Characterization of global internal tides at high horizontal resolution

Sam Kelly

University of Minnesota Duluth, Large Lakes Observatory & Dept. of Physics

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The Coupled-mode Shallow Water model (CSW)

Substitute $H\mathbf{u}'(\mathbf{x}, z, t) = \sum_{n=1}^{\infty} \mathbf{U}_n(\mathbf{x}, t)\phi_n(z)$ and $p'(\mathbf{x}, z, t) = \sum_{n=1}^{\infty} p_n(\mathbf{x}, t)\phi_n(z)$

Horizontal dependence (Shallow water equations)

$$\frac{\partial \mathbf{U}_n}{\partial t} - f\mathbf{k} \times \mathbf{U}_n = -H\nabla p_n$$

$$\frac{H}{c_n^2}\frac{\partial p_n}{\partial t} = -\nabla \cdot \mathbf{U}_n$$

Vertical dependence (an eigenvalue problem)

$$\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \phi_n}{\partial z}\right) + \frac{1}{c_n^2}\phi_n = 0$$

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Substitute $H\mathbf{u}'(\mathbf{x},z,t)=\sum_{n=1}^{\infty}\mathbf{U}_n(\mathbf{x},t)\phi_n(z)$ and $p'(\mathbf{x},z,t)=\sum_{n=1}^{\infty}p_n(\mathbf{x},t)\phi_n(z)$

Variable topography (Coupled shallow water equations)

$$\frac{\partial \mathbf{U}_n}{\partial t}-f\mathbf{k}\times\mathbf{U}_n=-H\nabla p_n-\sum_{m=1}^{\infty}p_m\mathbf{T}_{mn}$$

$$\frac{H}{c_n^2}\frac{\partial p_n}{\partial t}=-\nabla\cdot\mathbf{U}_n+\sum_{m=1}^{\infty}\mathbf{U}_m\cdot\mathbf{T}_{mn}+\mathbf{U}_0\cdot\nabla H\phi_n|_{-H}$$

$$\mathbf{T}_{mn}=\frac{1}{H}\int_{-H}^{0}\phi_n\nabla\phi_m\mathrm{d}z$$

Solving the system

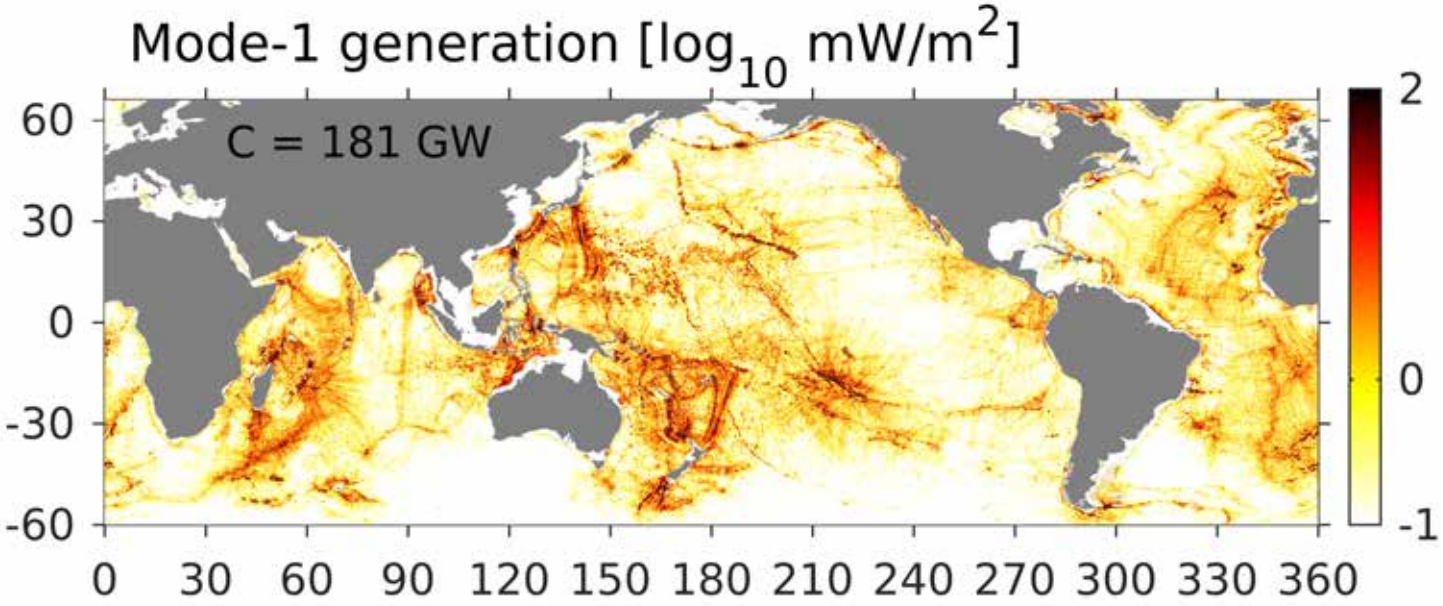
Coupled shallow water model* (CSW)

