(Sub)Mesoscale Transport in Idealized Southern Ocean Models

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Outline

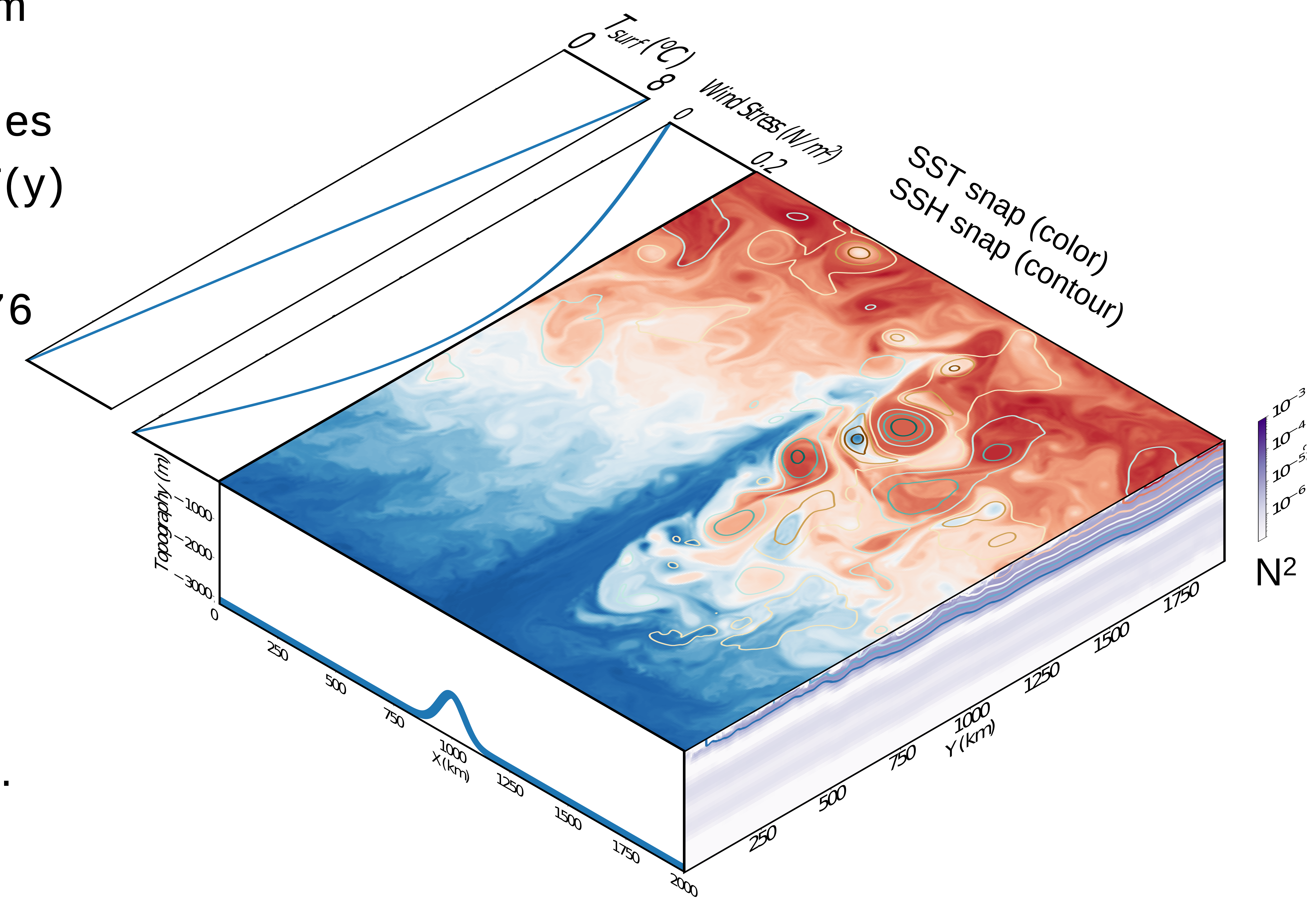
- (Sub)mesoscale vertical fluxes [Balwada et al 2018]
- Seasonal iron fluxes in idealized Southern Ocean [Uchida et al 2019]
- Ongoing work [reconstructing vertical fluxes from SSH]
- (Reconstruction of full eddy flux tensor in 3D [Balwada et al 2019])

Experimental Setup

- MITgcm, 2000^2km x 3km @ 50^{o}S
- Channel with no-slip sides
- SST restored to linear T(y)
- No salinity, linear EOS
- LLC4320 vertical grid (76 levs)
- LLC4320 params
- Quadratic drag, Leith dissipation
- 150 year spin-up

Resolution: **20, 5, 1 km**.

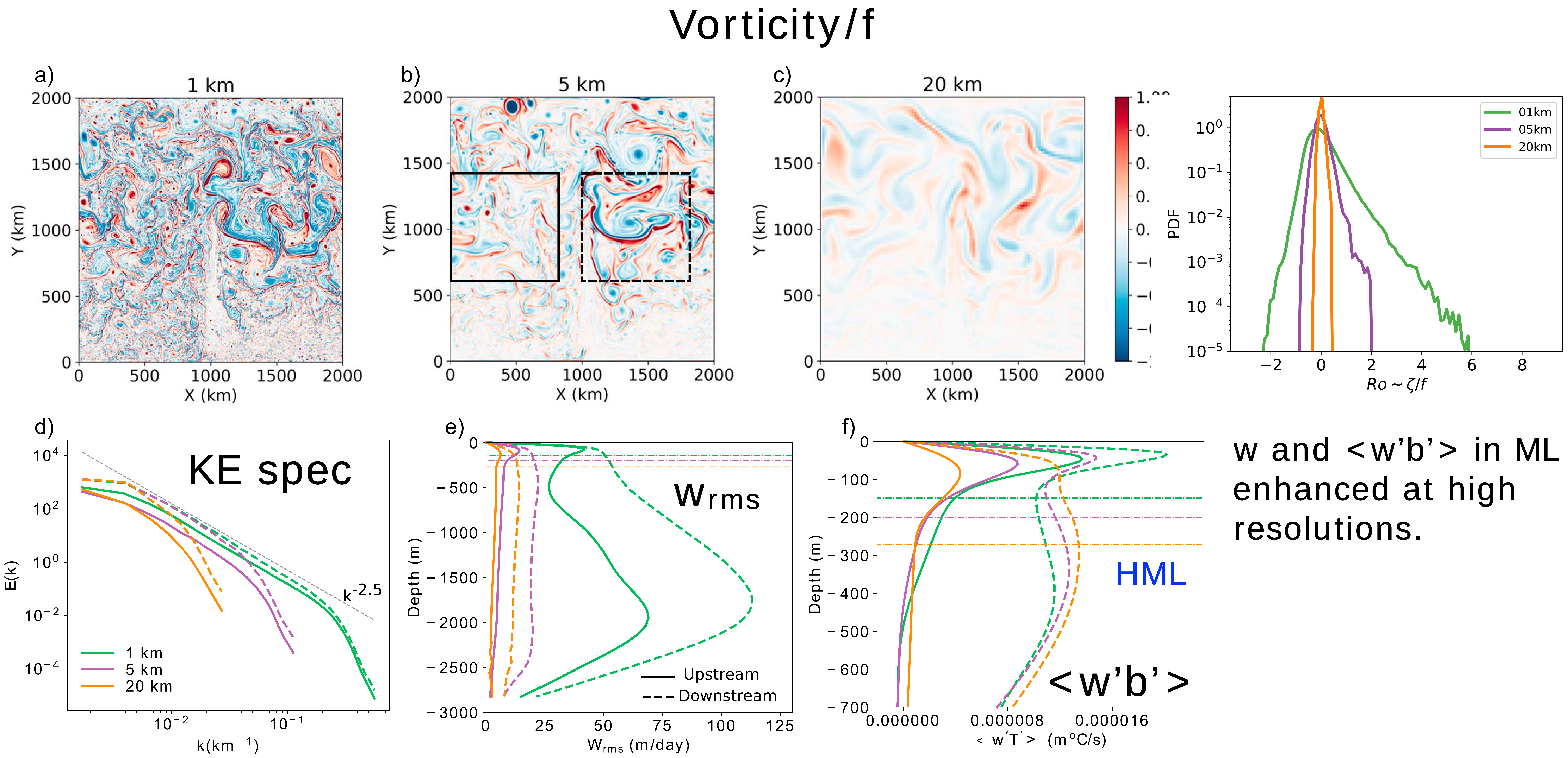
Tracer restored at surface.



