The Wavenumber Spectra and Standard Deviations of Uncorrelated Errors in SWOT Measurements of SSH for Various Footprint Diameters*

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The Wavenumber Spectra and Standard Deviations of Uncorrelated Errors in SWOT Measurements of Sea-Surface Height for Various Footprint Sizes

(2019) The science requirement specification of the errors in SWOT measurements of sea-surface height is expressed in the form of an along-track wavenumber spectrum after smoothing the data with a half-power filter cutoff wavelength of 15 km. This expression of the measurement errors is inconvenient for users who want to analyze SWOT data without applying 15-km smoothing. Higher-resolution SWOT data are primarily affected by the contribution of uncorrelated instrumental noise to the total measurement errors. The purpose of this note is to derive from the science requirement specification the corresponding wavenumber spectra and standard deviations of the uncorrelated measurement errors for footprint sizes of 0.5 km, 1.0 km and 2.0 km, without 15-km smoothing in the across-track dimension. The analysis assumes that the spectra of the uncorrelated errors are white at wavelengths shorter than 15 km for each of these footprint sizes.

Chelton_2019_SWOT_Measurement_Noise_190523.pdf (1.1 MB) ▶

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The Science Requirement for SWOT Measurement Errors

The along-track wavenumber spectrum of sea surface height errors after smoothing with a half-power filter cutoff wavelength of 15 km shall not exceed

$$S_{\text{error}}(l) = 2 + 0.00125\, l^{-2}\ \text{cm}^2/\text{cpkm}, \quad \text{for } 15\ \text{km} \leq \lambda \leq 1000\ \text{km},$$

where $l = \lambda^{-1}$ is the along-track wavenumber.

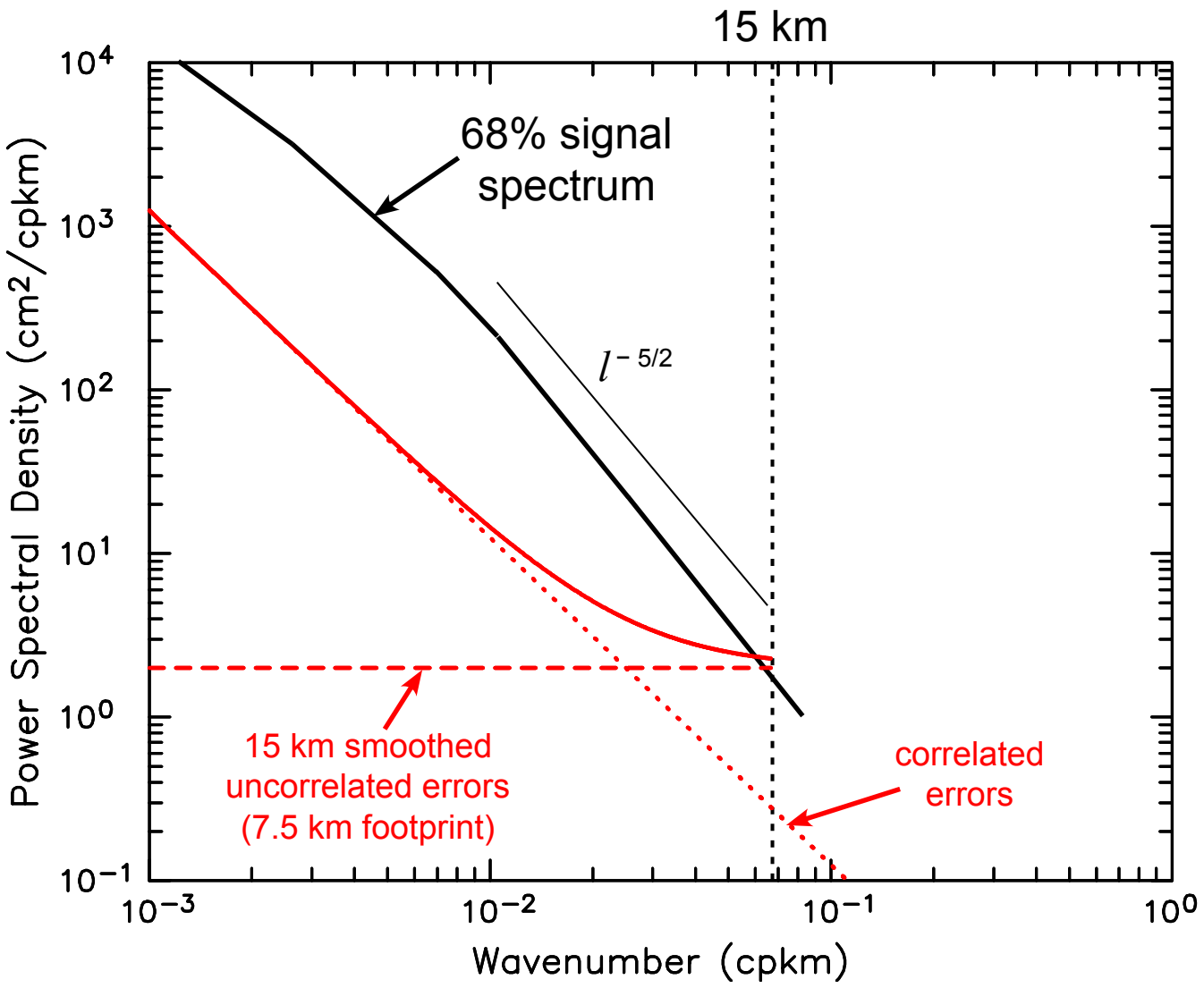
This consists of a constant "white noise" contribution

