BIOSWOT

Is the Omega equation the good framework for the experimental calculation of vertical velocities ?

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Vertical velocity in the ocean

Vertical exchanges in the ocean

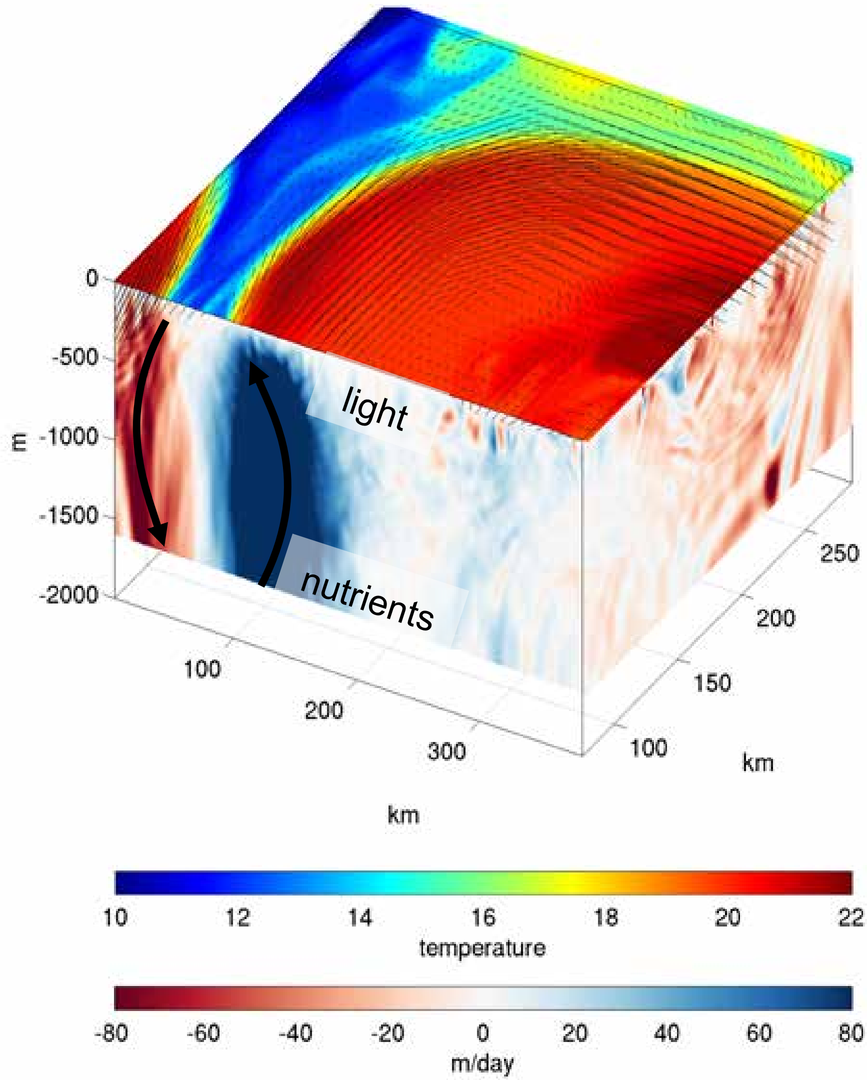
- supply nutrients to the euphotic zone
- subduct matter in the deep ocean
- can be strong when driven by meso and submesoscale dynamics

→ vertical velocity is driven by different sources:

- deformation of the main flow at different vertical and horizontal scales
- surface forcing
- Inertia-gravity waves
- …

→ it is difficult to observe

- localized, small spatial scale
- low intensity
- rapid variability



Vertical velocity in the ocean

w is usually inferred through calculation

- Surface Quasigeostrophy (Lapeyre and Klein 2006; Klein et al. 2009)
- Inverse method (Thomas et al. 2010)
- **the Omega equation** (the more widely used)

How well does the Omega equation represent the vertical circulation in terms of scale, intensity and pattern?

- how much depends on the dynamic of the flow ?
- how much depends on the method and the available data?

The Omega Equation

$$f^2 \frac{\partial^2 w}{\partial z^2} + \nabla_h (N^2 . \nabla_h w) = \nabla . Q$$

Different forcing can drive vertical velocity: $Q = Q_{TW} + Q_{AG} + Q_{FL} + Q_{TD}$

Giordani et al. (2006)

TW : « frontogenesis»

Deformation of the flow

- geostrophic velocity : $Q_{TWg}(\rho, \frac{\partial \rho}{\partial (x,y)})$
- total velocity : $Q_{TW}(\rho, \vec{v}_h, \frac{\partial \rho}{\partial (x,y)}, \frac{\partial \vec{v}_h}{\partial (x,y)})$

AG : Deformation of the thermal wind imbalance

$Q_{AG}(\frac{\partial \vec{v}_h}{\partial (x,y)}, \frac{\partial \vec{v}_{ag}}{\partial z})$

FL : Turbulent fluxes of momentum and buoyancy

Can be prescribed from atmospheric fluxes (wind, heat fluxes)

$Q_{FL}(\frac{\partial \mathbf{F}}{\partial (x,y)}, \frac{\partial \mathbf{X}}{\partial z})$

TD : Trend of the thermal wind imbalance

- Symmetric instability, inertial and sub inertial dynamics, …
- Can't be inferred from observations

$Q_{TD}(\frac{d}{dt}(\frac{\partial \vec{v}_{ag}}{\partial z}))$

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Different forcing can drive vertical velocity: $\boldsymbol{Q} = \boldsymbol{Q}_{TW} + \boldsymbol{Q}_{AG} + \boldsymbol{Q}_{FL} + \boldsymbol{Q}_{TD}$

Quasi Geostrophic Formulation : ω_{QG}

$$f^2 \frac{\partial^2 \omega_{QG}}{\partial z^2} + \nabla_h (N^2 . \nabla_h \omega_{QG}) = \nabla . \boldsymbol{Q}_{TWg}$$

Generalized Formulation : ω_{NG}

$$f^2 \frac{\partial^2 \omega_{QG}}{\partial z^2} + \nabla_h (N^2 . \nabla_h \omega_{QG}) = \nabla . (\boldsymbol{Q}_{TW} + \boldsymbol{Q}_{AG})$$

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(ρ, SSH)

Generalized Formulation : ω_{NG}

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$(\rho, \{\vec{v}_h, SSH)$ ¿

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The Omega Equation

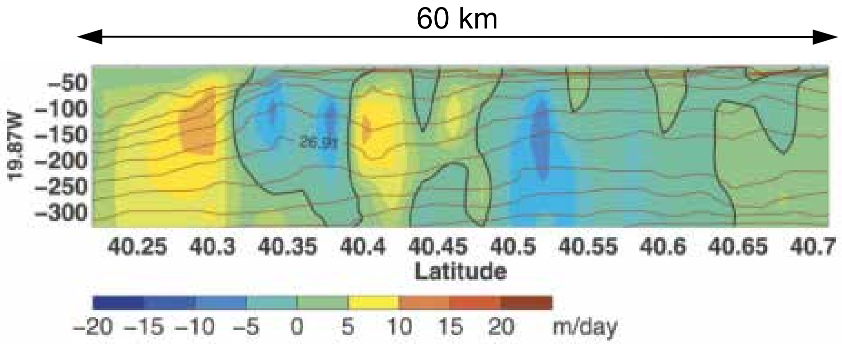
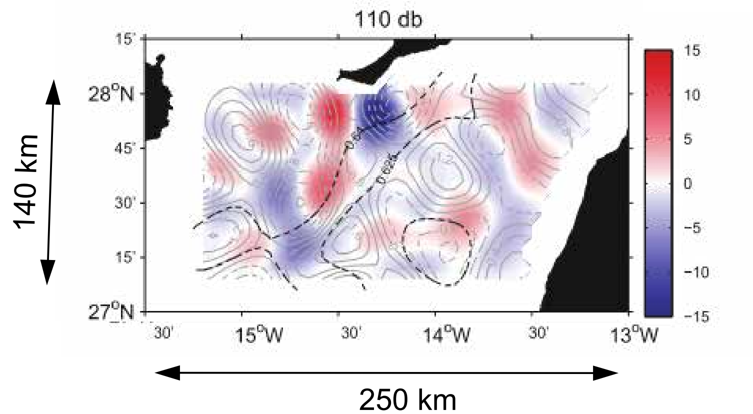
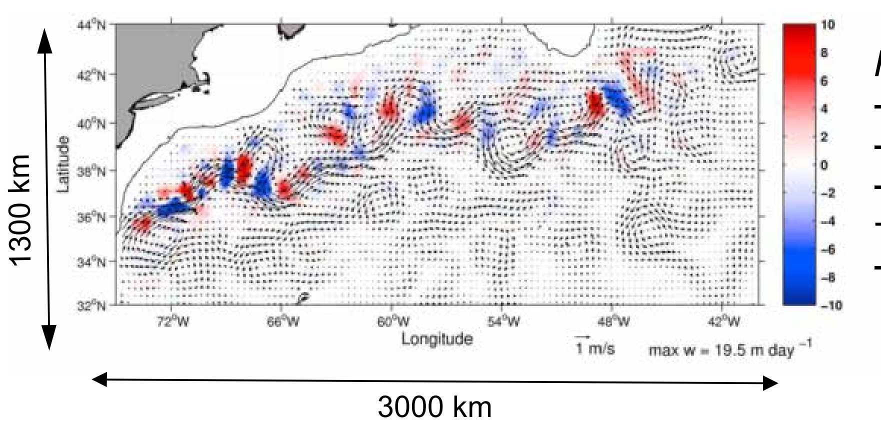