Vertical fluxes with increasing resolution,

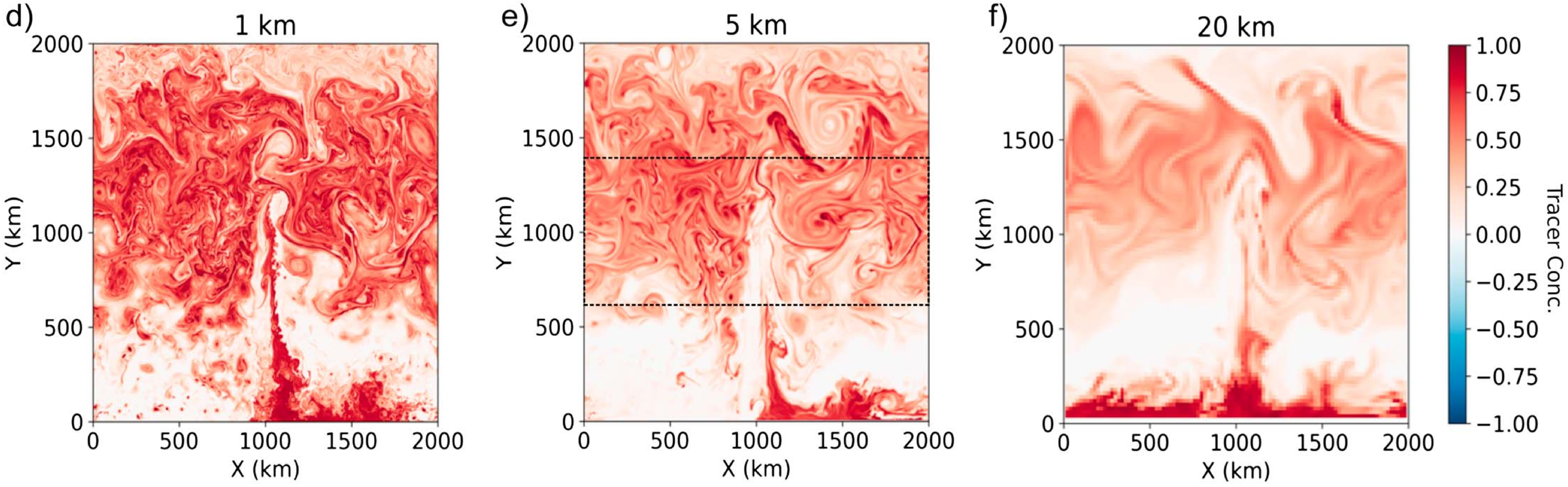
but no seasonal variation

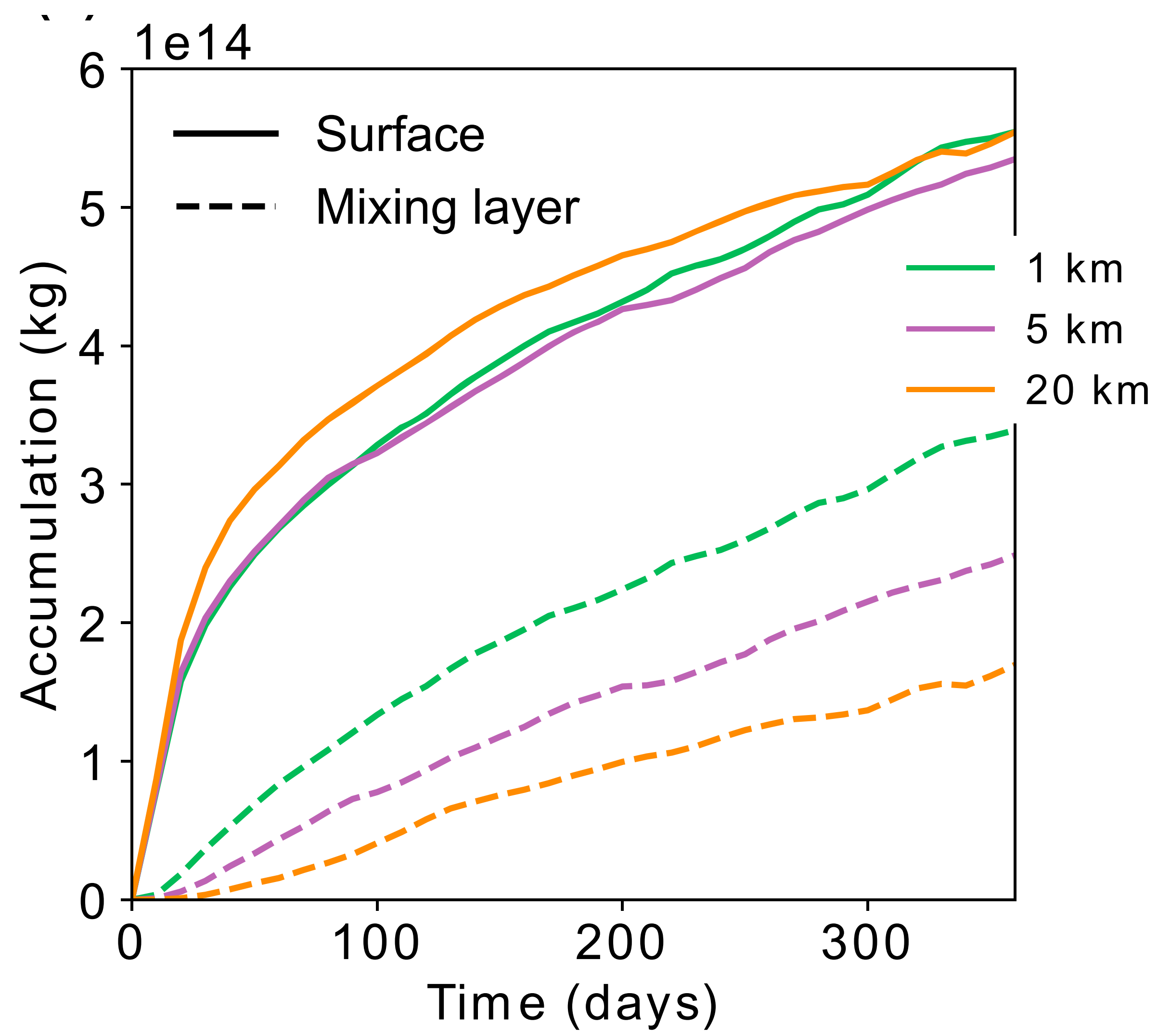
Flow characteristics



w and <w'b'> in ML enhanced at high resolutions.

