Dynamical separation of stationary and non-stationary internal tides

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Roadmap

1. Introduce an internal-tide model with first-order meanflow effects

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2. Show that meanflow effects are significant, but do not dissipate the tide
3. Present dynamical equations for the stationary tide
4. Show that meanflow effects explain the decay of the stationary tide
5. Parameterize meanflow effects with an eddy diffusivity, and apply the parameterization to the global ocean

The Coupled-mode Shallow Water model (CSW)

Substitute $H\mathbf{u}'(\mathbf{x}, z, t) = \sum_{n=1}^{\infty} \mathbf{U}_n(\mathbf{x}, t)\phi_n(z)$ and $p'(\mathbf{x}, z, t) = \sum_{n=1}^{\infty} p_n(\mathbf{x}, t)\phi_n(z)$

Horizontal dependence (Shallow water equations)

$$\mathbf{U}_{nt} + f\mathbf{k} \times \mathbf{U}_n = -H\nabla p_n$$

$$\frac{Hp_{nt}}{c_n^2} = -\nabla \cdot \mathbf{U}_n$$

Vertical dependence (a time-independent eigenvalue problem)

$$\frac{\partial}{\partial z}\left(\frac{1}{N^2}\frac{\partial \phi_n}{\partial z}\right) + \frac{1}{c_n^2}\phi_n = 0$$

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Variable topography (Coupled shallow water equations)

$$\mathbf{U}_{nt} + f\mathbf{k} \times \mathbf{U}_n = -H\nabla p_n - \sum_{m=0}^{\infty} p_m \mathbf{T}_{mn}$$

$$\frac{Hp_{nt}}{c_n^2} = -\nabla \cdot \mathbf{U}_n + \sum_{m=0}^{\infty} \mathbf{U}_m \cdot \mathbf{T}_{mn}$$

Topographic coupling coefficients (where H and ϕ vary with $\mathbf{x}$)

$$\mathbf{T}_{mn} = \frac{1}{H}\int_{-H}^{0} \phi_n \nabla \phi_m \mathrm{d}z$$

The Coupled-mode Shallow Water model (CSW)

Leading-order meanflow interaction

$$\mathbf{U}_{nt} + \sum_{m=0}^{\infty} \nabla \cdot \left(\overline{\mathbf{u}}_{mn}^{T} \mathbf{U}_{m}\right) + f\mathbf{k} \times \mathbf{U}_{n} = -H\nabla p_{n} - \sum_{m=1}^{\infty} p_{m} \mathbf{T}_{mn}$$

$$\frac{Hp_{nt}}{c_n^2} + \sum_{m=0}^{\infty} \nabla \cdot \left(\frac{\overline{\mathbf{u}}_{mn} H p_m}{c_n^2}\right) + \frac{\delta c_n^2}{c_n^2} \nabla \cdot \mathbf{U}_n = -\nabla \cdot \mathbf{U}_n + \sum_{m=0}^{\infty} \mathbf{U}_m \cdot \mathbf{T}_{mn}$$

Meanflow coupling coefficients

$$\overline{\mathbf{u}}_{mn}(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^{0} \overline{\mathbf{u}}(\mathbf{x}, z, t) \phi_m \phi_n \mathrm{d}z$$

$$\delta c_n^2(\mathbf{x}, t) = \frac{1}{H} \int_{-H}^{0} \delta N^2(\mathbf{x}, z, t) \Phi_n \Phi_n \mathrm{d}z$$

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$$\frac{Hp_{nt}}{c_n^2} + \sum_{m=0}^{\infty} \nabla \cdot \left(\frac{\overline{\mathbf{u}}_{mn} Hp_m}{c_n^2} \right) + \frac{\delta c_n^2}{c_n^2} \nabla \cdot \mathbf{U}_n = -\nabla \cdot \mathbf{U}_n + \sum_{m=0}^{\infty} \mathbf{U}_m \cdot \mathbf{T}_{mn}$$

Assumptions

- Small Rossby and Froude number ($\epsilon = U/fL \ll 1$)*
- Geometric approximation (for dynamical stability)
- Simultaneous meanflow and topographic effects are weak

*See asymptotic derivation by Wagner, Ferrando, and Young (2017)

Solving the system

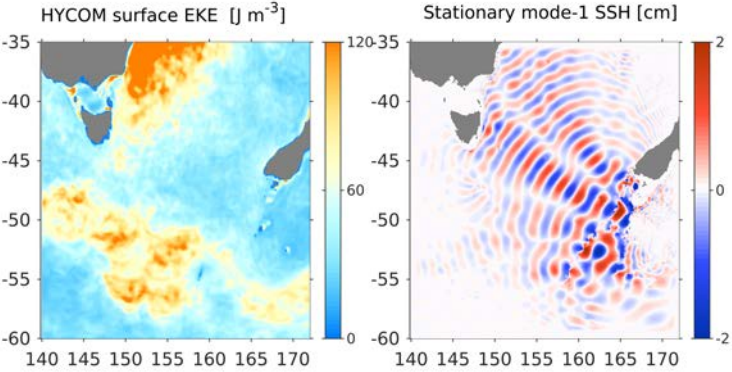
Coupled shallow water model* (CSW)