$$\overline{S}_{\text{white}}(15\ \text{km}) = 2\ \text{cm}^2/\text{cpkm}$$

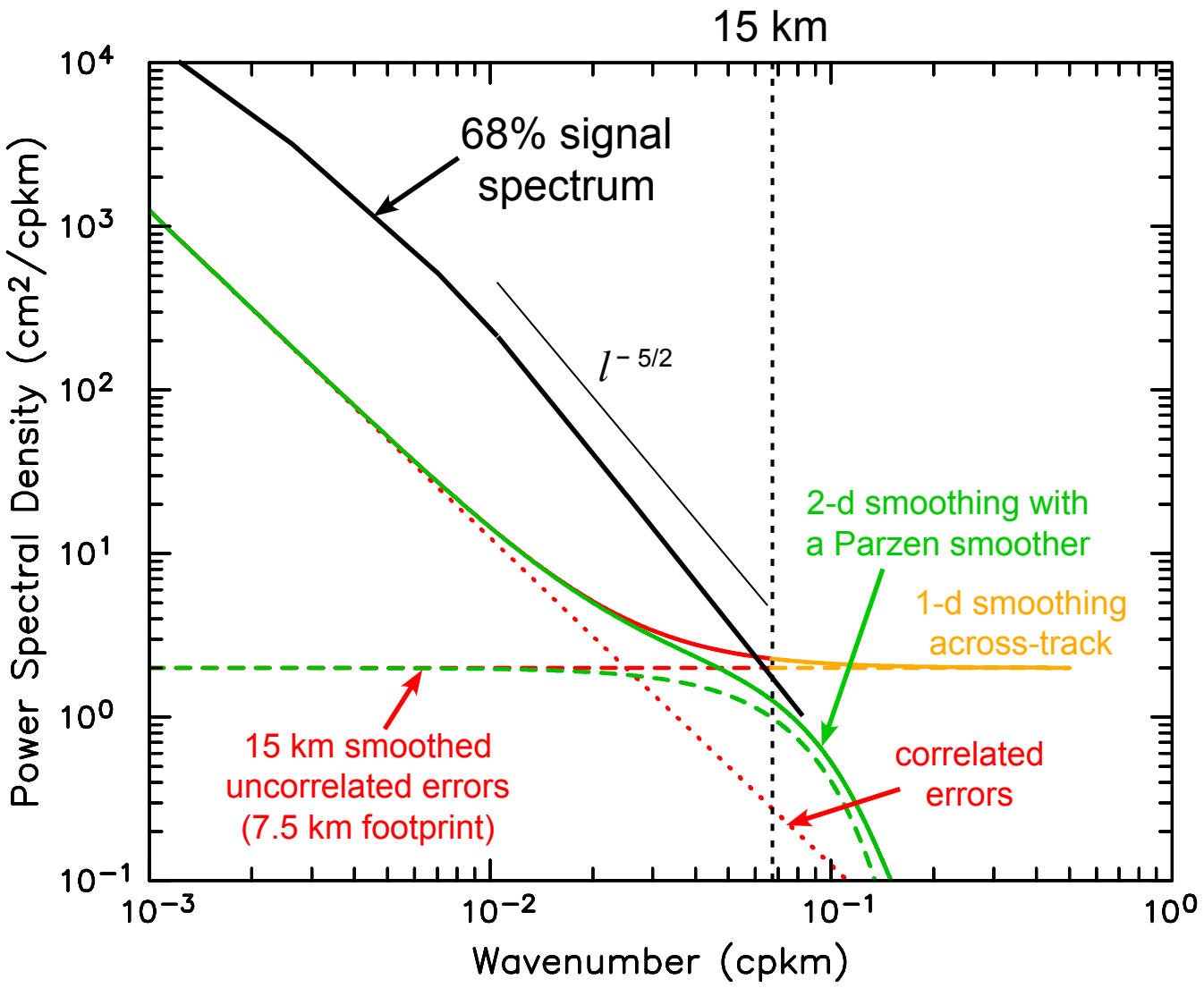
that dominates at short wavelengths, and a "red noise" contribution from correlated errors that dominate at long wavelengths.

The smoothing with a half-power filter cutoff wavelength of 15 km is variously described in the SWOT documentation as 2-dimensional or only 1-dimensional in the across-track dimension.

The Science Requirement for SWOT Measurement Errors



The Science Requirement for SWOT Measurement Errors



Onboard Processing of SWOT Data

In its low-resolution mode over the ocean, the raw radar measurements of SSH by the KaRIn instrument on SWOT will have a footprint size of about 100 m.

To reduce the measurement noise, these raw data will be smoothed in onboard processing to achieve a footprint diameter of Δx.

- The original plan was to smooth the raw data with a footprint diameter of $\Delta x = 1$ km onto a 1 km $\times$ 1 km grid on which the errors are uncorrelated.
- The new baseline plan is to smooth the raw data with a footprint diameter of $\Delta x = 0.5$ km onto an oversampled 0.25 km $\times$ 0.25 km grid.
- The SWOT Project will also smooth the onboard-processed data in ground-based post-processing to achieve a footprint diameter of $\Delta x = 2$ km on a 2 km $\times$ 2 km grid.