FIG. 6. Vertical velocity field estimated from the SeaSoar and SOPRANE data (in color). Thin lines are density isocontours, and the thick line is the *w*-zero contour.

Legal et al., 2007 :
- 2D QG
- 1-2km resolution
- North East Atlantic

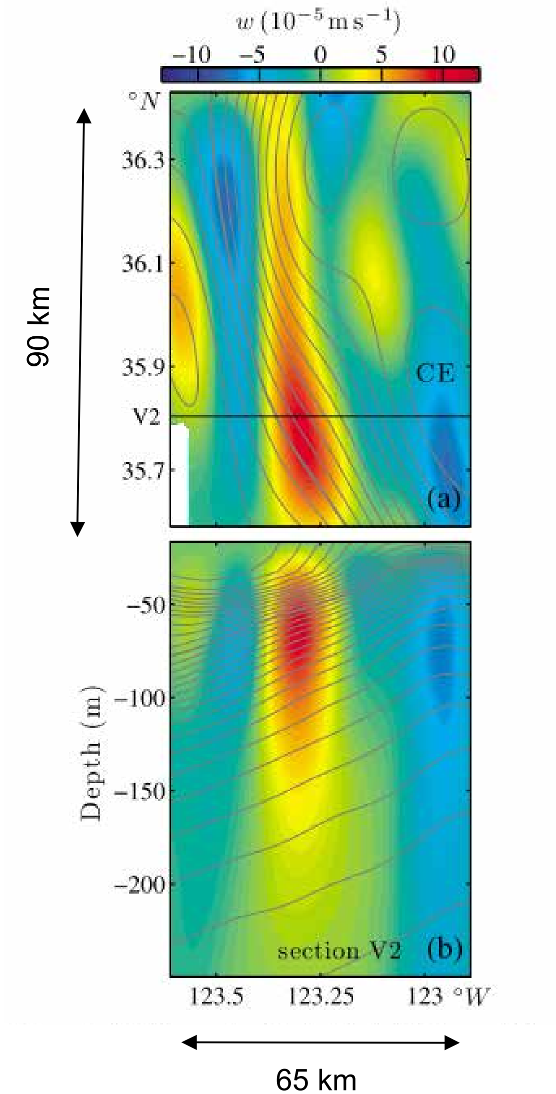


Benitez-Barrios et al., 2011 :
- QG
- 20km resolution
- SouthEast of the Canary islands



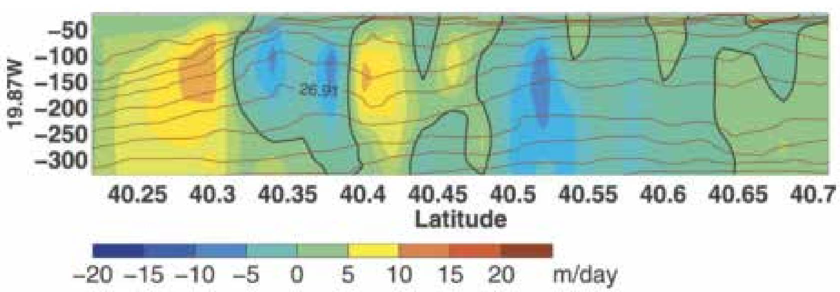
Pascual et al., 2015 :
- QG
- Gulf Stream
- ARMOR3D (satellite + in situ product)
- 35km resolution

Pallàs-Sanz et al. 2010:
- Generalized
- 3-11 km resolution
- California Current System



The Omega Equation

60 km



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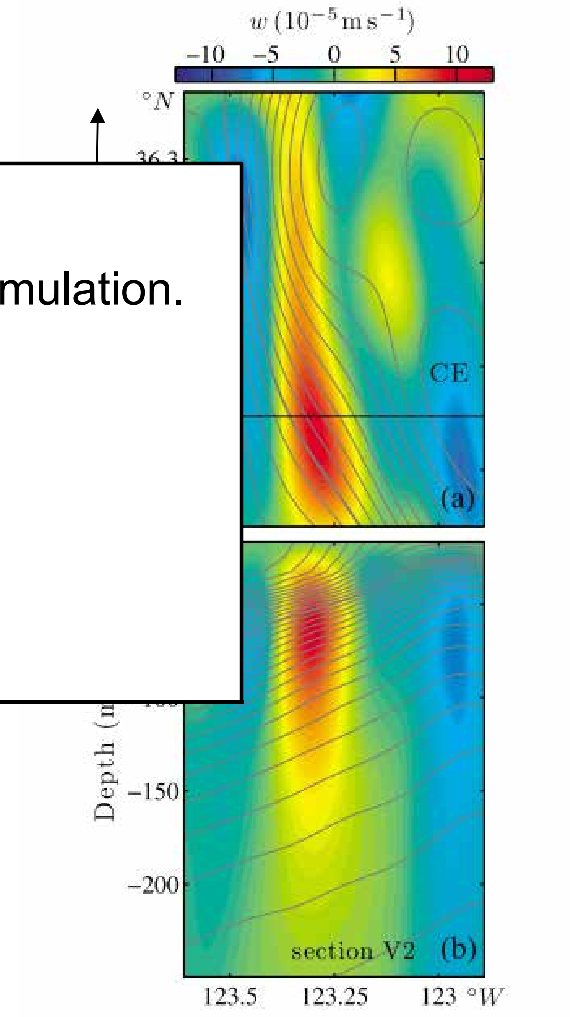
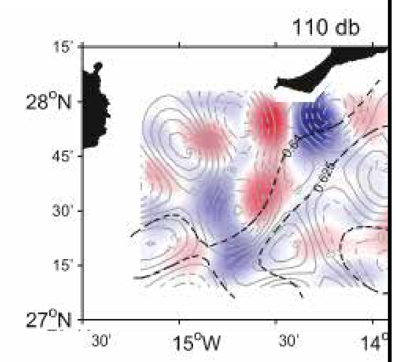


65 km

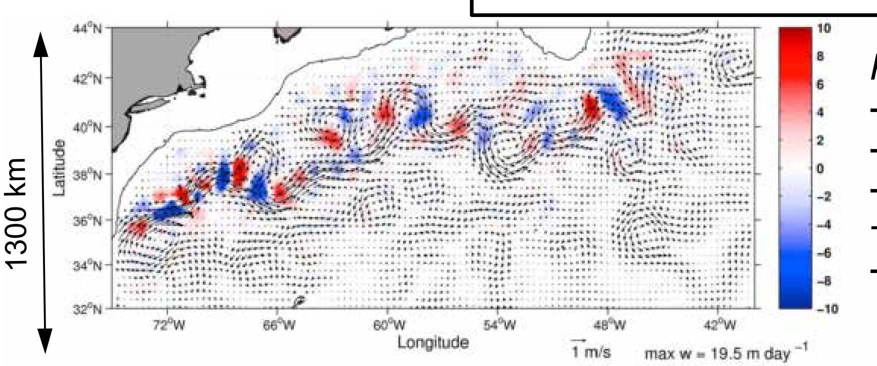
140 km



250 km

- the majority of the papers use the QG formulation.
- dataset resolution: (O(1-10km))
- wmax: (O(10 m/day))
- finest scale on the horizontal: (O(10 km))