- Finite differences on a spherical C grid
- Adams-Bashforth time-stepping algorithm
- Damped by linear/quadratic drag, viscosity, or sponge
- Forced by prescribed surface tide velocities
- C code available at Bitbucket.org

Resolution	# modes	cores	RAM [GB]	speed [cycles/hr]
1/10°	4	4	6	60
1/25°	4	16	30	10
1/50°	4	128	150	11
1/100°	4	256	750	2.3

*The model is described in Kelly et al. (2016) and Griffiths and Grimshaw (2007)

How robust is internal-tide generation?



Global integrals: $C_1 = 0.18$ and $\nabla \cdot \mathbf{F}_1 = -0.09$ TW below 1000 m

Parameters: $\Delta x = 1/25°$, $N = 4$ modes, $r^{-1} = 3$ days
Inputs: TPXO tides, Smith & Sandwell bathy., HYCOM stratification

Margins are a **source** of mode-1 energy flux.

Internal-tide generation vs parameters

Horizontal resolution, Δx:

	$1/10^\circ$	$1/25^\circ$	$1/50^\circ$
C_1 [GW]	102	181	220
$-\nabla \cdot \mathbf{F}_1$ [GW]	59	93	50

Vertical resolution, # modes:

	2	4	6	8
C_1 [GW]	180	181	182	181
$-\nabla \cdot \mathbf{F}_1[GW]$	94	93	94	94

Decay time scale, r^{-1} [days]:

	0.5	1	2	4	8	16	32
C_1 [GW]	184	183	182	181	180	179	179
$-\nabla \cdot \mathbf{F}_1$ [GW]	105	105	99	88	74	61	51

Default parameters

$\Delta x = 1/25^\circ$, $N = 4$ modes, $r^{-1} = 3$ days

Internal-tide generation vs inputs

Surface tide velocities:

	TPXO	GOT	FES	HAMTIDE
C_1 [GW]	181	289	152	205
$-\nabla \cdot \mathbf{F}_1$ [GW]	93	221	56	58

Bathymetry:

	Smith & Sandwell	GEBCO
C_1 [GW]	181	211
$-\nabla \cdot \mathbf{F}_1$ [GW]	93	108

Stratification:

	HYCOM	WOA
C_1 [GW]	181	184
$-\nabla \cdot \mathbf{F}_1$ [GW]	93	88

Default inputs

TPXO tides, Smith & Sandwell bathy., HYCOM stratification

Summary

1. The Coupled Shallow Water model (CSW) code is available for download (bitbucket.org)
2. The margins are a source of mode-1 energy flux
3. $C_1 \approx 0.2$ TW and $\nabla \cdot \mathbf{F}_1 \approx -0.1$ TW, but both quantities vary by $\pm 25\%$ due to parameters and inputs

Further results (see poster):

- The energy and decay time scale determine each other

Data sources: Richard Ray (GOT), Jim Richman & Jay Shriver (HYCOM), volkov.oce.orst.edu (TPXO), www.aviso.altimetry.fr (FES), icdc.cen.uni-hamburg.de (HAMTIDE), www.nodc.noaa.gov (WOA), topex.ucsd.edu (Smith & Sandwell bathy.), www.gebco.net (GEBCO)