The spectral level $S_{\Delta x}$ and standard deviation $\sigma_{\Delta x}$ of the uncorrelated measurement errors for the footprint diameters $\Delta x = \{0.5,\ 1.0,\ 2.0\}$ km can be derived from the science requirement $\overline{S}_{\text{white}}(15\ \text{km}) = 2\ \text{cm}^2/\text{cpkm}$ for 15-km smoothed noise.

What does the SRD white-noise spectrum imply for the standard deviation of uncorrelated errors of the L2 gridded products (0.5 km and 2 km grids)?

1. Integrate the SRD white-noise spectrum to obtain the equivalent variance of uncorrelated errors on the 7.5-km SRD grid:

   $\sigma^2(7.5) = S(7.5) \times l_N(7.5) = 2\ \text{cm}^2/\text{cpkm} \times (1/15)\ \text{cpkm} = 2/15\ \text{cm}^2$.

2. Imagine that the 7.5-km SRD data is obtained by block-averaging the 0.5-km L2 data. Then each 7.5-km SRD point is an average of $(7.5/0.5)^2 = 15^2$ 0.5-km points.

3. The SRD noise variance must then be related to the 0.5-km L2 white-noise variance by

   $\sigma^2(7.5) = \sigma^2(0.5)/15^2$.

4. Then it follows that the 0.5-km L2 white-noise standard deviation is

   $\sigma(0.5) = 15\ \sigma(7.5) = (30)^{1/2}\ \text{cm} = 5.48\ \text{cm}$.

5. Also, the equivalent 0.5-km L2 white-noise spectral level is then

   $S(0.5) = \sigma^2(0.5) / (2 \times 0.5\ \text{km}) = 30\ \text{cm}^2/\text{cpkm}$.

The Spectrum and Standard Deviation of Uncorrelated Measurement Errors for a Footprint Diameter of Δx

For a footprint diameter of Δx, and hence a grid spacing of Δx between uncorrelated measurement errors, across-track smoothing with a half-power filter cutoff wavelength of λ_c attenuates the (assumed) white along-track wavenumber spectrum $S_{\Delta x}$ at all wavenumbers according to

$$\overline{S}_{\text{white}}(\lambda_c) = \frac{2\Delta x}{\lambda_c} S_{\Delta x}. \quad \text{For } \lambda_c = 15 \text{ km, this is } \overline{S}_{\text{white}}(15 \text{ km}) = \left(\frac{\Delta x}{7.5 \text{ km}}\right) S_{\Delta x}.$$

This relation can be inverted to obtain the (assumed) white spectrum of uncorrelated measurement errors without 15-km smoothing from the science requirement $\overline{S}_{\text{white}}(15 \text{ km})$ for 15-km smoothed noise:

$$S_{\Delta x} = \left(\frac{7.5 \text{ km}}{\Delta x}\right) \underbrace{\overline{S}_{\text{white}}(15 \text{ km})}_{2 \text{ cm}^2/\text{cpkm}} = \left(\frac{15 \text{ km}}{\Delta x}\right) \text{cm}^2/\text{cpkm}.$$

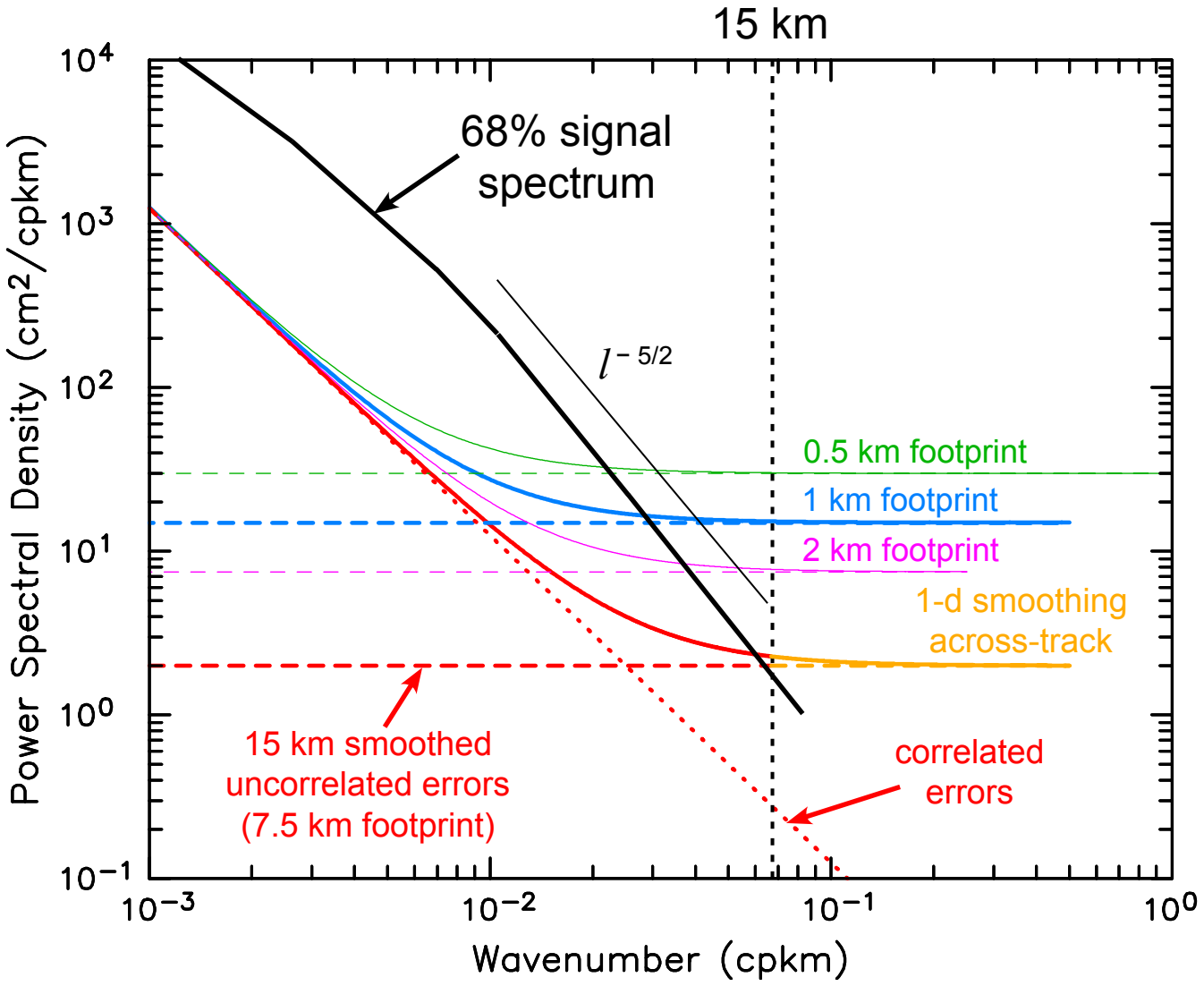
From Parseval's Theorem, the variance of the uncorrelated measurement errors for a footprint diameter and grid spacing of Δx with associated Nyquist wavenumber $l_{\mathcal{N}} = (2\Delta x)^{-1}$ is