1300 km

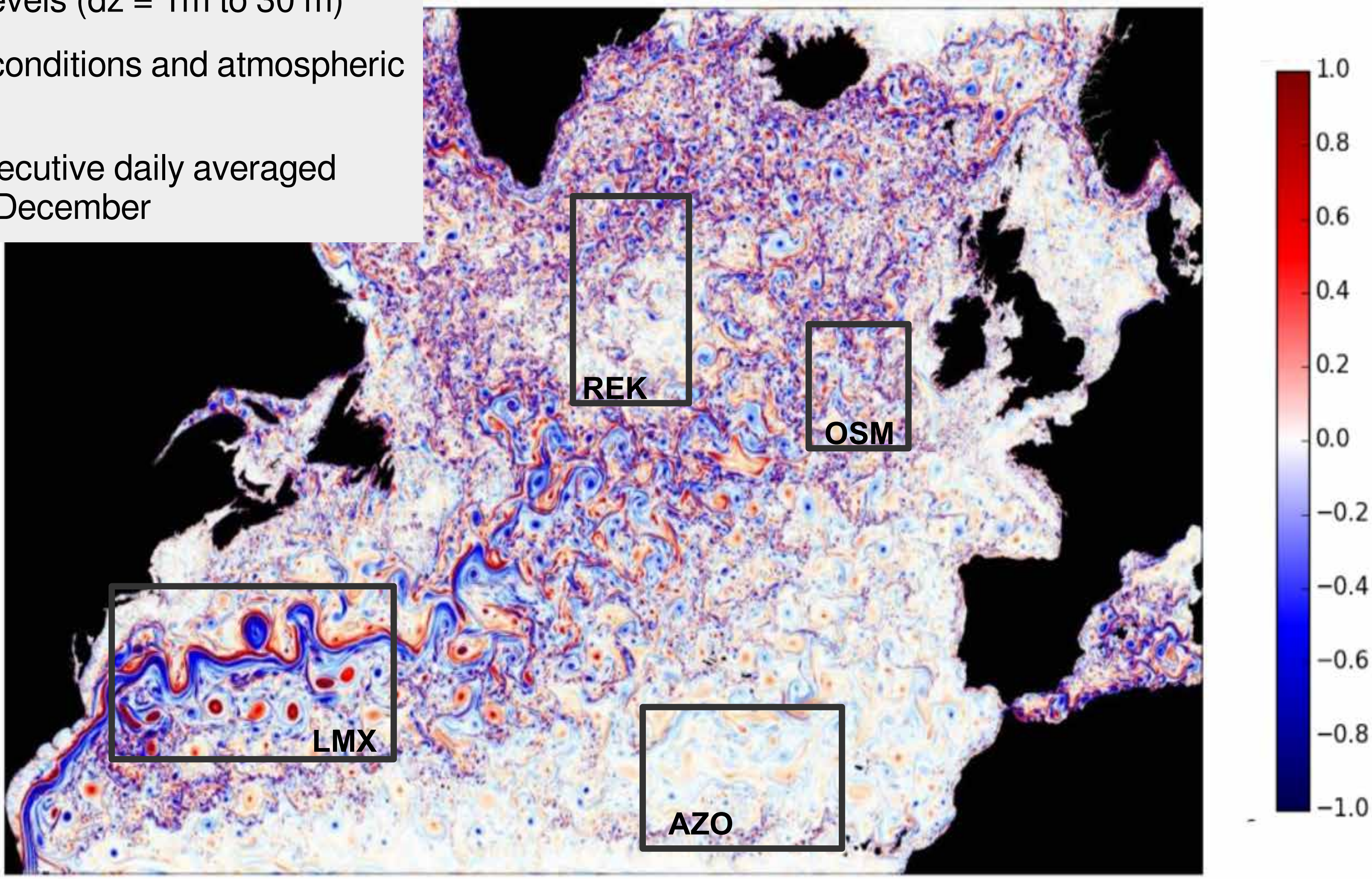


3000 km

Pascual et al., 2015 :
- QG
- Gulf Stream
- ARMOR3D (satellite + in situ product)
- 35km resolution

NATL 60 Model configuration and numerical experiment

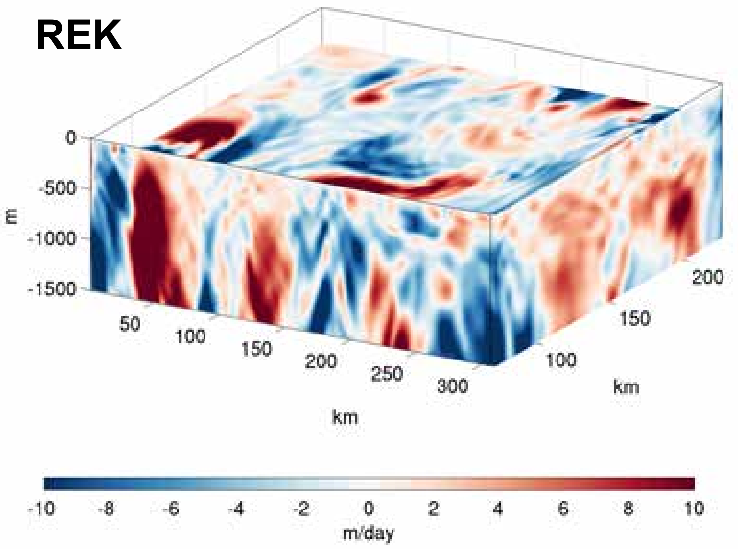
- numerical code : NEMO v3.5"
- horizontal grid : 1/60° (dx = 0.8-1.6 km)
- vertical grid : 300 levels (dz = 1m to 30 m)
- realistic boundary conditions and atmospheric forcing
- 2 series of 11 consecutive daily averaged outputs in June and December



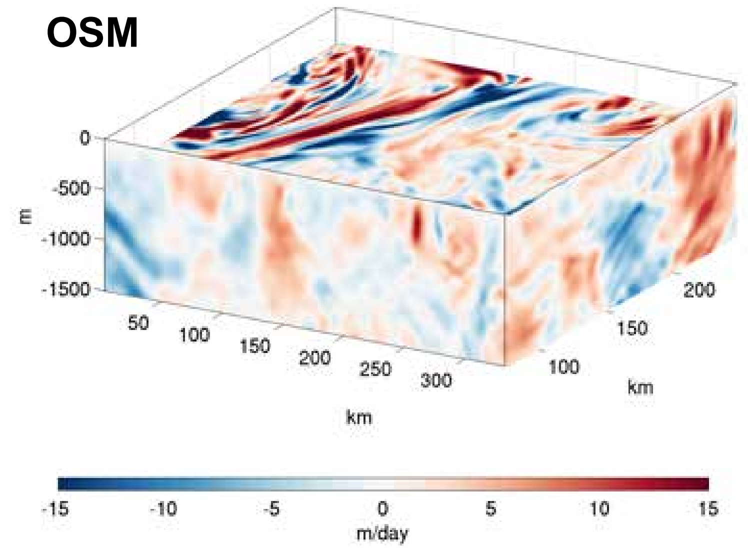
Surface relative vorticity in winter Courtesy of J. LeSommer