Tracer accumulation

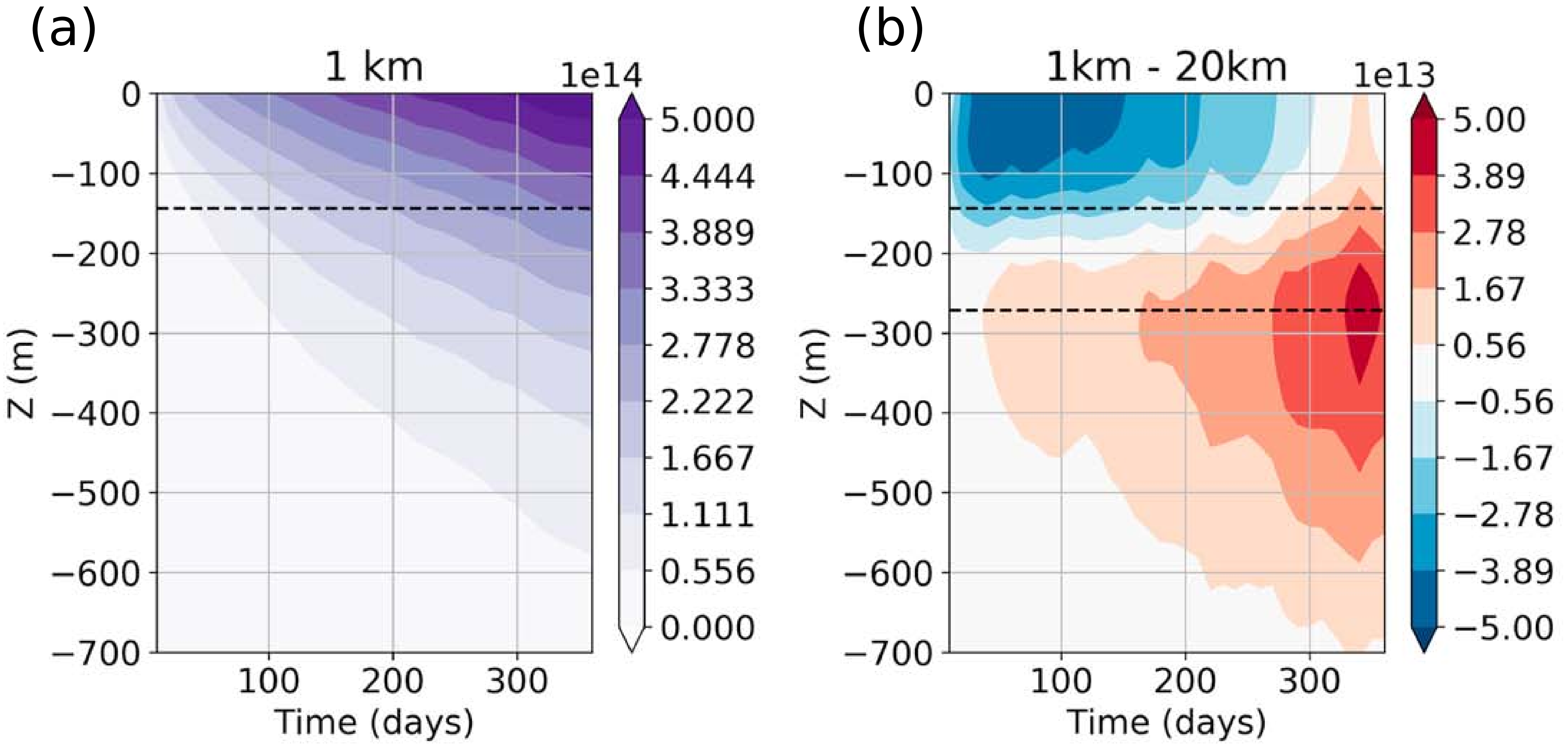




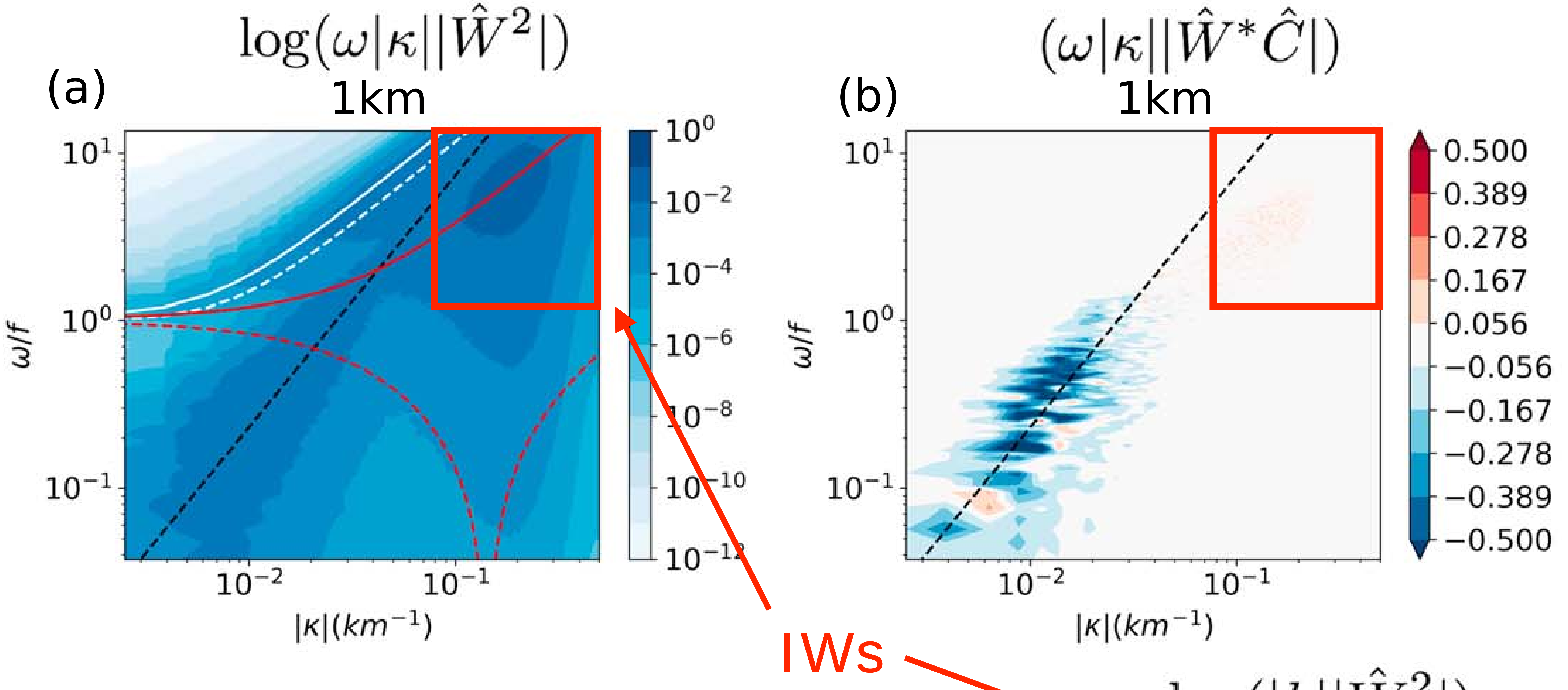
$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = \nabla \cdot (\kappa \nabla C) + \frac{\partial F_c}{\partial z},$$

$$F_c(z=0) = k_i[C(z=0) - C_{atm}]$$

k_i ~ piston velocity for CO_2

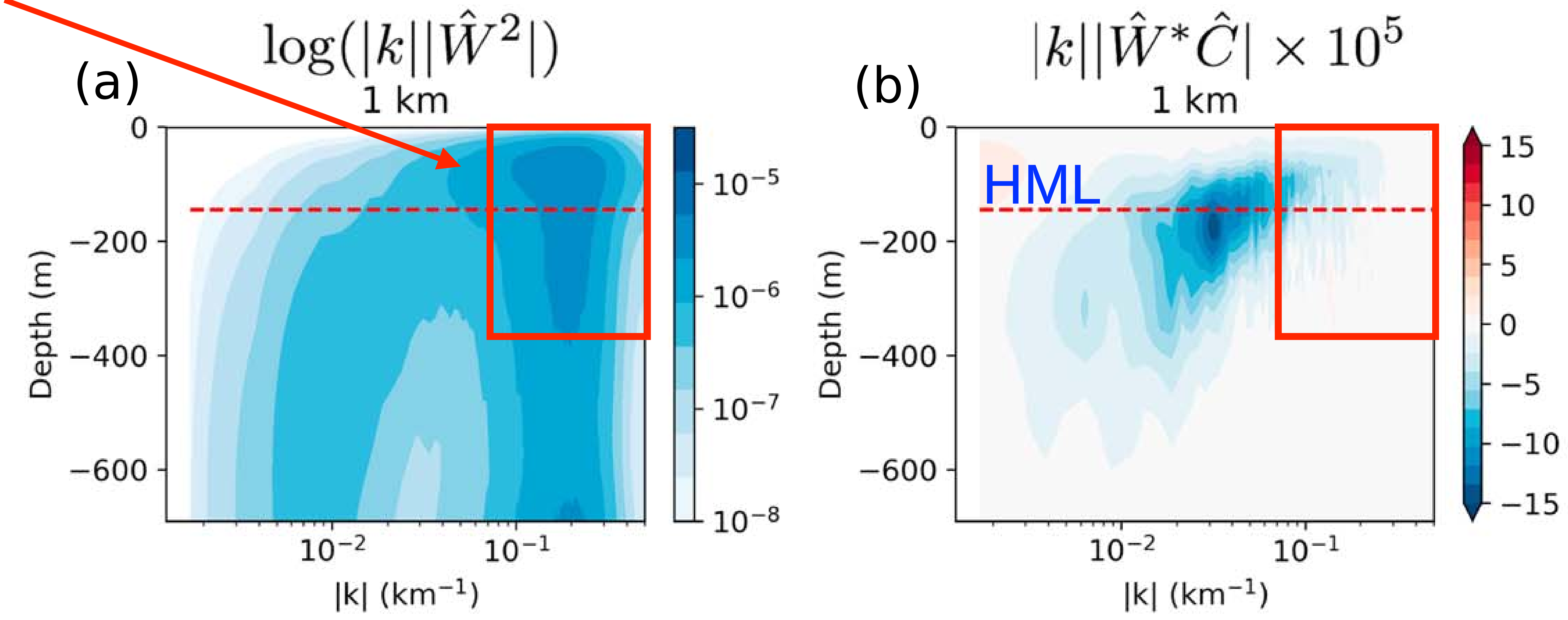


Cross-spectra of vertical velocity and tracer flux



Variance-preserving spectra of w2 and w'C' at 400 m, averaged over days 40–180 (after tracer release), in upstream region, as a function of wavenumber and frequency/f

Azimuthally and time-averaged (over days 40–180) variance-preserving spectra as a function of wavenumber and depth