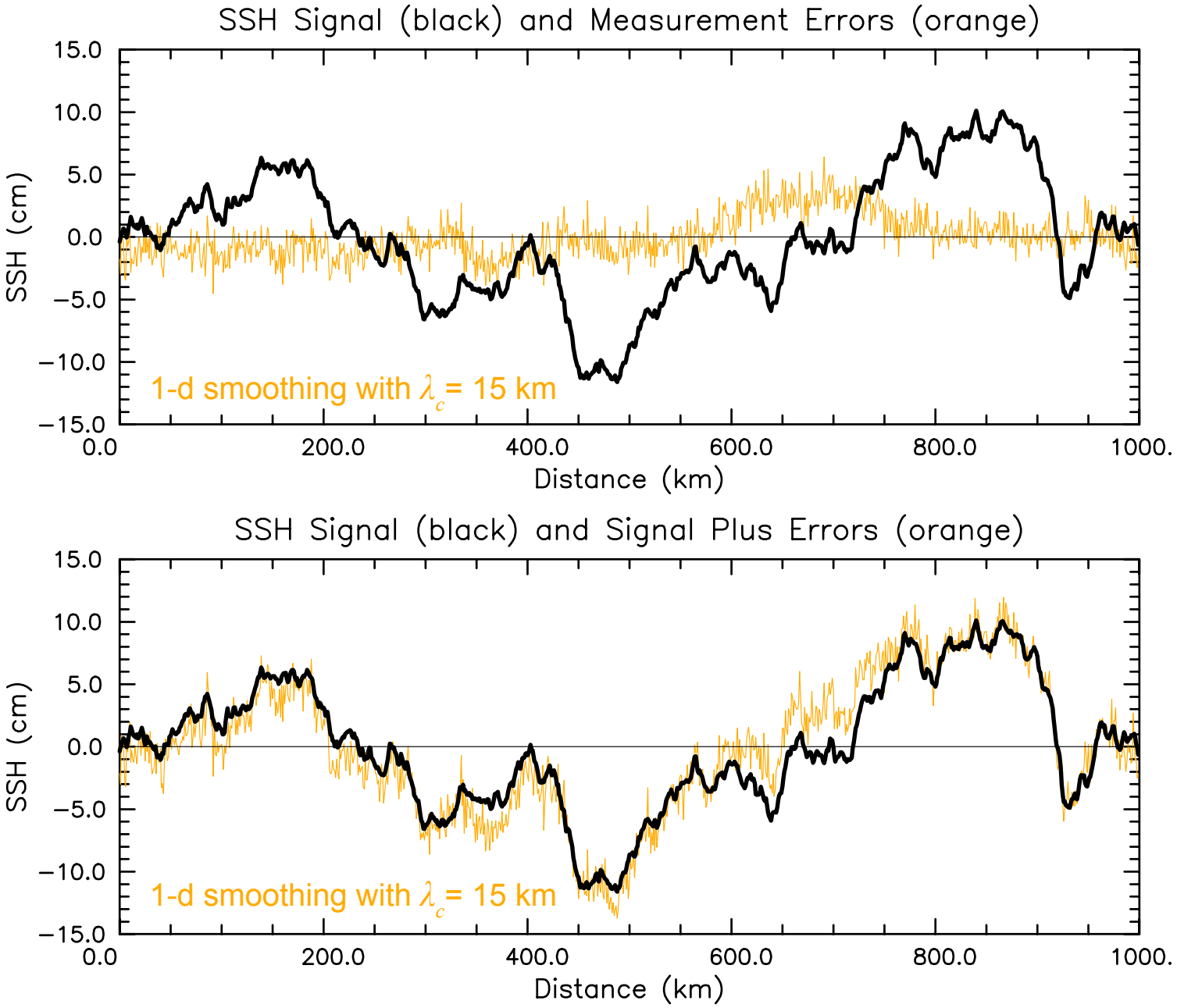
$$\sigma^2_{\Delta x} = \int_0^{l_{\mathcal{N}}} S_{\Delta x}\, dl = l_{\mathcal{N}}\, S_{\Delta x} = \left(\frac{7.5 \text{ km}^2}{\Delta x^2}\right) \text{cm}^2.$$