NATL 60 : model vertical velocity on June 10th 2008

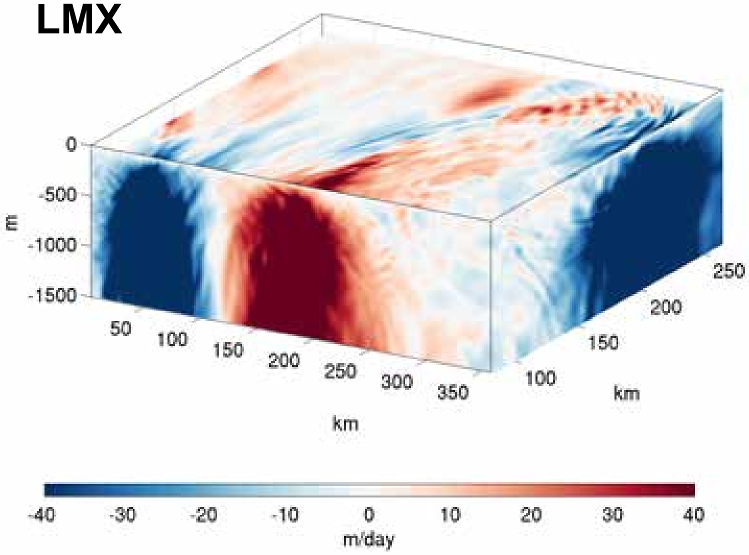
REK



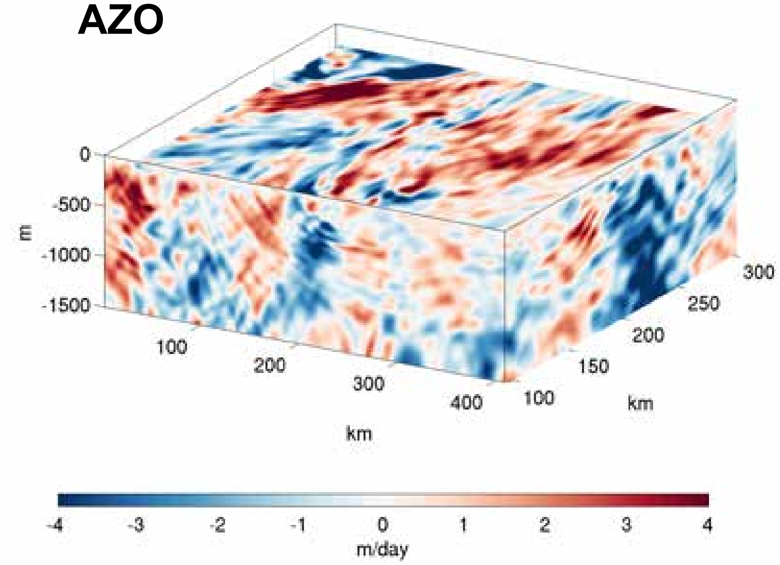
OSM



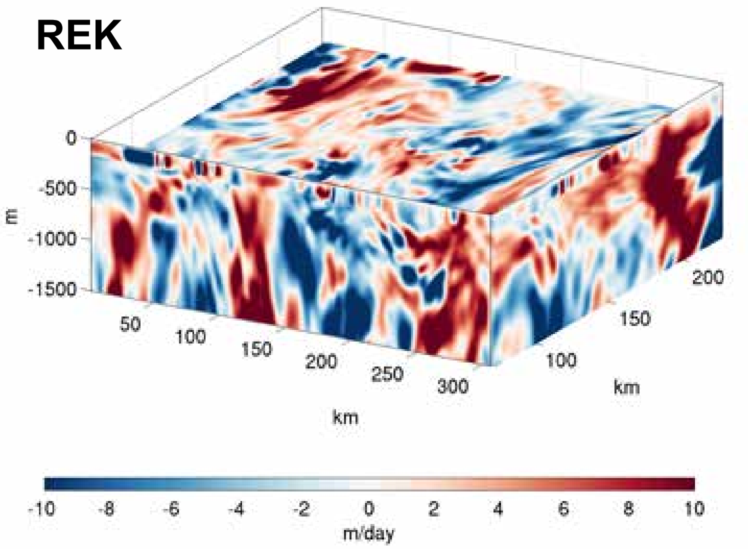
LMX

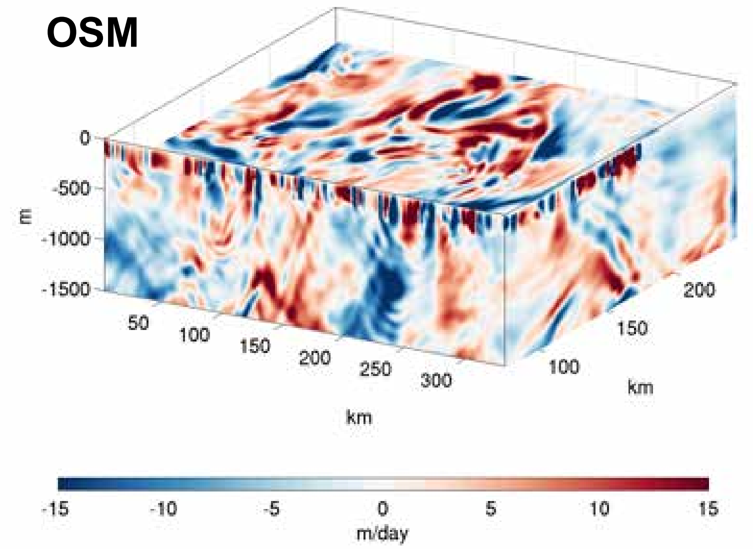


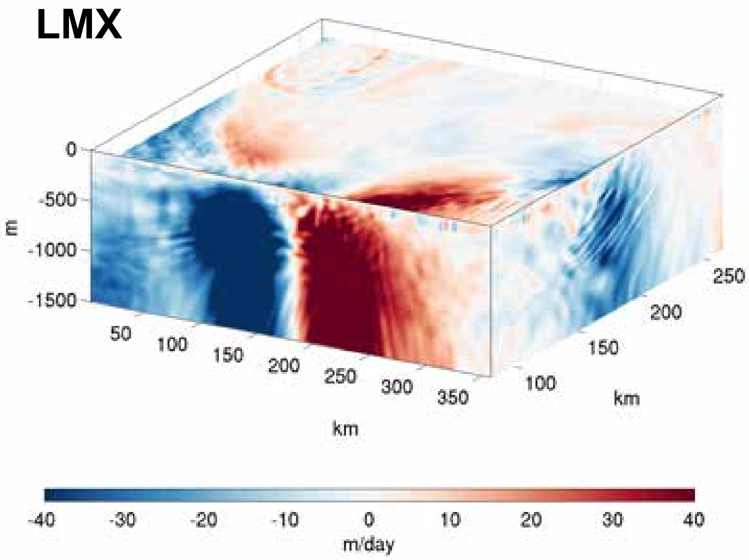
AZO

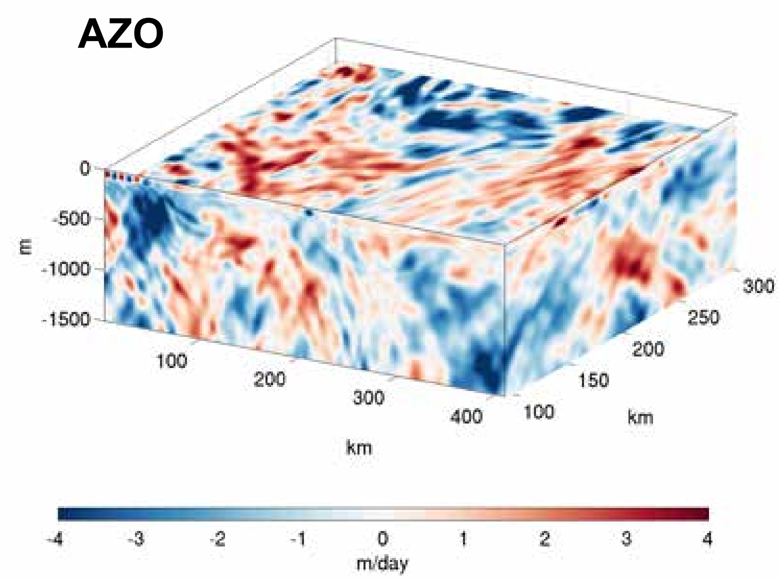