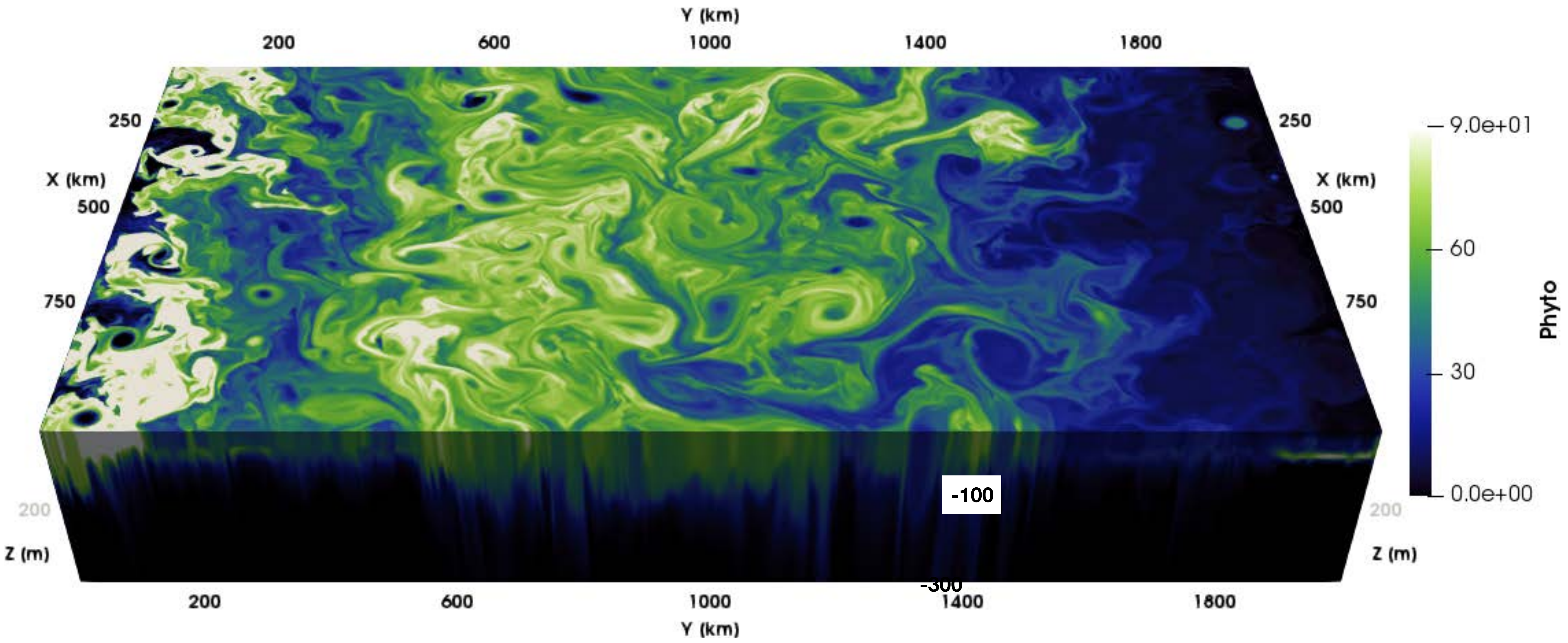


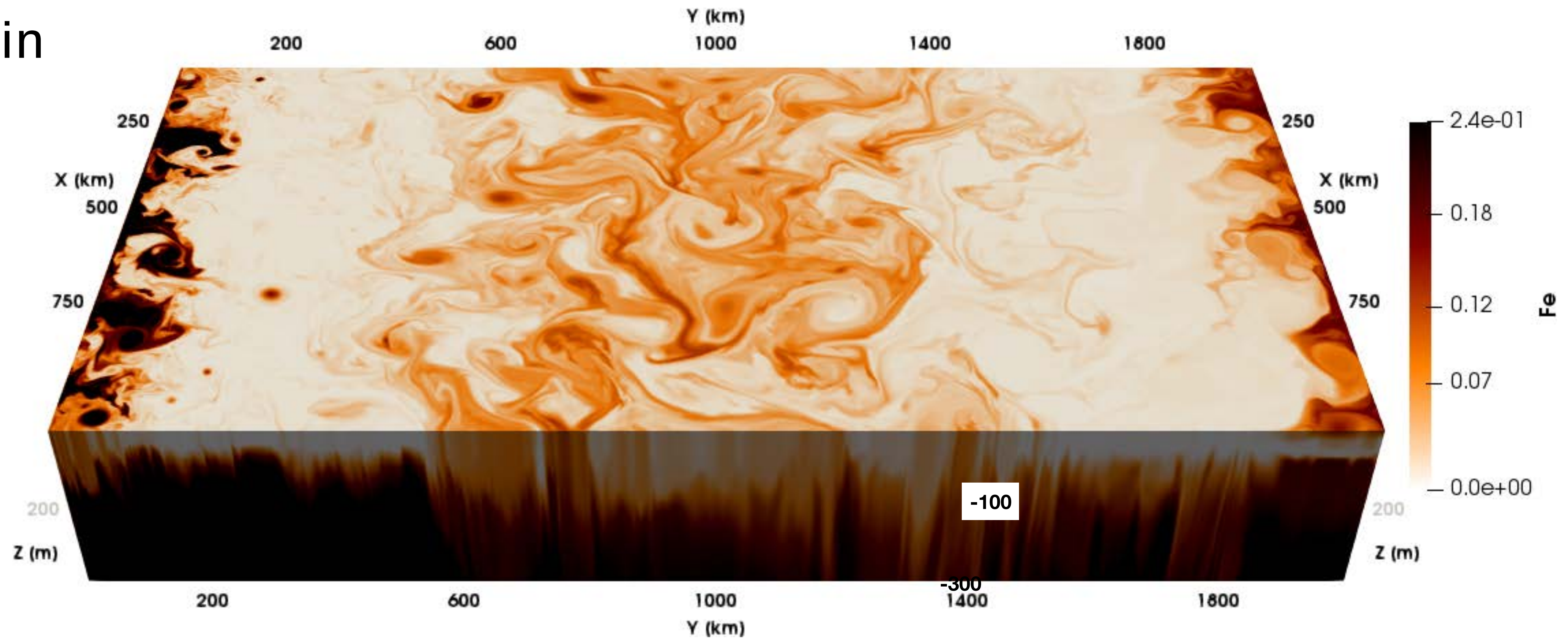
Vertical fluxes with increasing resolution,

with seasonality and a biogeochemistry model

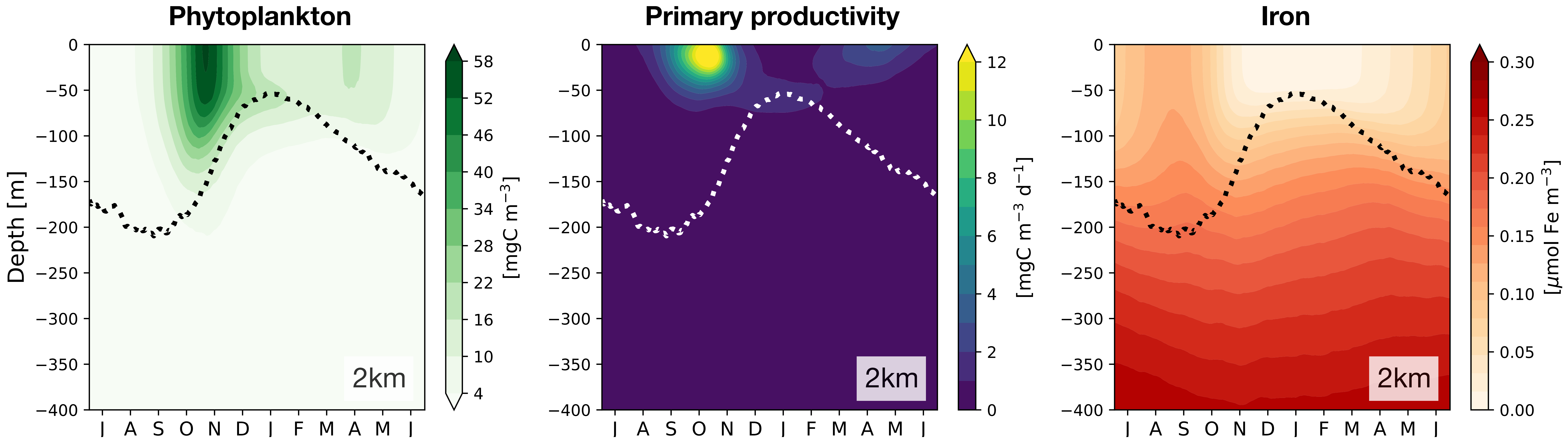
Biomass & iron

- 2000^2km x 3km @ 50°S
- Focus on 2km resolution
- Seasonally-varying temperature restoring and windstress
- Simplified Darwin BGCM
- Nutrient forcing by restoration in sponge layer in northern 100km of domain
- Iron-limited throughout
- Basically, seasonal, and tracer forced from below instead of surface