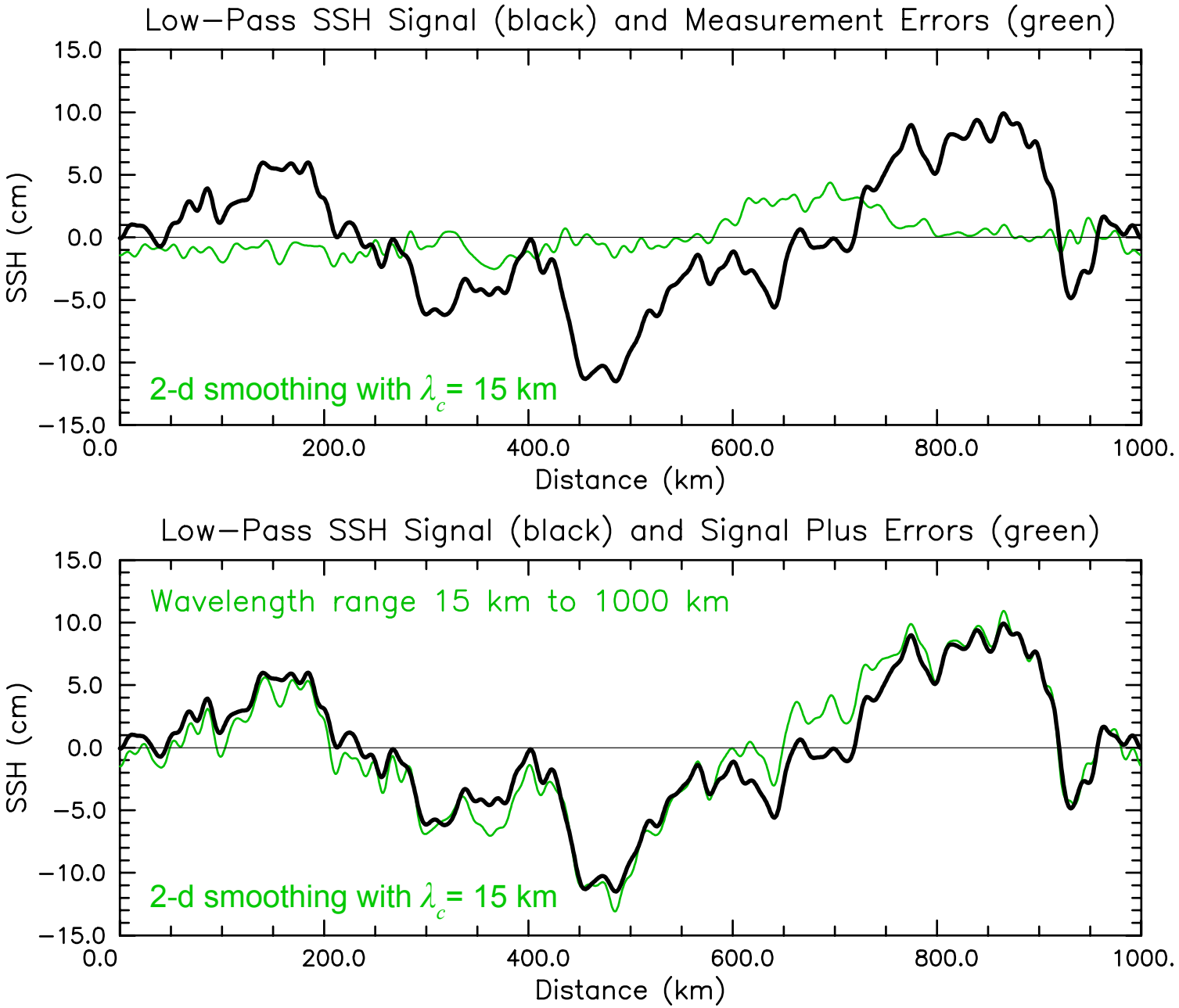
SWOT Measurement Error Characteristics for Footprint Diameters of 0.5 km, 1.0 km and 2.0 km