NATL 60 : model vertical velocity on December 10th 2008





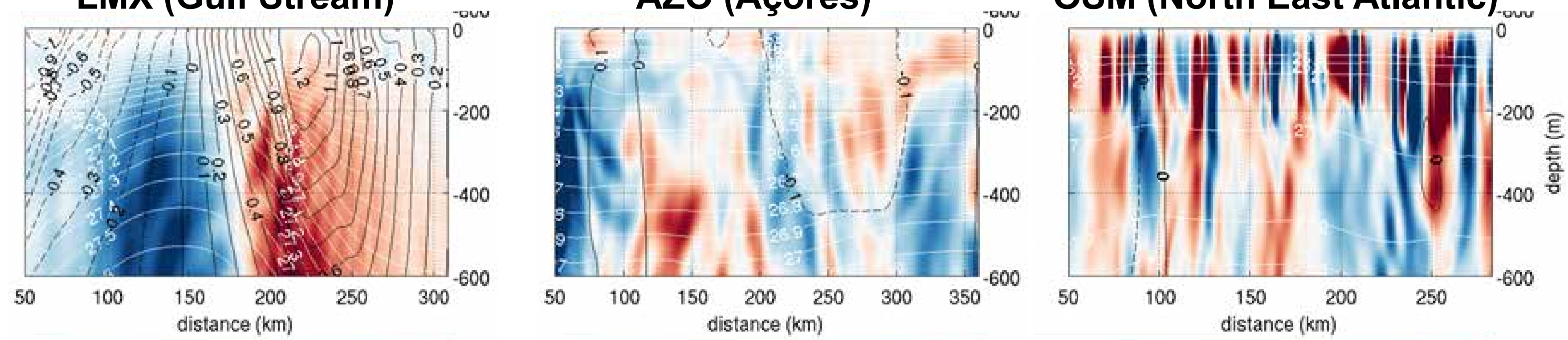




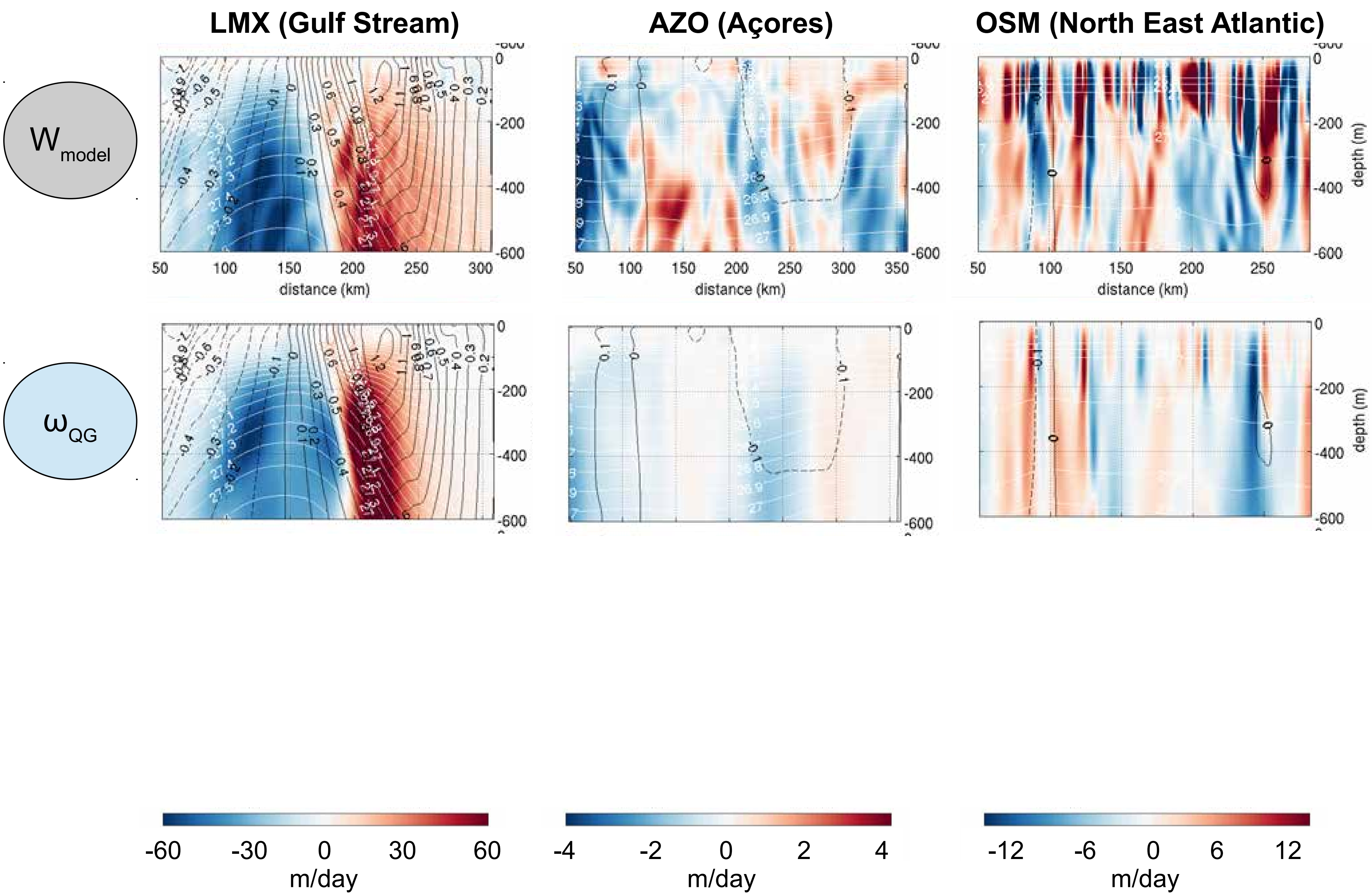
Vertical velocity on December 10th 2008

W_{model}

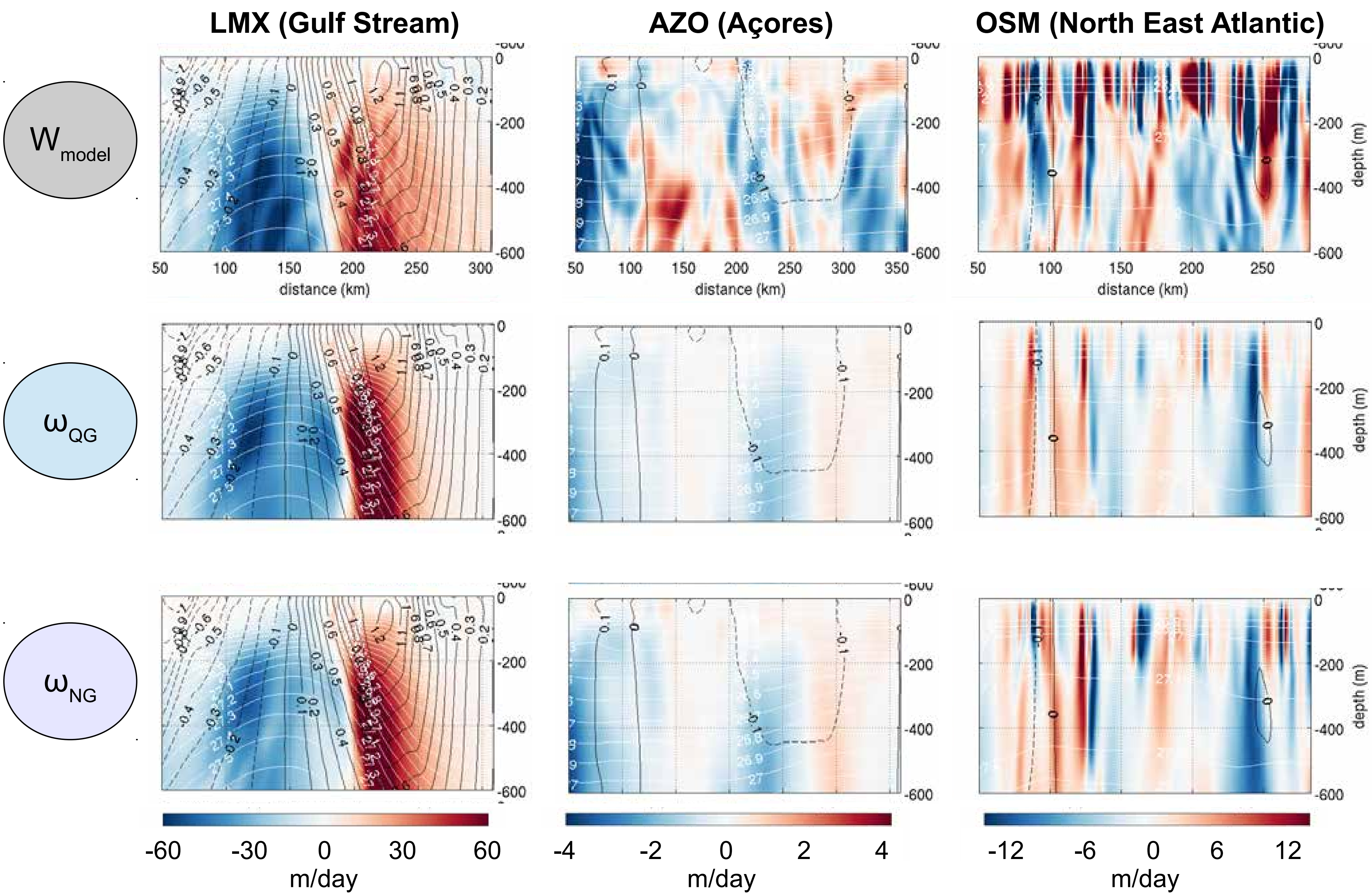
LMX (Gulf Stream) | AZO (Açores) | OSM (North East Atlantic)