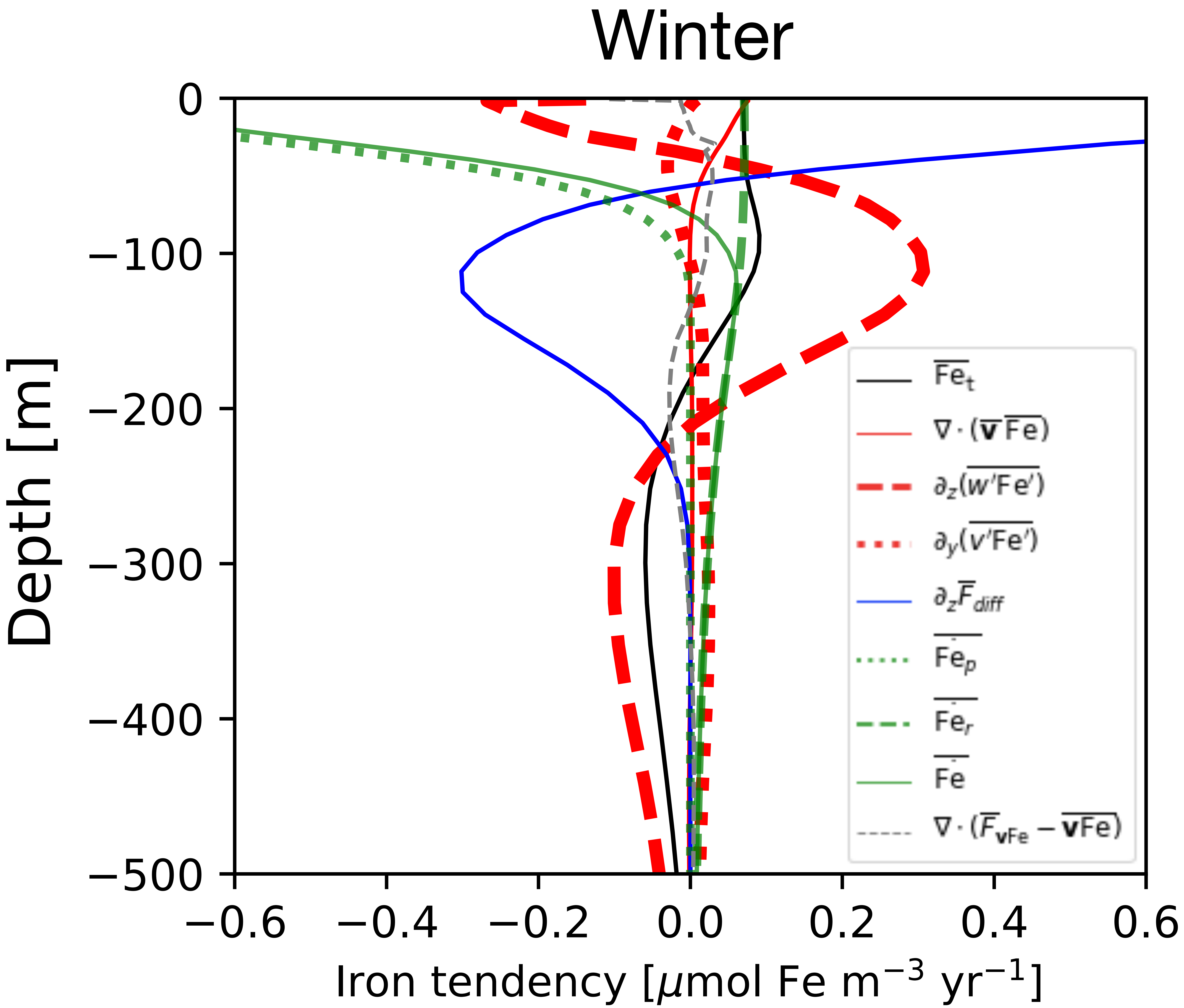


Seasonal productivity & uptake



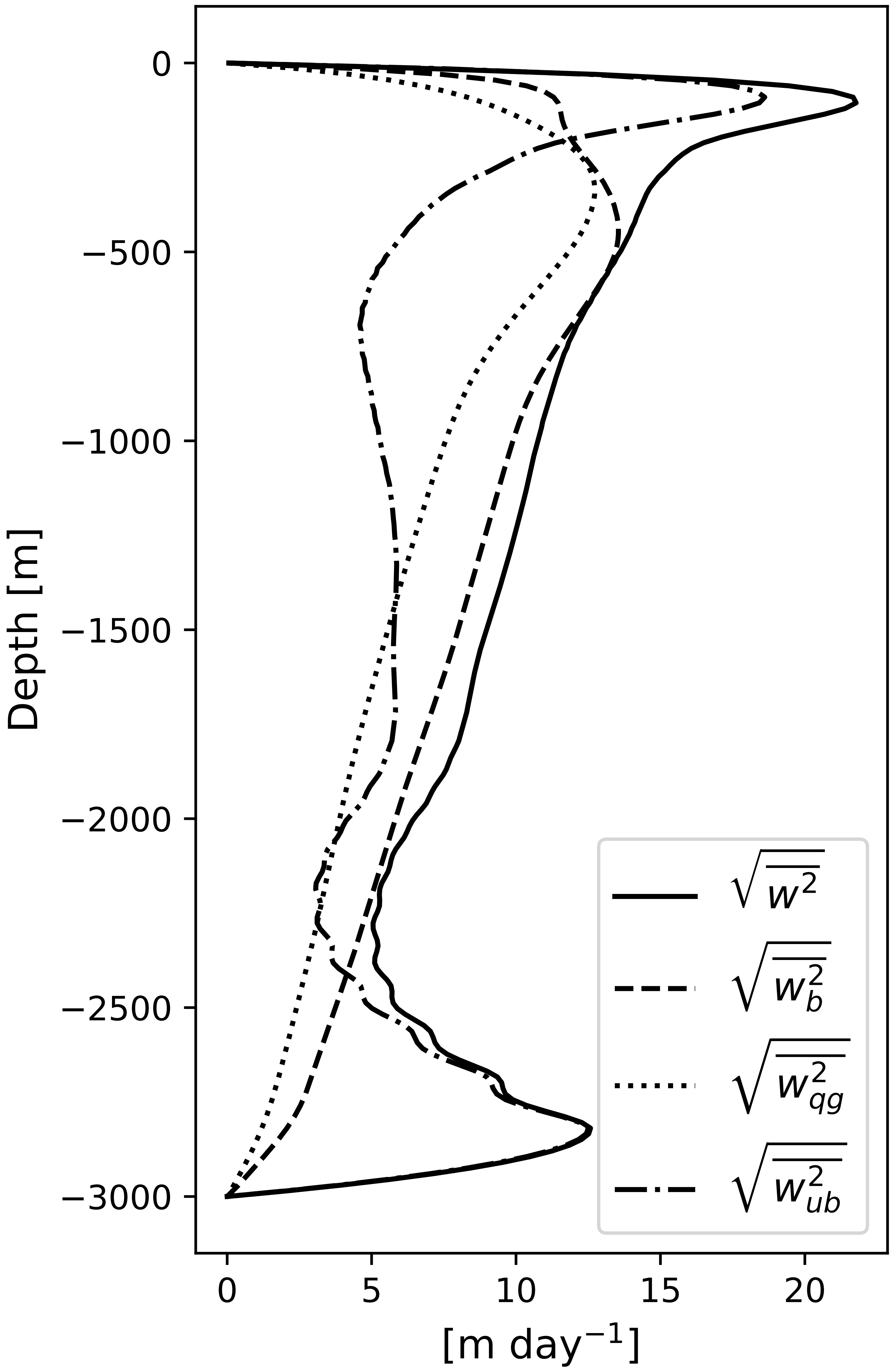
- Configured to represent the iron-limited ecosystem in the Southern Ocean.
- A strong spring bloom around Oct.-Dec.
- Our interest is in quantifying the eddy transport of iron.

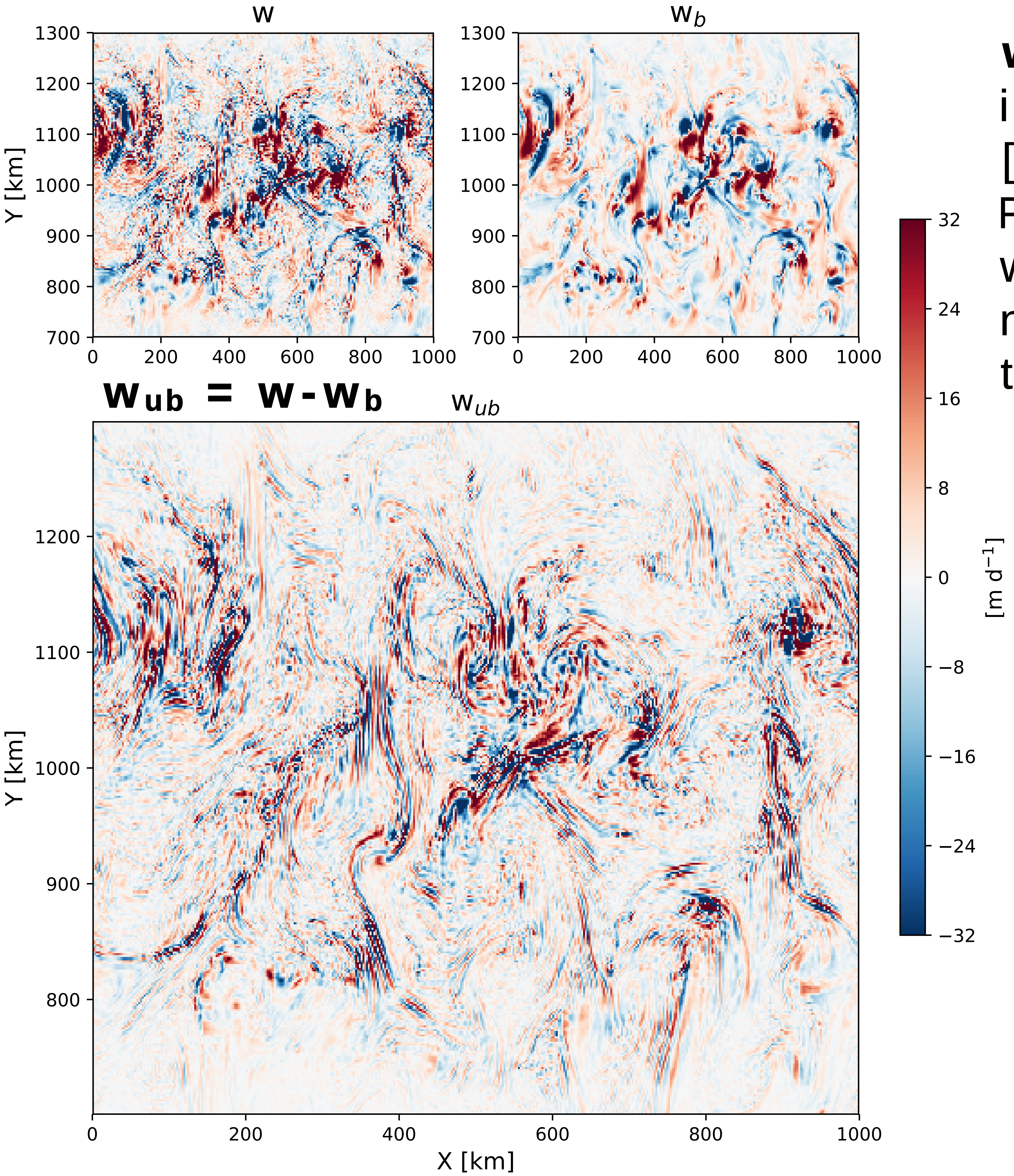
Iron budget



- Vertical eddy iron transport (red dashed) is first-order importance in calculating the iron budget.
- Diffusive flux (blue solid) is large within the mixing layer (top 200m).