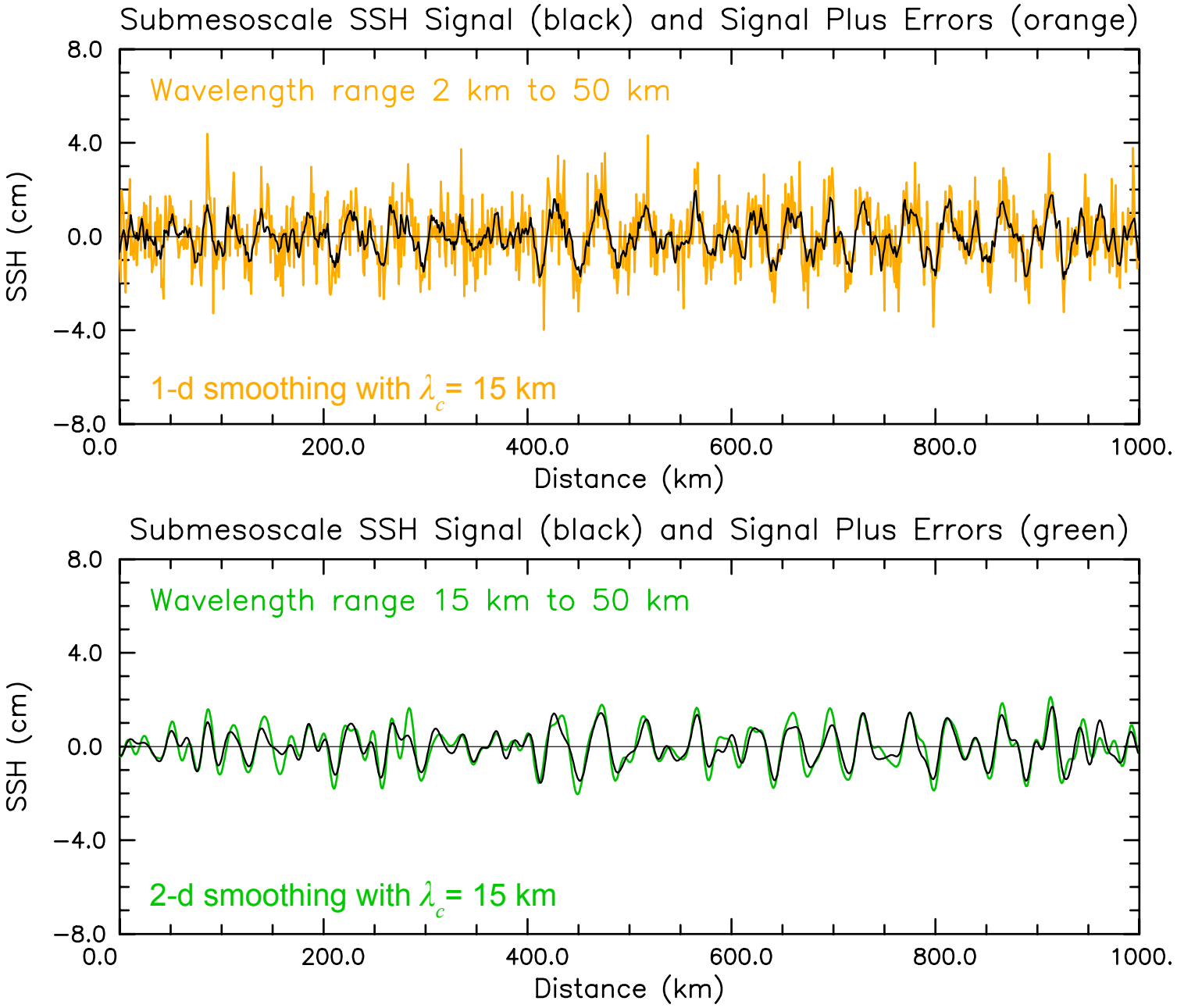
footprint diameter	Δx	$S_{\Delta x}$	$\sigma^2_{\Delta x}$	$\sigma_{\Delta x}$
0.5 km	0.5 km	30 cm^2/cpkm	30 cm^2	5.48 cm
1.0 km	1.0 km	15 cm^2/cpkm	7.5 cm^2	2.74 cm
2.0 km	2.0 km	7.5 cm^2/cpkm	1.875 cm^2	1.37 cm

Spectra of SWOT Measurement Errors for Footprint Diameters of 0.5 km, 1.0 km and 2.0 km