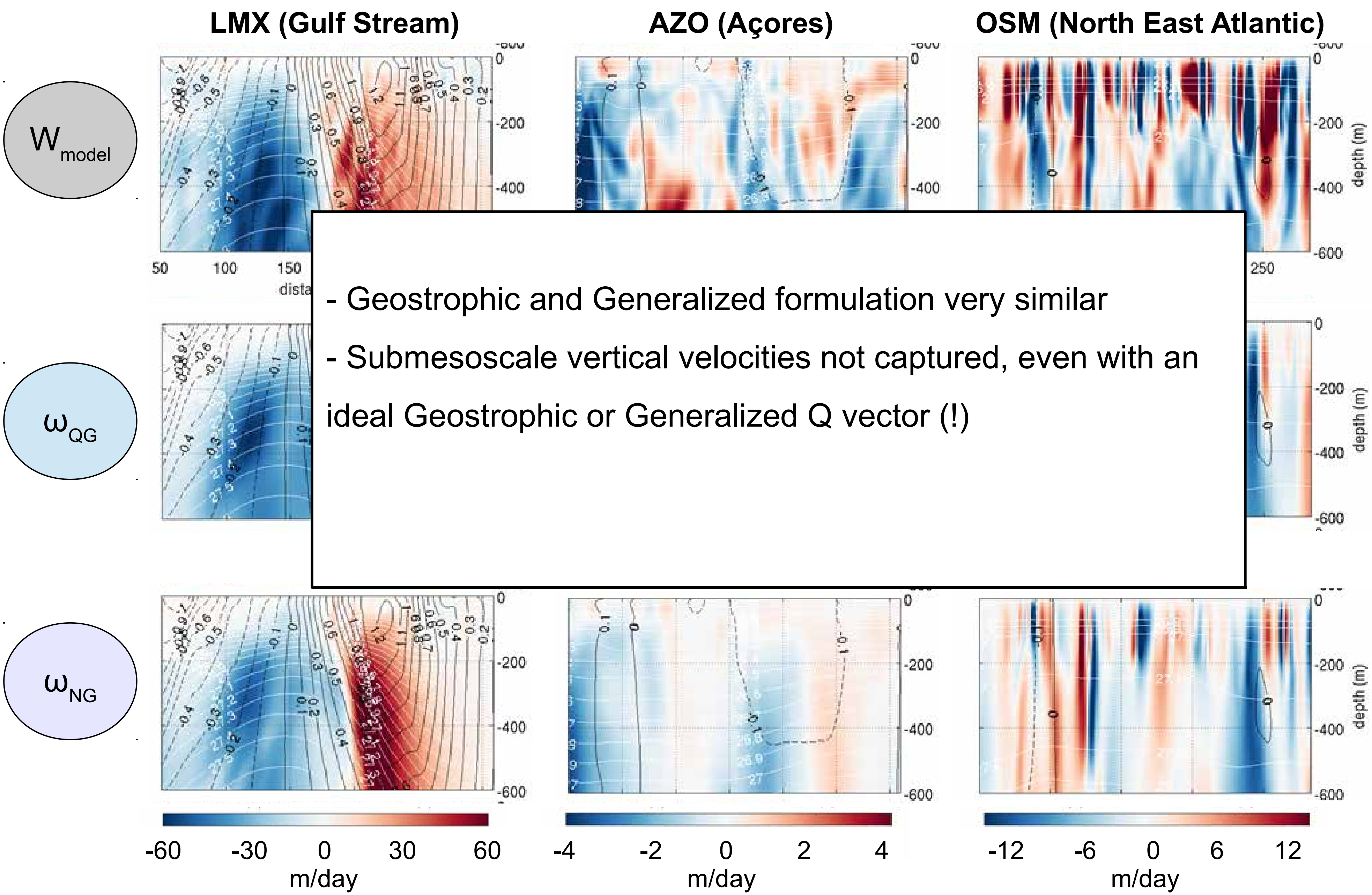
Vertical velocity on December 10th 2008



Vertical velocity on December 10^{th} 2008

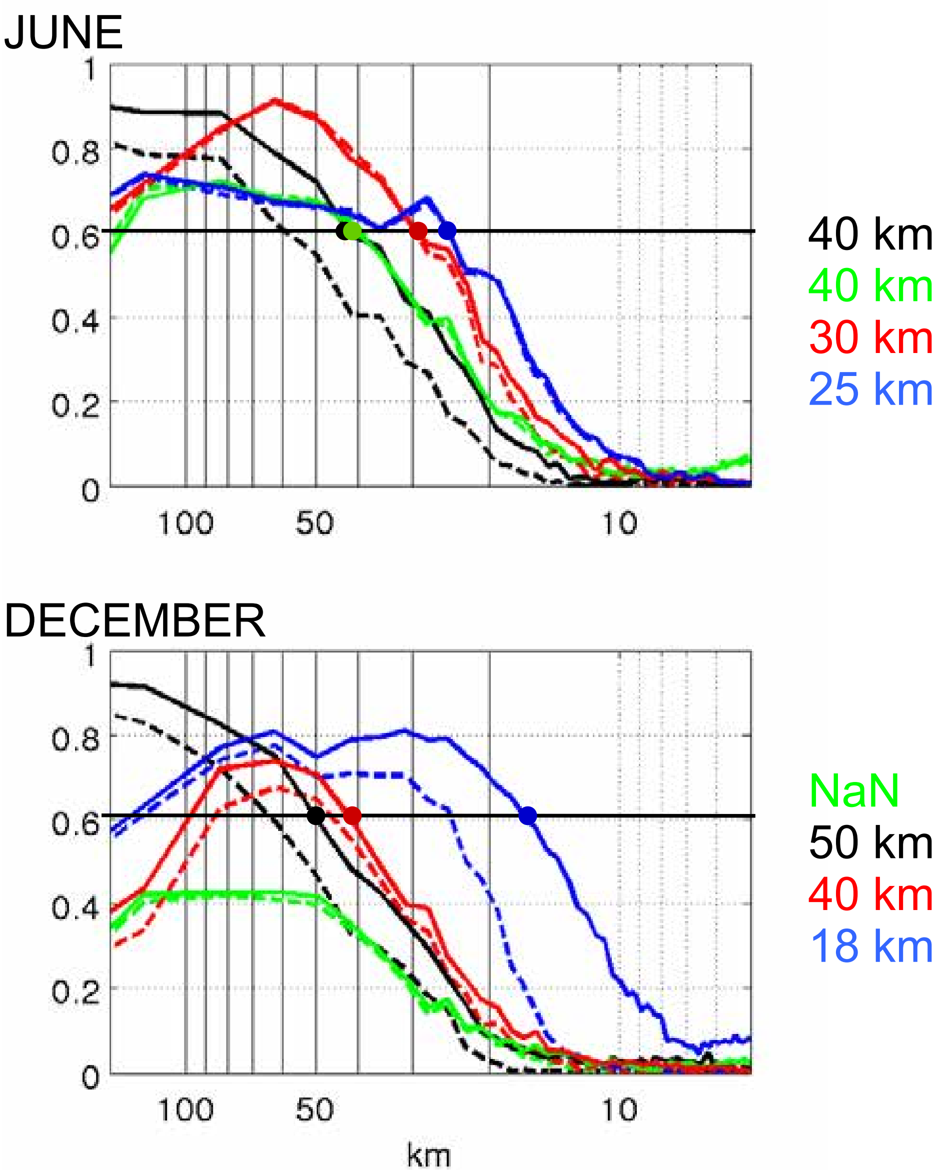


Vertical velocity on December 10th 2008



- Geostrophic and Generalized formulation very similar
- Submesoscale vertical velocities not captured, even with an ideal Geostrophic or Generalized Q vector (!)