- Finite differences on a spherical C grid
- Adams-Bashforth time-stepping algorithm
- Damped by linear/quadratic drag, viscosity, or sponge
- Forced by prescribed surface tide velocities
- C code without meanflow available at Bitbucket.org
- Matlab code with meanflow available by email

Resolution	# modes	cores	RAM [GB]	speed [cycles/hr]
$1/25^\circ$	4	16	30	10
$1/50^\circ$	4	128	150	11
$1/100^\circ$	4	256	750	2.3

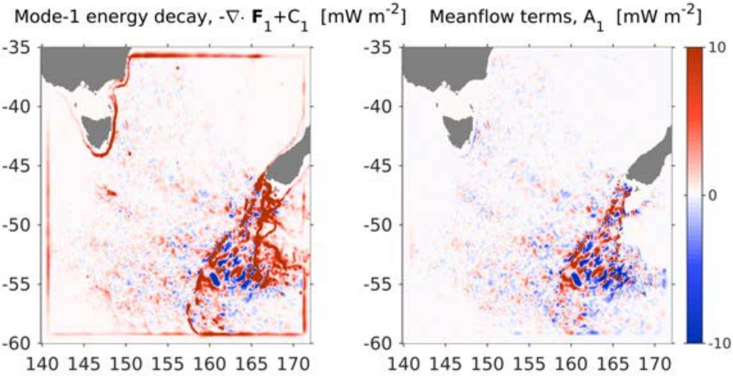
*The model is described in Kelly et al. (2016) and Griffiths and Grimshaw (2007)

Simulation of Tasman Sea for 2015



- TPXO *M*2 surface tides
- Smith and Sandwell bathymetry
- HYCOM meanflow
- Horizontal viscosity $\nu_T = 27.5$ m^2 s^{-1} (for stability)

Do meanflow effects dissipate the mode-1 tide?...No



- Lateral sponges dissipate most mode-1 energy (meanflow terms nearly average to zero)

Stationary tide equations

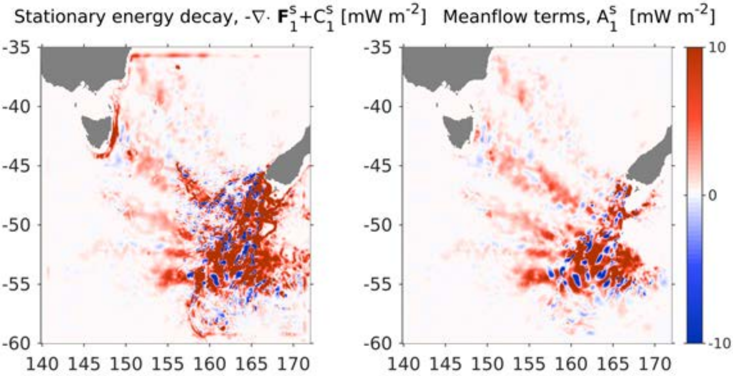
$$\mathbf{U}^s_{nt} + f\mathbf{k}\times\mathbf{U}^s_n = -H\nabla p^s_n - \sum_{m=1}^{\infty} p^s_m \mathbf{T}_{mn} - \left[\sum_{m=0}^{\infty} \nabla\cdot\left(\overline{\mathbf{u}}^T_{mn}\mathbf{U}_m\right)\right]^s$$

$$\frac{Hp^s_{nt}}{c_n^2} = -\nabla\cdot\mathbf{U}^s_n + \sum_{m=0}^{\infty} \mathbf{U}^s_m\cdot\mathbf{T}_{mn} - \left[\sum_{m=0}^{\infty} \nabla\cdot\left(\frac{\overline{\mathbf{u}}_{p,mn}Hp_m}{c_m c_n}\right) + \frac{\delta c_n^2}{c_n^2}\nabla\cdot\mathbf{U}_n\right]^s$$

Notes:

- Stationary variables (e.g., $\mathbf{U}^s_n$ and p^s_n) are harmonic fits or ensemble averages
- Signals are orthogonal with respect to time averaging: $\langle \mathbf{U}^s_n p_n\rangle = \langle \mathbf{U}^s_n p^s_n\rangle$ and $\langle \mathbf{U}^s_n (p_n - p^s_n)\rangle = 0$.
- Meanflow terms (square brackets) are fixed, but unclosed (depend on non-stationary tide).