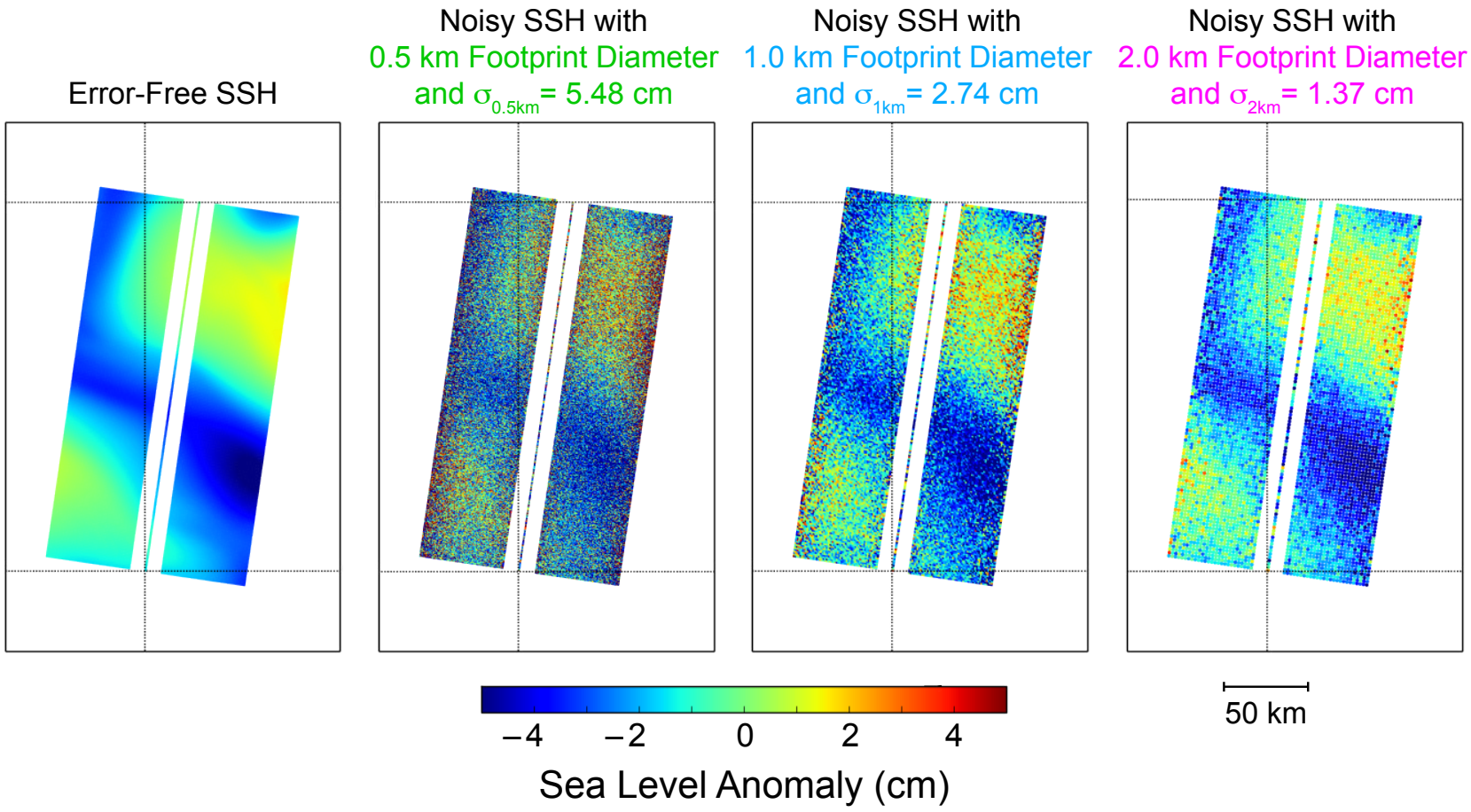



SSH Signal (black) and Measurement Errors (orange)
15.0
10.0
5.0
0.0
−5.0
−10.0
−15.0
SSH (cm)
1-d smoothing with λ_c = 15 km
0.0
200.0
400.0
600.0
800.0
1000.
Distance (km)
SSH Signal (black) and Signal Plus Errors (orange)
15.0
10.0
5.0
0.0
−5.0
−10.0
−15.0
SSH (cm)
1-d smoothing with λ_c = 15 km
0.0
200.0
400.0
600.0
800.0
1000.
Distance (km)


Low−Pass SSH Signal (black) and Measurement Errors (green)
15.0
10.0
5.0
0.0
−5.0
−10.0
−15.0
SSH (cm)
2-d smoothing with λ_c = 15 km
0.0
200.0
400.0
600.0
800.0
1000.
Distance (km)
Low−Pass SSH Signal (black) and Signal Plus Errors (green)
Wavelength range 15 km to 1000 km
2-d smoothing with λ_c = 15 km