Vertical velocity





$\mathbf{w_{ub}} = \mathbf{w} - \mathbf{w_b}$

$\mathbf{w_b}$: H.O. inversion [Giordani & Planton 2000 with some negl. terms]

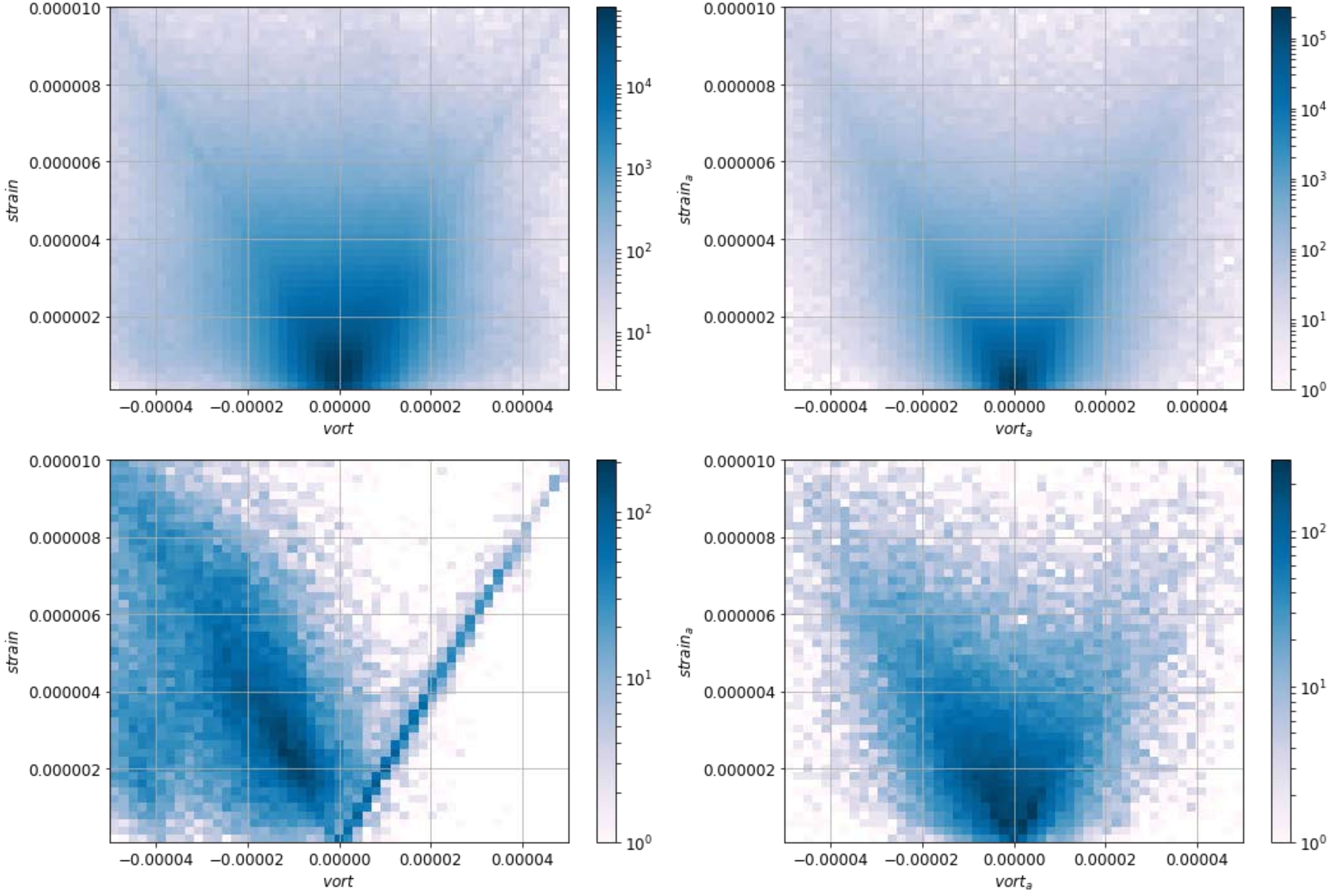
Cross spectra

Strain-Vorticity

Full

Ageostrophic

Strain



Vorticity

Full domain

Within fronts (defined by sharpness threshold)

Currently working on associating submesoscale features with fluxes

3D reconstruction of eddy flux tensor

Measuring eddy fluxes

Consider a modeled tracer c(x,y,z,t) advected by non-divergent flow **v**(x,y,z,t):

$$\partial_t c + \nabla \cdot (\mathbf{v} c) = 0, \quad \text{with} \quad \nabla \cdot \mathbf{v} = 0$$

Reynold's averaged equation is

$$\boxed{\partial_t \overline{c} + \nabla \cdot (\overline{\mathbf{v}}\,\overline{c}) = -\nabla \cdot \mathbf{F}^c \qquad \mathbf{F}^c \equiv \overline{\mathbf{v}' c'}}$$

$$\overline{\overline{(\ )}} = \overline{(\ )} \text{ and } \overline{(\ )'} = 0.$$

Mean fields are resolved fields. **Affected only by divergence of flux.** Though eddy variance is affected by full flux:

$$\partial_t \left(\frac{\overline{c'^2}}{2} \right) + \nabla \cdot \left(\overline{\mathbf{v} \frac{c'^2}{2}} \right) = -\nabla \overline{c} \cdot \mathbf{F}^c$$

Parametrizations of divergent flux assume down-gradient diffusion. Full flux has rotational part:

$$\mathbf{F}^c = \nabla \chi + \nabla \times \boldsymbol{\phi}$$

Connecting 'measured' flux to parameterization: remove rotational part? **No unique solution**

Measuring eddy fluxes: Method of Multiple Tracers

N tracers $c_j(x,y,z,t)$, j = 1:N, each advected by non-divergent resolved flow:

$$\partial_t \bar{c}_j + \bar{\mathbf{v}} \cdot \nabla \bar{c}_j = -\nabla \cdot (\overline{\mathbf{v}' c_j'}) \equiv \nabla \cdot (\mathbf{K} \nabla \bar{c}_j)$$

Measure fluxes & mean gradients => over-determined problem for **K**:

$$\underset{3 \times 3}{\mathbf{K}} \; \underset{3 \times N}{\nabla \bar{c}_j} = \underset{3 \times N}{-\overline{\mathbf{v}' c_j'}}$$

If non-parallel mean tracer gradients can be maintained, then least-squares provides an optimal solution (Plumb & Mahlman 1987; Bachman, Fox-Kemper & Bryan 2015).

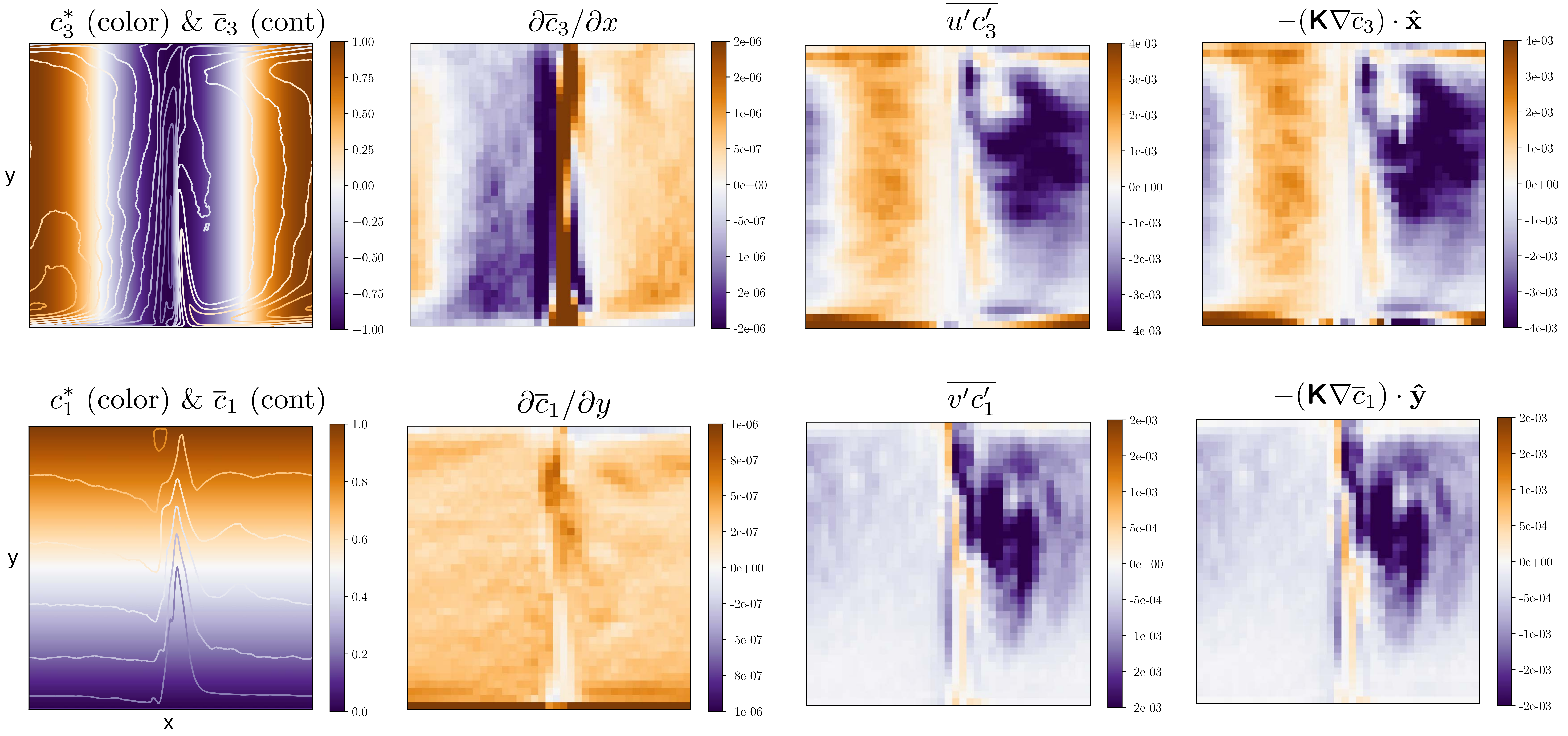
N=10 tracers, run 50 years, restored to a **target fields:** RHS term $-\tau^{-1}(c_j - c_j^*)$, τ = 6 years

$$c_1^* = y/L \qquad c_2^* = -z/H \qquad c_3^* = \cos(2\pi x/L) \qquad c_4^* = \sin(2\pi x/L) \qquad c_5^* = \sin(4\pi x/L)$$
$$c_6^* = \sin(\pi y/L) \qquad c_7^* = \cos(2\pi y/L) \qquad c_8^* = \sin(2\pi y/L) \qquad c_9^* = \cos(\pi z/H) \qquad c_{10}^* = \sin(\pi z/H)$$

Average: Full time average + lateral spatial coarse-graining over 50km boxes.