Do meanflow effects dissipate the stationary mode-1 tide?...Yes



- Meanflow terms explain most stationary tide decay

A_1 is also the generation map for non-stationary tides

Parameterizing meanflow effects on the stationary tide

Greatly simply the model with an eddy viscosity

$$-\left[\sum_{m=0}^{\infty} \nabla \cdot \left(\overline{\mathbf{u}}_{mn}^{T} \mathbf{U}_m\right)\right]^s \approx \nu_T \nabla^2 \mathbf{U}_n^s$$

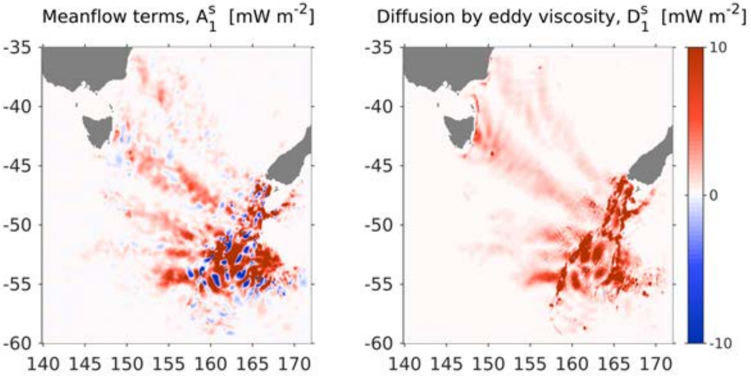
where*

$$\nu_T = \Gamma L \sqrt{2\mathrm{EKE}}$$

- EKE is observed eddy kinetic energy
- Γ is the mixing efficiency (a free constant)
- L is the mixing length (Rossby radius or eddy diameter)

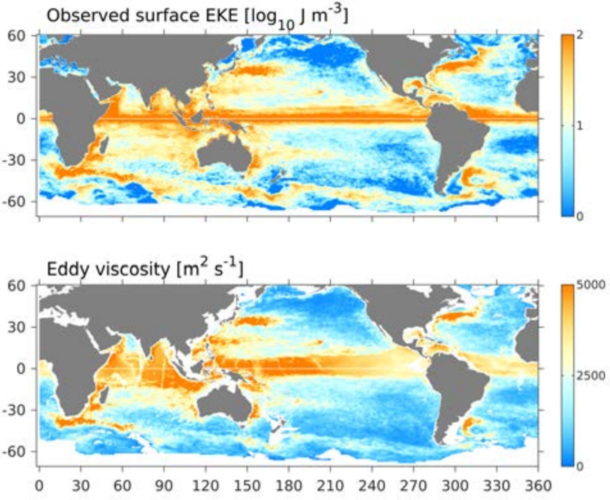
*See Klocker and Abernathy (2014) for details on ν_T.

Parameterizing meanflow effects on the stationary tide



- 47% of A_1 variance explained when averaged to 1°

Global parameterization of meanflow effects



AVISO EKE courtesy of Bo Qiu

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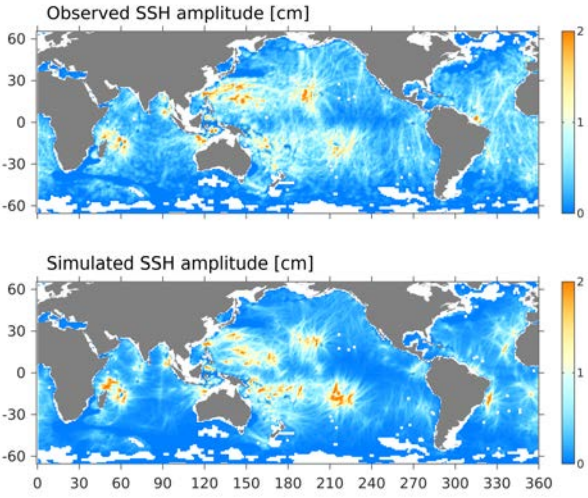

- 68% of $M2$ amplitude variance explained when averaged to 1°

HRET SSH courtesy of Ed Zaron

Summary

Conclusions

1. Meanflow effects explain stationary mode-1 tide decay
2. Meanflow effects can be parameterized by eddy viscosity
3. Stationary mode-1 tide only depends on surface tides, bathymetry, and long-term means of N^2 and EKE

Additional thoughts

1. Determine the stationary and non-stationary tide separately?
2. Still no obvious dominant source of final mode-1 dissipation

Data sources: Jim Richman & Jay Shriver (HYCOM), volkov.oce.orst.edu (TPXO), topex.ucsd.edu (Smith & Sandwell bathy.), Ed Zaron (HRET $M2$ tides), Bo Qiu (AVISO EKE)