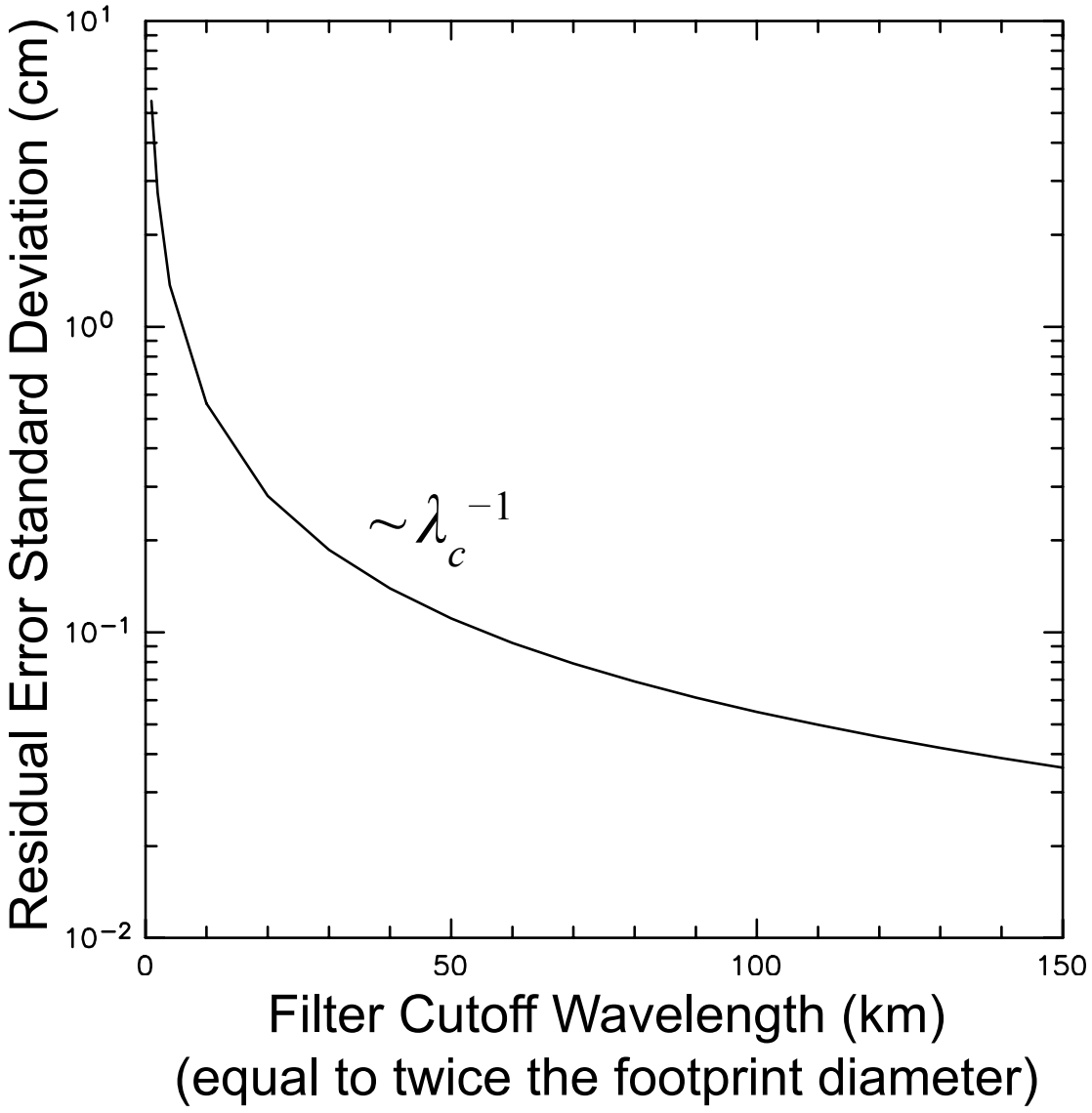

Submesoscale SSH Signal (black) and Signal Plus Errors (orange)
Wavelength range 2 km to 50 km
1-d smoothing with λ_c = 15 km
SSH (cm)
Distance (km)
8.0
4.0
0.0
−4.0
−8.0
0.0
200.0
400.0
600.0
800.0
1000.
Submesoscale SSH Signal (black) and Signal Plus Errors (green)
Wavelength range 15 km to 50 km
2-d smoothing with λ_c = 15 km
SSH (cm)
Distance (km)

Examples of Simulated SWOT Measurements of SSH with Uncorrelated Measurement Errors

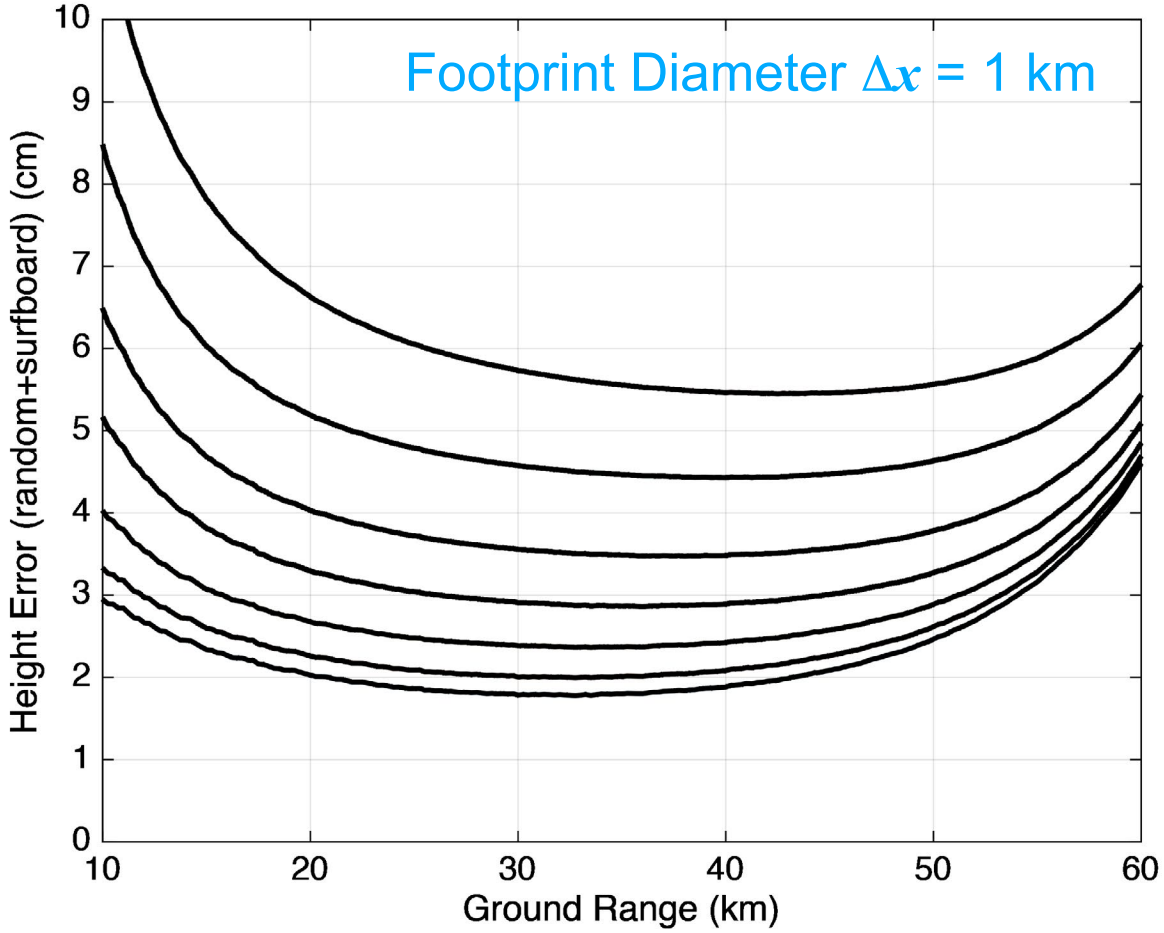


(figure from Nathalie Steunou)

Residual Standard Deviation of 2-Dimensionally Smoothed Uncorrelated SWOT Measurement Errors



Dependencies of the Standard Deviation of Uncorrelated SWOT Measurement Errors on Swath Location and SWH



Residual Standard Deviations of 2-Dimensionally Smoothed Errors of SWOT Estimates of Velocity Components and Vorticity