Spectral coherence between w_{model} and ω below the mixed layer in the four regions :



→ Structures larger than 40 km are well reproduced

→ They represent 60 to 90% of the variance depending on the region

→ the reconstruction from deformation has different skills depending on the region

→ improvement due to the inclusion of the others terms (QG vs NG) is also region dependant

Conclusions

Is the Omega equation the good framework for the experimental calculation of vertical velocities ?

- ***Not really !***

→ *The omega equation doesn't reproduce well the submesoscale vertical velocity (below few tens of kilometers) in any dynamical regime.*

→ *In some regimes these small mesoscale and submesoscale (below 40 km) features account for up to 30 % of the variance of the field.*

→ The vertical velocity inferred from the omega equation represents well the mesoscale energetic patterns. Structures larger than 40 km tend to have a spectral coherence above 0.6

- ***Who is the culprit ?***

→ IGWs seem to be strongly coupled to balanced motion in the low energetic, finer scale regimes. Their contribution in the Q vector is extremely difficult to quantify (and transforms the problem in a prognostic equation).

$$\mathbf{Q}_{TD}\left(\frac{d}{dt}\left(\frac{\partial \vec{v}_{ag}}{\partial z}\right)\right)$$

- ***Consequences for SWOT-based in situ experiments of vertical velocities***

→ Energetic, « large mesoscale » region : Classical Omega equation OK. Neglecting ageostrophic (i.e., non-SWOT) contribution seems also OK.

→ « Small and sub- mesoscale » : Omega equation approach may be misleading. Possibly, the in situ strategy should be built for optimally constraining an assimilation scheme, not the omega equation.

→ Need for «ground true » of vertical velocities or fluxes (swarms of 3D drifters look quite promising) .

Conclusions

- How important are the differences between w and ω for the estimation of vertical fluxes?

- Can we investigate the (missing) **trend term** of the generalized omega equation ? $\mathbf{Q}_{TD}(\frac{d}{dt}(\frac{\partial \vec{v}_{ag}}{\partial z}))$

- ***New observationnal networks :***

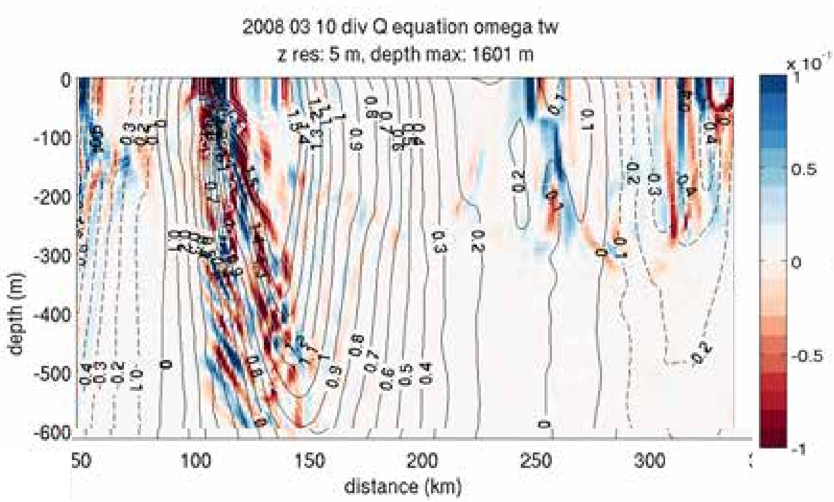
→ how is the solution impacted by a reduced resolution in subsurface while the surface information stays high resolution.

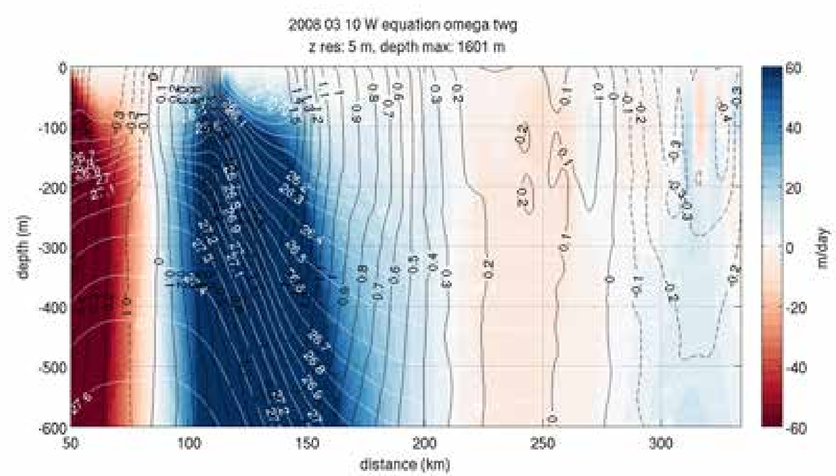
→ what kind of *in situ* information would be needed to resolve *w* depending on the regime.

- ***Q vertical variability***

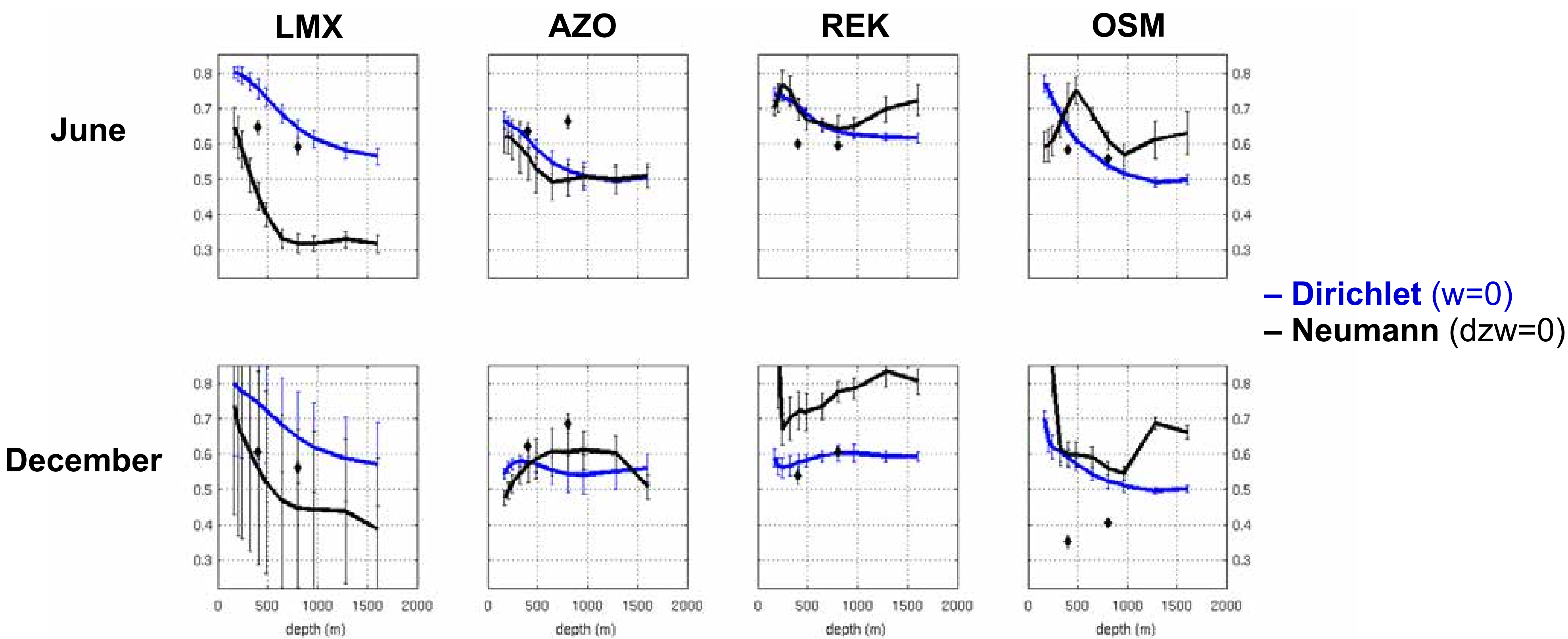
→ how to propagate the inforation on the subsurface ?

