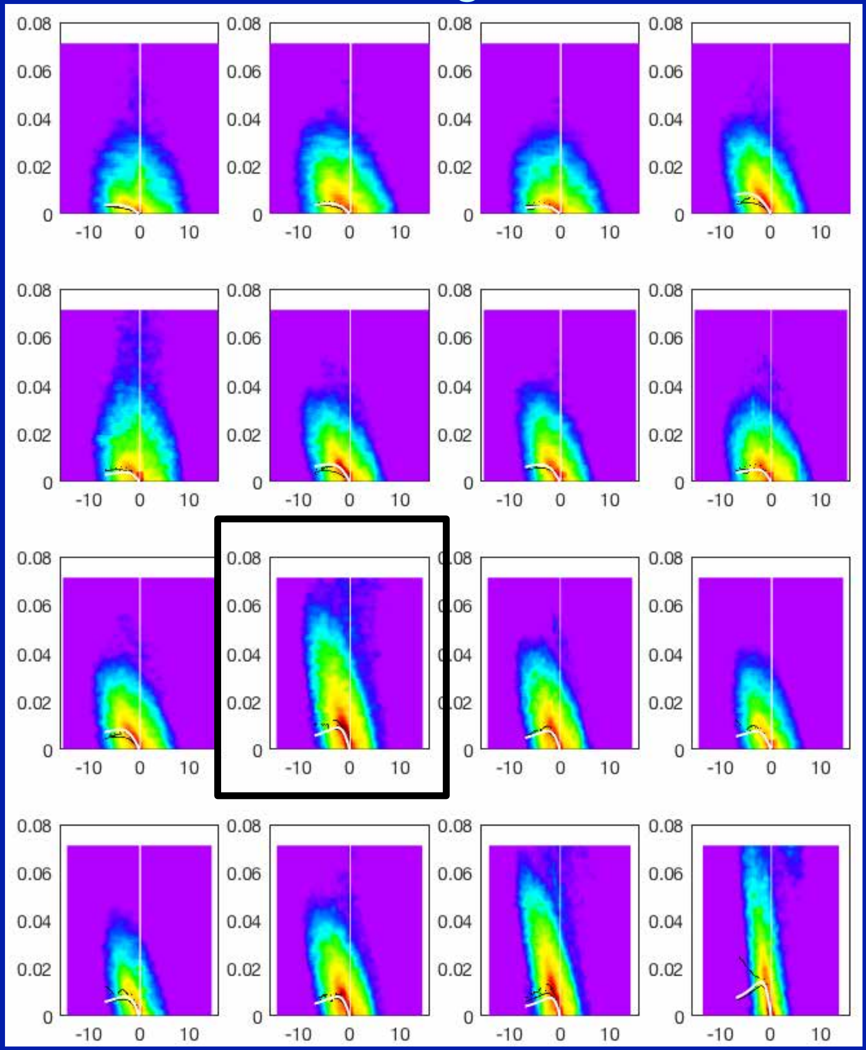
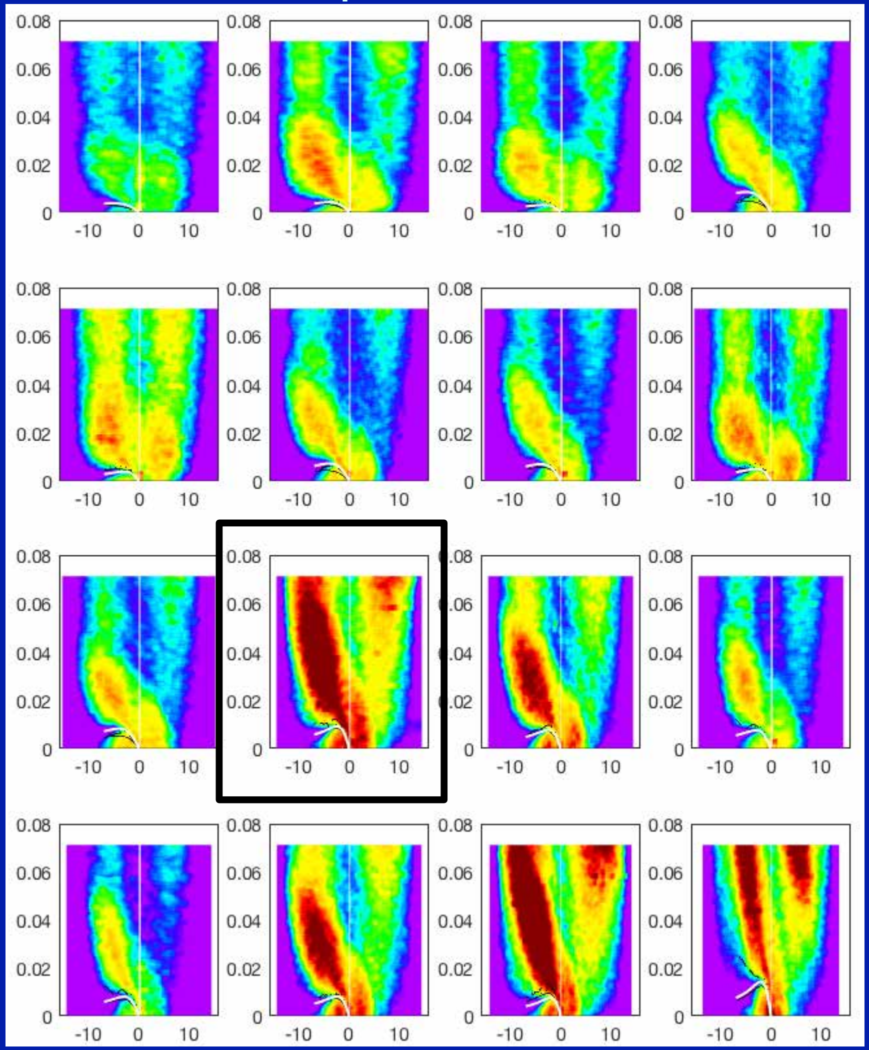
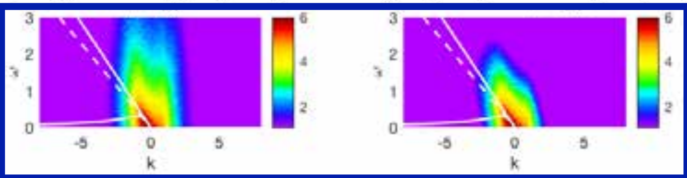
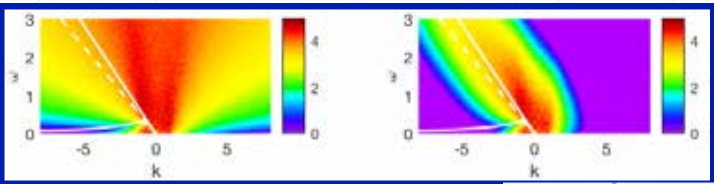
Original and linear-inverted AVISO spectra







model



τ=10, *r* =0.005

Conclusions

1. A connection is made between linear stochastic and nonlinear quasi-geostrophic turbulence models of SSH variability.

2. Observed autocorrelation and spectral structures are broadly reproduced by the nonlinear simulations when the model is forced by stochastic fluctuations near the deformation radius.

3. The flux of energy into the gravest mode of the ocean mesoscale can be represented as a stochastic forcing with $\sigma_W \approx 2 \times 10^{-5} \text{m s}^{-1/2}$.

4. The ocean mesoscale is nonlinear: nonlinearity removes energy along the linear dispersion relation and deposits it elsewhere.

5. There appears to be a visible signature of signal propagation characteristics assumed by the objective analysis procedure in the AVISO altimeter SSH dataset.

6. Much remains to be learned – SWOT will help!