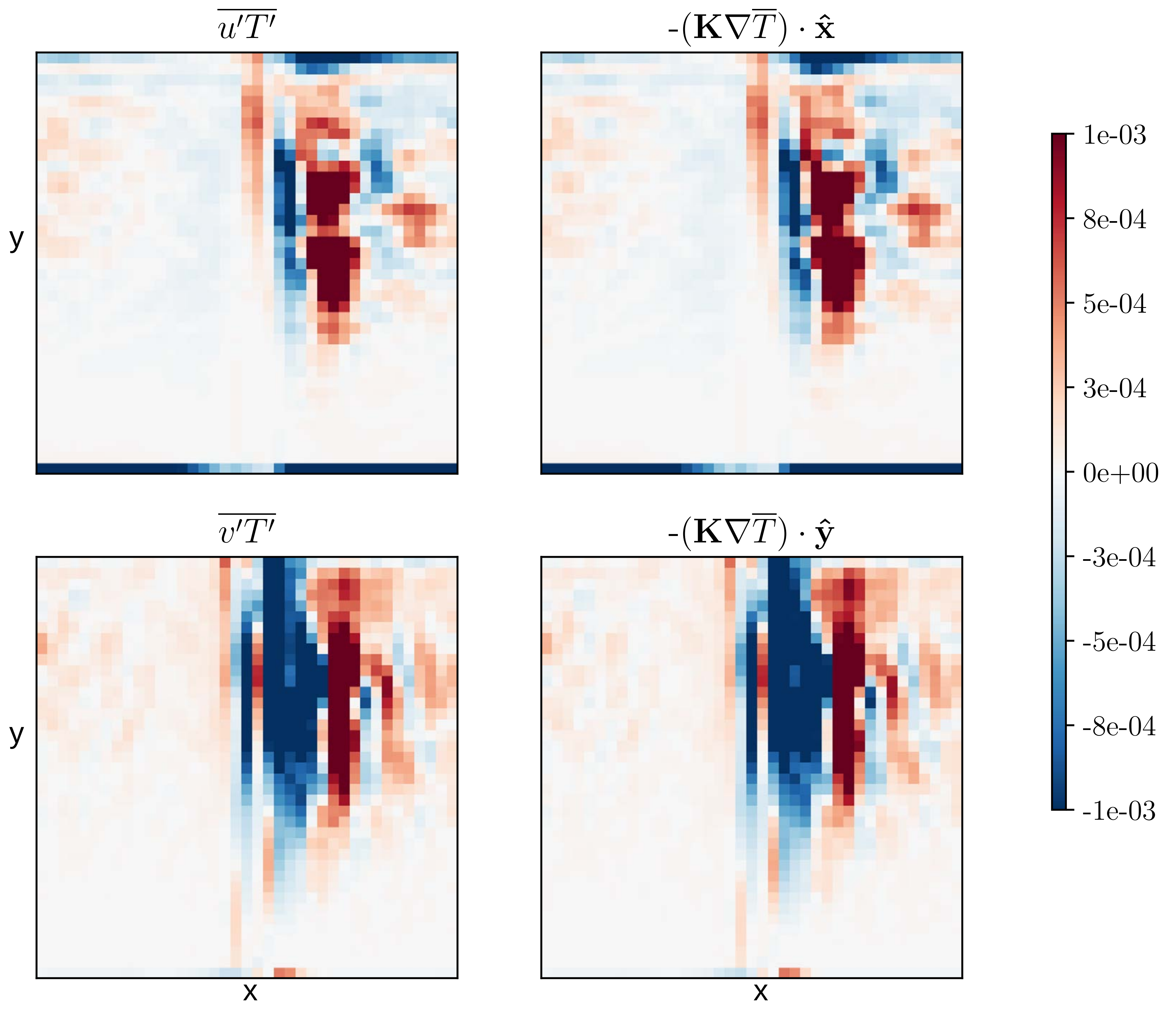
Can measured K reconstruct fluxes?

Consider c_3 and c_1: target fields have x and y gradients, resp. Mean gradients are retained (col 2).
Eddy fluxes in dominant gradient directions (col 3) are well-reconstructed by **K** (col 4).   [z = 1500m]

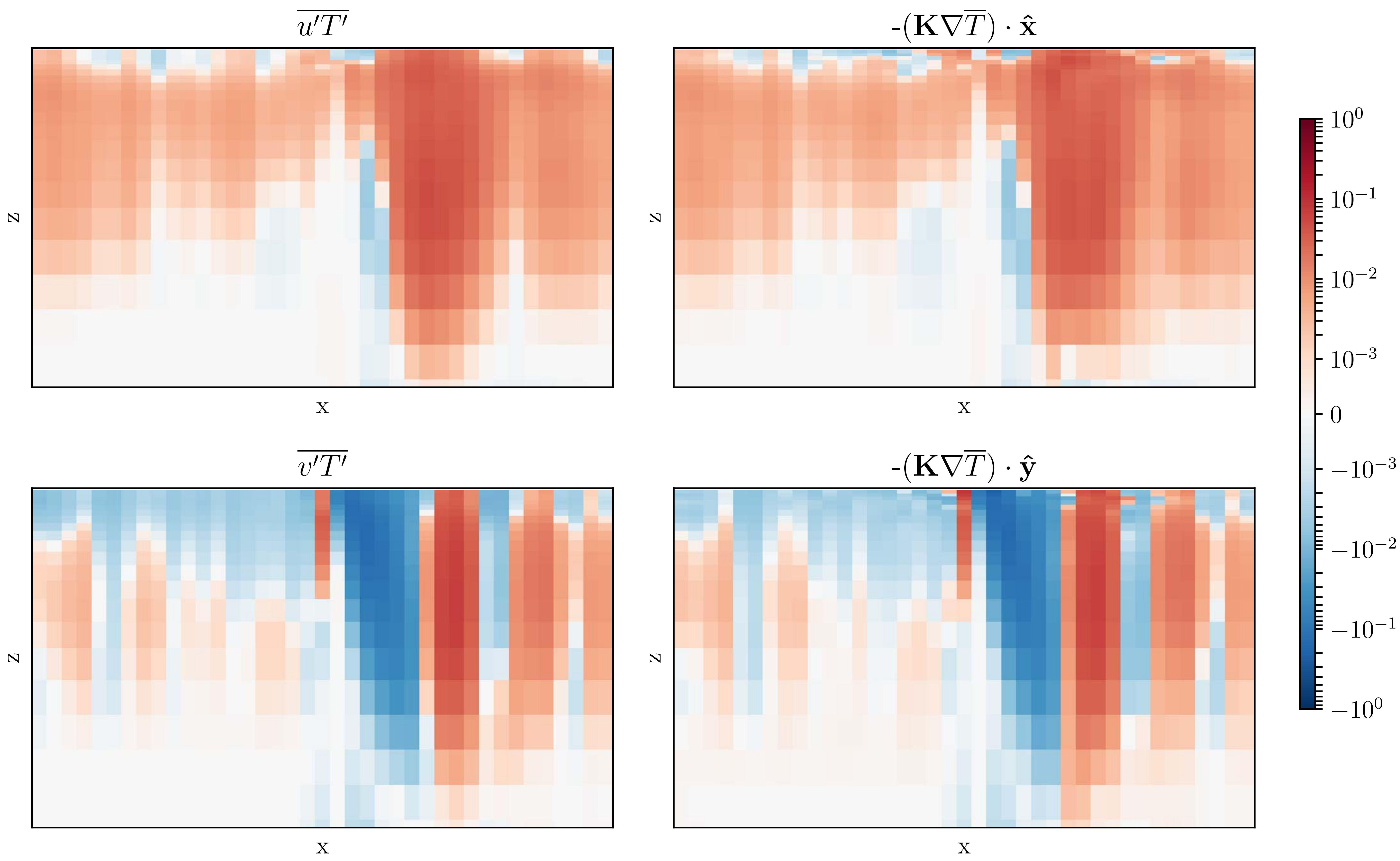


Harder test: Can K reconstruct buoyancy flux?