→ can vertical modes of variability be identified ?



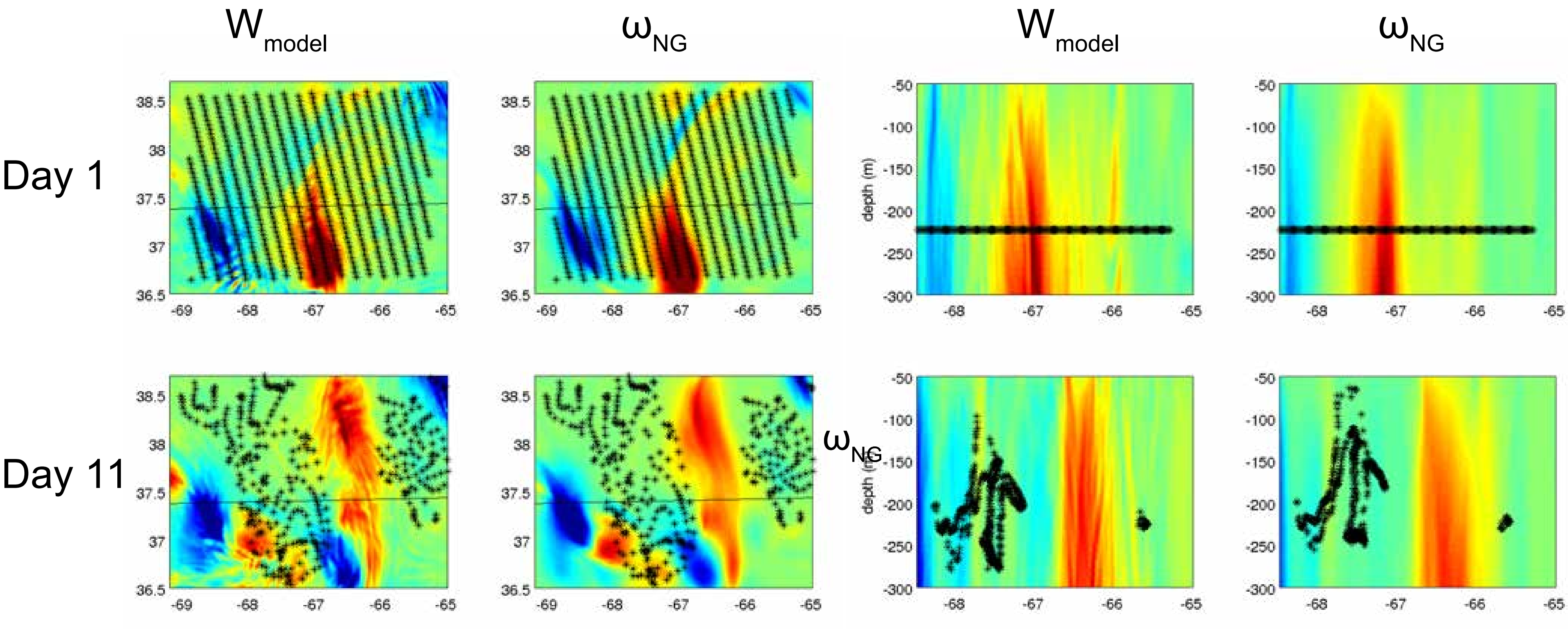


Boundary conditions



- Depending on the region, boundary conditions account for 20% to 60% of error
- Dirichlet bottom condition (w=0) is more predictable (the deeper the better)
- Neuman bottom condition ($d_z w=0$) can be much better (LMX: Gulf Stream)

Preliminary work: particule advection. M. Van Hove & A. Riad



Horizontal sections at 225 m depth

Vertical sections

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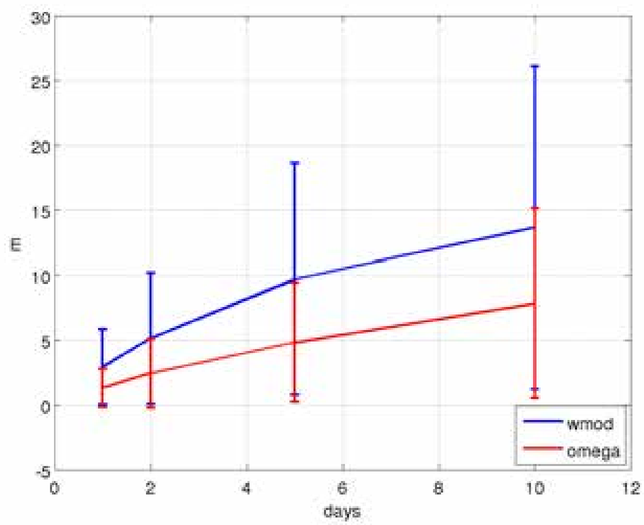
REK



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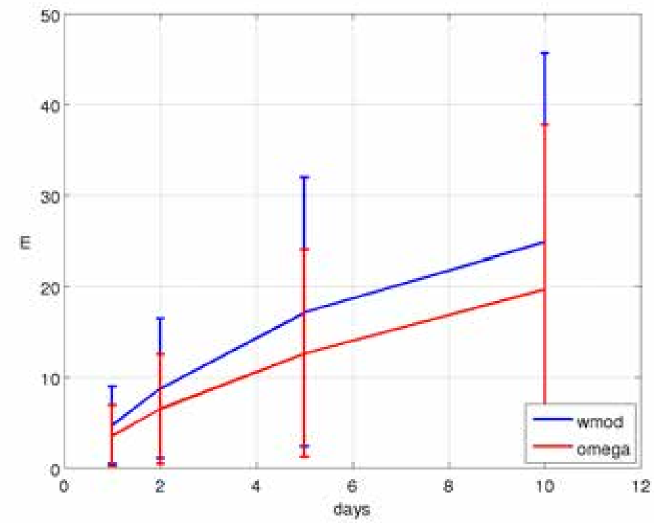

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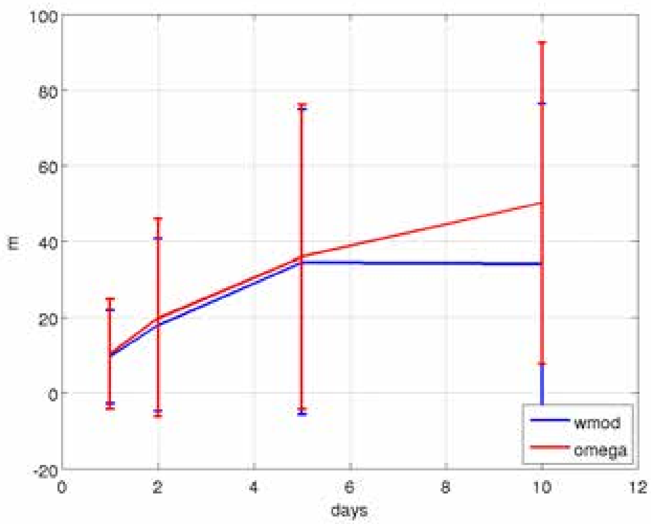

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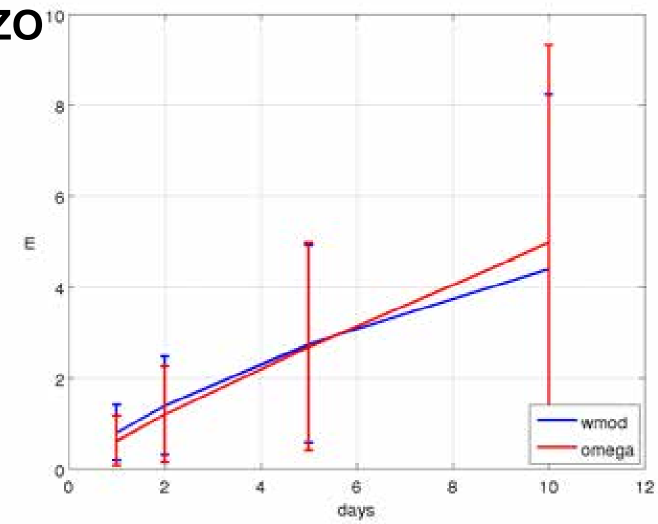

Ongoing work

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