T-fluxes at z = -1500m



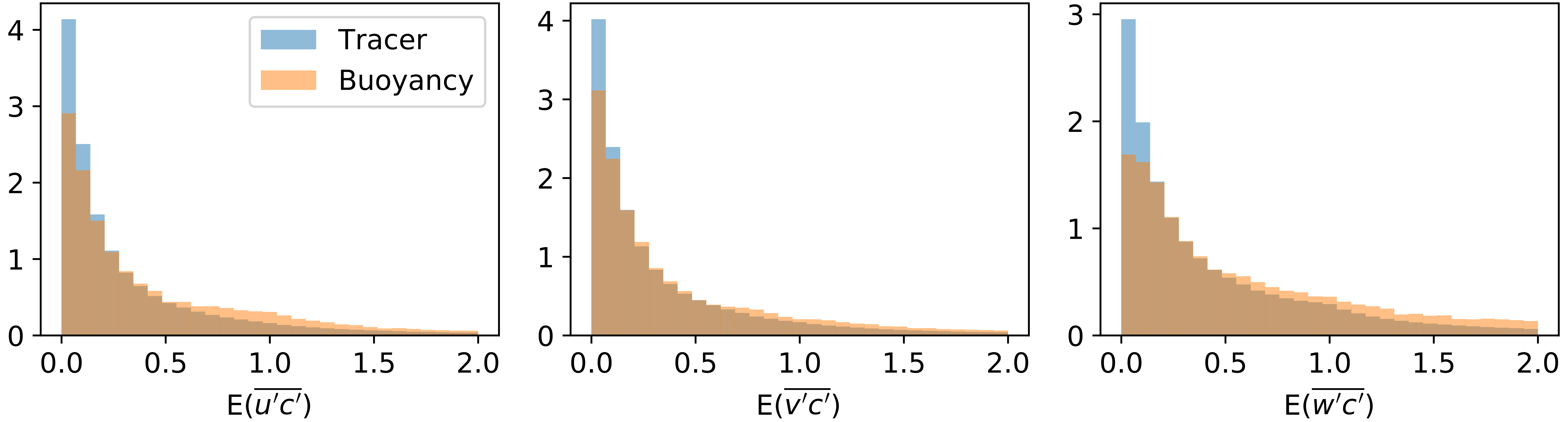
T-fluxes at y = 1000 km



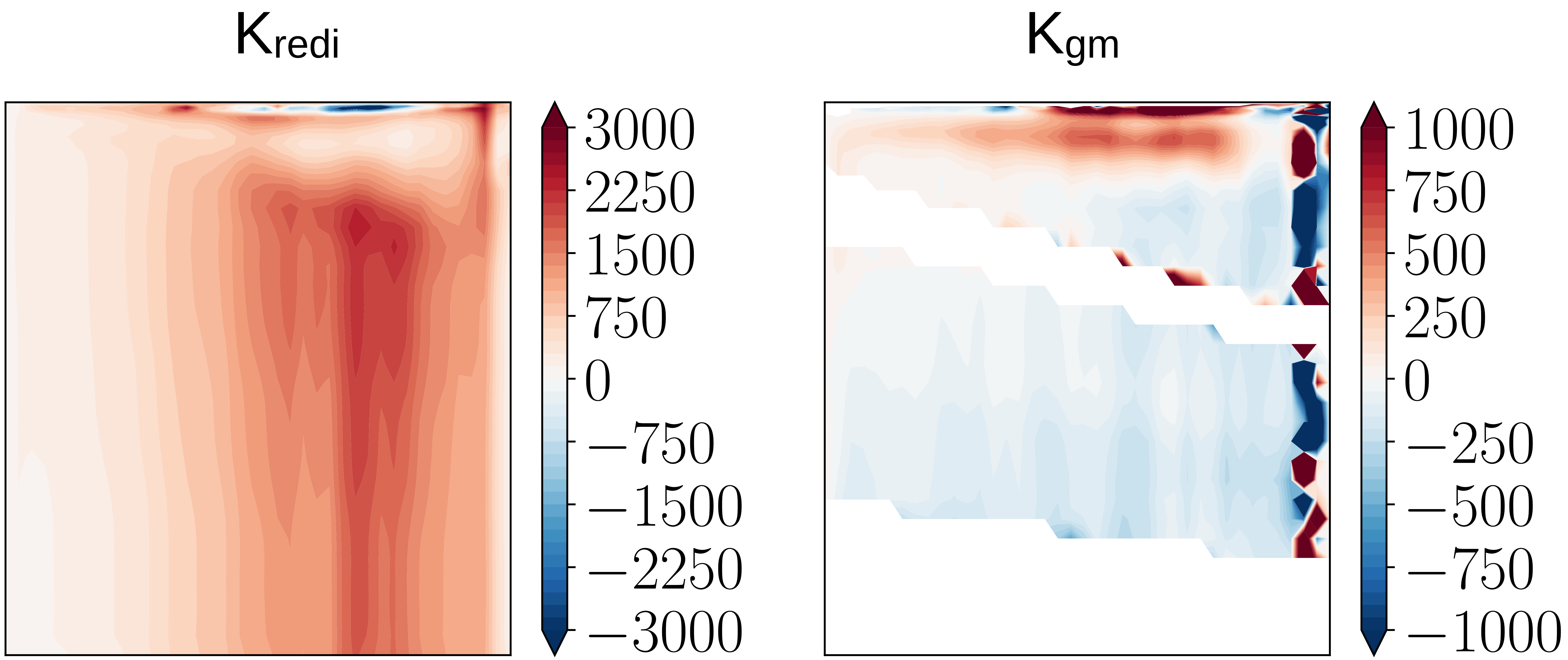
But … is it *really* good enough?

Flux reconstruction error: For each tracer c_j, and buoyancy (temperature T), flux error is computed at each point in domain as:

$$E(\text{Flux}) = |\text{Flux} - \text{Flux}_{\text{recon}}|/|\text{Flux}|$$

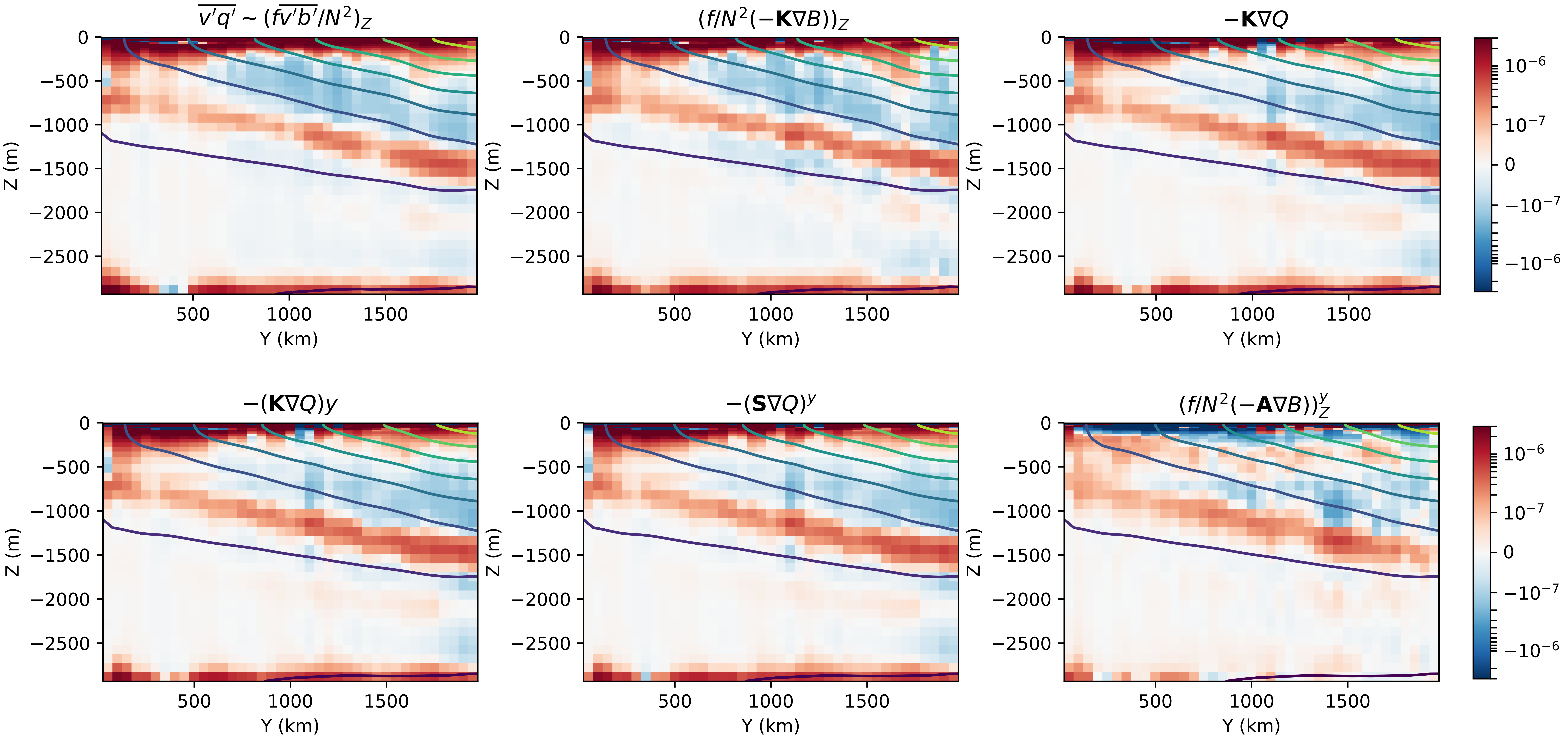


What do diffusivities look like?



Check whether PV flux mostly due to buoyancy flux…

Check whether predicted dynamical connection holds



Breadcrumb trail

- Need eddy fluxes to get oxygen (& other climate tracers) right
- Parameterized climate models with $\kappa_{\text{redi}} = \kappa_{\text{gm}}$ have κ_{redi} too small
- Tuning GCM to get stratification right requires $\kappa_{\text{gm}} \sim 500$ m/s^2
- Tracer diffusivity estimates from models and obs: $\kappa_{\text{redi}} \sim 5000$ m/s^2
- Model and obs: Both diffusivities strongly depth-dependent
- From above, $\kappa_{\mathrm{redi}} \partial_z \boldsymbol{s} \approx \partial_z \left(\kappa_{\mathrm{gm}} \boldsymbol{s}\right)$ and $\kappa_{\mathrm{redi}} \approx \kappa_q$
- >> Set κ_{redi} via theory for QGPV flux, and integrate to get κ_{gm}

$$\boxed{\kappa_{\mathrm{gm}}(z)\boldsymbol{s}(z) = \kappa_{\mathrm{gm}}(0)\boldsymbol{s}(0) - \int_z^0 \kappa_{\mathrm{redi}}(z')\partial_z \boldsymbol{s}\, dz'}$$